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ABSTRACT

A microcell is a cell with 1-km or less radius which is suitable for heavily urbanized area such as a metropolitan city. This paper deals with the microcell prediction model of propagation loss which uses probabilistic techniques. The RSL (Receive Signal Level) is the factor which can evaluate the performance of a microcell and the LOS (Line-Of-Sight) component and the blockage loss directly effect on the RSL. We are combining the probabilistic method to get these performance factors. The mathematical methods include the CLT (Central Limit Theorem) and the SPC (Statistical Process Control) to get the parameters of the distribution. This probabilistic solution gives us better measuring of performance factors. In addition, it gives the probabilistic optimization of strategies such as the number of cells, cell location, capacity of cells, range of cells and so on. Specially, the probabilistic optimization techniques by itself can be applied to real-world problems such as computer-networking, human resources and manufacturing process.

Key words: Microcell, propagation loss, optimization, central limit theorem, statistical control.

1. INTRODUCTION

In present article, we study a class of propagation loss with prediction model of microcells. A microcell is a cell with 1-km or less radius which is suitable for heavily urbanized area such as a metropolitan city. This classical prediction model with deterministic method has been studied by Lee [5]. This paper uses the analytic and probabilistic approach to solve the microcell prediction model. The RSL (receive signal level) is a very important factor to evaluate the performance of the microcell antenna. The RSL consist of two parts, which are the LOS (Line-Of-Sight) component and the blockage loss. Unlike [5], we use RSL as an evaluation factor and it is not point-to-point parameter. The average blockage loss $\widehat{\alpha}_B$ considers all destinations for a microcell antenna and this modified RSL can represent the performance of an antenna

The CLT and the SPC are main techniques to find the explicit formulas. From the CLT [6], the sum of random variables approaches the normal distribution. To get the unknown parameters, SPC and the law of large numbers are used [] and this paper represents all these mathematical techniques. The probabilistic optimization technique of this paper is a part of the stochastic optimization which has been studied [1,3] and it can be applied any kinds of real-world applications such as computer-networking, human resources and manufacturing process and secure backup system [3].

Explicit formulas obtained demonstrate a relatively effortless use of functionals of the main characteristics and optimization of their objective function.

Section 2 presents preliminary mathematical techniques (CLT and SPC). These techniques are used to get the explicit formulas of the performance factor. The terminology of microcell prediction model is shown at section 3 and section 4 show the general idea to get the reasonable parameters. The probabilistic optimization is dealt by section 5.

2. PRELIMINARIES: Mathematical Techniques

In this section, there are two main mathematical techniques to solve the probabilistic microcell prediction model. The CLT (Central Limit Theorem) gives us the explicit distribution the total blockage length which consist of a random length of blocks. Even though the distribution of each block length is not known, but identically and independently

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distributed (iid). We can find the parameters, such as mean μ and variance σ, of each block using the SPC (Statistical Process Control). We also assume that the number of block is practically many and the total length of blockage is the sum of length blocks. Under these assumption, The distribution of the total blockage length yields the normal random distribution (or normal distribution) with the parameters $(n\mu, \sigma\sqrt{n})$ because of the CLT. The SPC analyze data to find the unknown mean and variance. Next two subsections show the more detailed explanation of these mathematical techniques

2.1 Central Limit Theorem (CLT)

The Central Limit Theorem (CLT) is one of the most remarkable result in probability theory []. Briefly, it says that the sum of a large number of independent random variables approaches normal (normal) random variable. This theorem is based on the law of large number [6].

Let given a sequence $X_1, X_2, \ldots$ of iid random variables with (μ, σ). Then the distribution of

$$\xi_n := \frac{X_1+X_2+\cdots+X_n-n\mu}{\sigma\sqrt{n}} \tag{2.1}$$

approaches to the standard normal as $n \to \infty$, then $|x| < \infty$,

$$P\{\xi_n \leq x\} \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{x^2}{2}}dx, \tag{2.2}$$

In other words, the CLT is as follows:

$$\lim_{n\to\infty} P\{\xi_n \leq x\} = \Phi(x) \tag{2.3}$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{x^2}{2}}dx. \tag{2.4}$$

The strong law of large number (i.e., $\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n} \cdot \sum_{i=1}^{n} X_i\right] = \mu$) gives us the parameter μ which is the mean value of X_i and be used for the further section.

2.2 Statistical Process Control (SPC)

The statistical process control (SPC) is the classical method for analyze the quality and the SPC charts is one of the fundamental quality control. If the statistical data are known by an observation or an experiment, then we can find the important measure such as a mean, a variance and so on. The sampling variance is the one kind of SPC (statistical process control). Let assume that the statistical data are exist, such as $X_1, X_2, \ldots, X_n,$, then the sample variance is

$$\sigma^2 = \frac{1}{n-1}\sum_{k=1}^{n}(X_k - \mu)^2. \tag{2.5}$$

where n is the number of the sample data. This variance σ is used by the parameters of the distribution, which we will use it for the further section.

3. MICROCELL PREDICTION MODEL

A microcell is small, less than 1 km, and suitable for heavily urbanized area. Since the blocks (most of them are buildings) make noticeable deference in reception at distance less than 1 km between two point, we use a microcell prediction model [2]. From Lee [5], the microcell prediction model breaks the process down into a RSL (receive signal level, P_r) for the LOS (line-of-sight, P_{LOS}) and then the attenuation due to the building blockage component ($\alpha(B)$). The performance measures will be these factors (i.e., $P_r, P_{LOS}, \widehat{\alpha}_B$). The RSL P_r is defined as follows:

$$P_r = P_{LOS} - \widehat{\alpha}_B. \tag{3.1}$$

where $\widehat{\alpha}_B$ is the average total loss of blockage by buildings.

Let $X_1, X_2, \ldots, X_n$ are the blockage of one building and each of them has the random length and identically and independently distributed (iid) with the mean μ and the variance σ. The section 4 will give you reasonable parameters (μ, σ) of $X_i, i = 1, \cdots, n$ and B [ft] is the total length of the blockage due to buildings and it yields:

$$B = X_1 + X_2 + \cdots + X_n. \tag{3.2}$$

Since $X_i, i = 1, \cdots, n$ are iid, we have

$$\mathbb{E}[B] = n \cdot \mathbb{E}[X_k]. \tag{3.3}$$

Because of CLT (2.2), the probability distribution function of B is

$$F(x) := P\{B \leq x\} \simeq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} exp\big(-(x-\mu)^2 n/\sigma^2\big) dx \tag{3.4}$$

then $\alpha(B)$ [dB] is as follows:

$$\begin{aligned} \alpha(B) = &\big(1 + 0.5log(B/10)\big)\mathbf{1}_{\{1 \leq B < 25\}} + \big(1.2 + 12.5log(B/25)\big)\mathbf{1}_{\{25 \leq B < 600\}} \\ &+ \big(17.95 + 3log(B/600)\big)\mathbf{1}_{\{600 \leq B < 3000\}} + 20 \cdot \mathbf{1}_{\{3000 \leq B\}} \end{aligned} \tag{3.5}$$

and the average loss of blockage buildings:

$$\widehat{\alpha}_B = \mathbb{E}\big[\alpha(B)\big].$$

From (3.4), it yields

$$\widehat{\alpha}_B = \int_{\mathbb{R}} \alpha(x) dF(x).$$

The ERP (Effective Radiatio Power) is power transmitted from an antenna. Good design of a transmitting antenna at the cell site can ensure good reception [2]. Let P_t is the ERP in [dB] and then P_{LOS} is as follows:

$$P_{LOS} = \begin{cases} P_t - 77 - 21.5log\frac{d}{100} + 30log\frac{h}{20}, & 100 \leq d < 200, \\ P_t - 83.5 - 14log\frac{d}{200} + 30log\frac{h}{20}, & 200 \leq d < 1000, \\ P_t - 93.3 - 36.5log\frac{d}{1000} + 30log\frac{h}{20}, & 1000 \leq d < 5000, \end{cases}$$

where d [ft] is the total distance and h [ft] is the antenna height. From (3.1) and (3.5), we have RSL P_r:

$$\begin{aligned} P_r &= P_{LOS} - \widehat{\alpha}_B \\ &= P_{LOS} - \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \alpha(x) \cdot exp\big(-(x-\mu)^2 n/2\sigma^2\big) dx. \end{aligned} \tag{3.6}$$

From (3.5), it yields:

$$P_r = P_{LOS} - \frac{1}{\sqrt{2\pi}}\Big(\int_{x=1}^{25}\alpha(x)\cdot exp\big(-(x-\mu)^2 n/2\sigma^2\big)dx + \int_{x=25}^{600}\alpha(x)\cdot exp\big(-(x-\mu)^2 n/2\sigma^2\big)dx$$

$$+\int_{x=600}^{3000}\alpha(x)\cdot exp\big(-(x-\mu)^2 n/2\sigma^2\big)dx + 20\int_{x=3000}^{\infty}\alpha(x)\cdot exp\big(-(x-\mu)^2 n/\sigma^2\big)dx\Big).$$

This probabilistic microcell prediction model is another approach of the original microcell prediction model. The CLT is used for getting the better performance measures. It may not accurate as the original prediction model, but it can give us more compact solution and reduce the time for collecting data.

4. GENERAL IDEA OF DATA COLLECTION

4.1 Fundamentals

This section shows the general idea of collecting data. The blockage length of each building does not need to be measured . Only shape and scale of buildings on the map or the picture are used to measure the length of blockage. We assume that all buildings are rectangular and count the number of buildings within 1-km radius area and measure the width and length of a block which is covered by a radius of a microcell (see Figure 4.1).

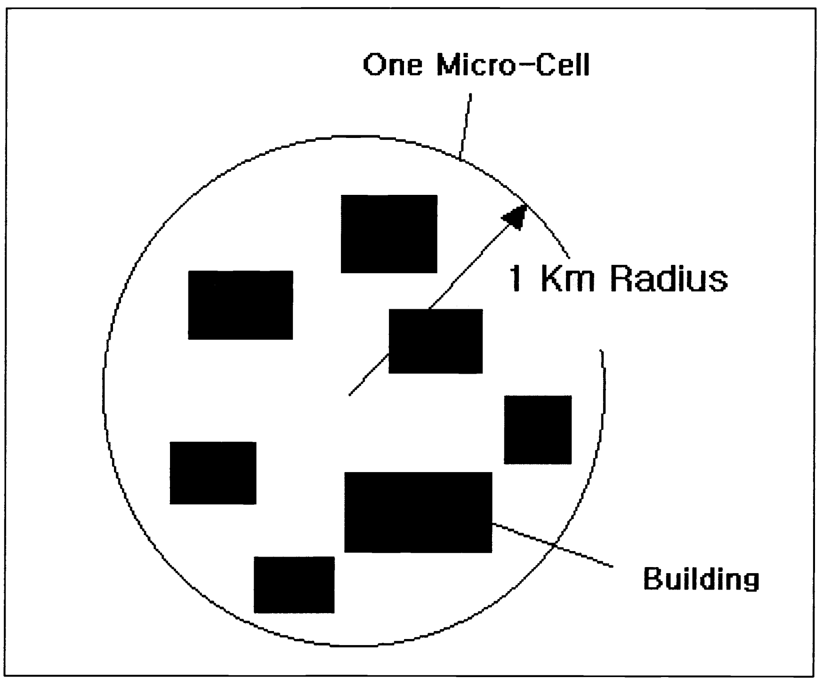

Figure 4.1: Example of One Microcell Model

Let a_k and b_k are the width and the length of the kth building, then the length of blockage X_k is

$$X_k = \frac{\rho}{2\cdot m}\cdot\sqrt{a_k^2 + b_k^2},\ k = 1,\ldots,n \tag{4.1}$$

where ρ is the scale of buildings in the map (if we have the actual numbers of a_k and b_k, then $\rho = 1$) and m $(m \geq 4)$ is the number of sample direction.

Because of CLT (2.3), the distribution of blockage length is the normal distribution and the law of large number gives the mean μ of X_k and SPC shows how to get the variance σ of X_k (2.5).

4.2 Sample Experiment

Let assume that we have a 1-km microcell antenna (Figure 4.2) and buildings are within this area (Figure 4.3). We measure the length and width of all buildings and find the X_ks from (4.1) (See the Table 4.1)

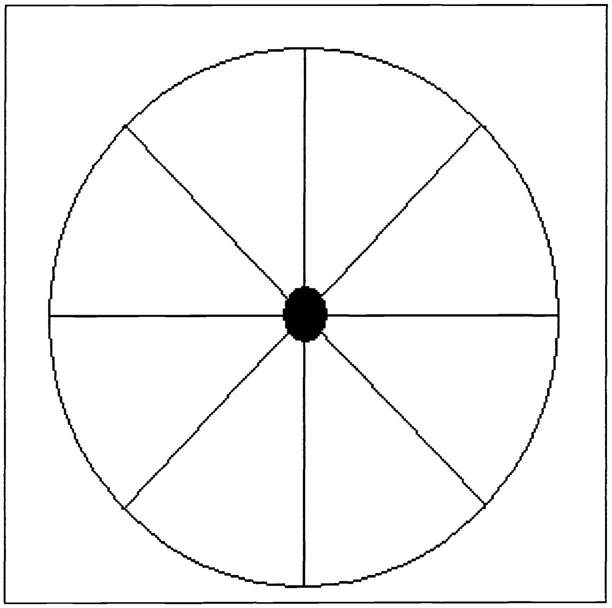

Figure 4.2: Microcell with 8-sample directions

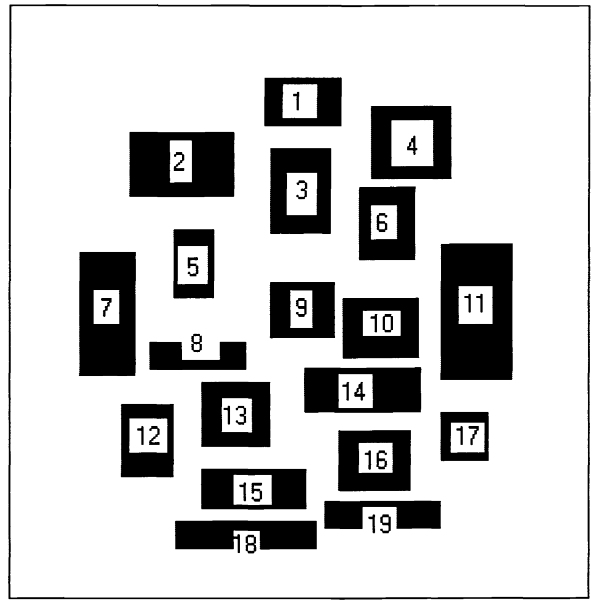

Figure 4.3: Sample buildings within a microcell

Table 4.1 shows the collection of data of microcells and calculations from our sample buildings from Figure 4.3.

Building Number	Length (a_k)	Width (b_k)	Area $(a_k \cdot b_k)$	X_k
1	15	10	150	1.1267
2	21	14	294	1.5774
3	12	17	204	1.3005
4	15	15	225	1.3258
5	8	15	120	1.0625
6	11	15	165	1.1626
7	11	26	286	1.7644
8	20	6	120	1.3050
9	11	11	121	0.9723
10	16	12	192	1.2500
11	14	27	378	1.9009
12	11	14	154	1.1128
13	14	12	168	1.5424
14	23	9	207	1.5436
15	21	8	168	1.4045
16	14	12	168	1.1524
17	10	10	100	0.8839
18	28	6	168	1.7897
19	23	6	138	1.4856

Table 4.1: Sample data collection from buildings (figure 4.3)

The length of blockage X_k of buildings can be find from (4.1) and the average of X_k is 1.3302. From (3.3), we have

$$\mathbb{E}[B] = n \cdot \mathbb{E}[X_k] = 25.2733, \tag{4.2}$$

where $n = 19$. From Lee [5] and Freeman [2], the average length of blockage ($\widetilde{B}$) through 8-directions (see figure (4.4)) as follows:

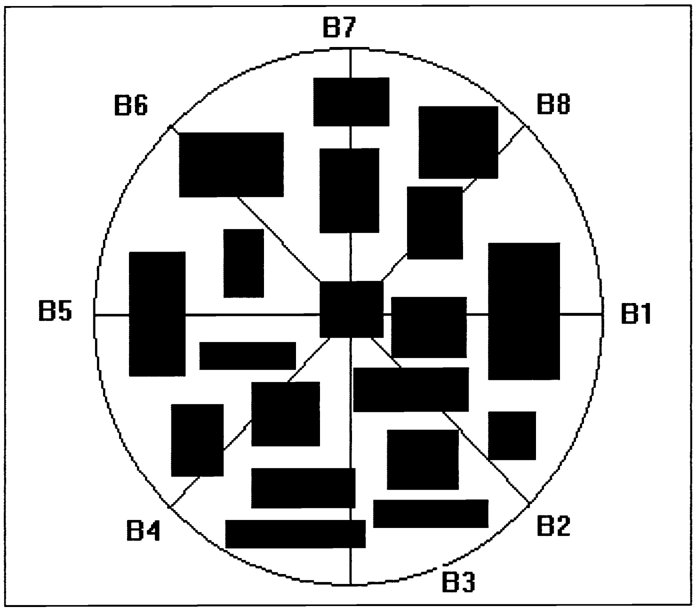

Figure 4.4: 8 directions of the microcell direction

Using the statistical calculation, the average length of blockage $\widetilde{B}$ is

$$\widetilde{B} = \frac{B_1+B_2+\cdots+B_8}{m} = 26, \tag{4.3}$$

where $m = 8$. From (4.2)-(4.3), the error between models is

$$\frac{||\mathbb{E}[B]-\widetilde{B}||}{\widetilde{B}} = 0.028. \tag{4.4}$$

As (4.4) shows, it is only 2.8% difference between the actual measure and probabilistic measure.

5. PROBABILISTIC OPTIMIZATION

5.1 Fundamentals

In this section we deal with a class of optimization problems that arise in probabilistic microcell model. We use the commonly used stochastic optimization method (see [3],[4] and [6]). Let a strategy Σ is a set of action we impose on the system. A system can be subject to a set, say C, of cost functions. Denote by $\phi(\Sigma, C, t)$ the expected costs within $[0, t]$, due to the strategy Σ costs C and define the expected cumulative cost rate over an infinite horizon:

$$\phi(\Sigma, C) := \lim_{t\to\infty} \frac{1}{t}\phi(\Sigma, C, t). \tag{5.1}$$

5.2 Optimality of the sample model

We formulate our problem for a special case. Let $g(p)$ be the cost rate function for p [dB] signal level. Then the expected rate for the RSL is $\mathbb{E}[g(P_r)]$ and the objective function for the RSL with a strategy of the length of an antenna h [ft] in this example. Formally,

$$\phi(\Sigma(h_0), C) = min\{\phi(\Sigma(h), C)\}. \tag{5.2}$$

Recall from section 3, $P_r(B)$ gives RSL with random variables and the cost rate is

$$\phi(\Sigma(h_0), C) = min\{\mathbb{E}[g(P_r)] : 0 < h < \infty\}, \tag{5.3}$$

where

$$\mathbb{E}[g(P_r)] = \int_{\mathbb{R}} g(P_{LOS} - \alpha(x))dF(x), \tag{5.4}$$

and we can add the constraint of an antenna height (such as a regulation of city and so on):

$$h \le \beta \tag{5.5}$$

where β is the constraint height of a microcell antenna.

In addition, a feasible radius of a microcell is also an important strategy. In this case, the parameter of the objective function (strategy) is different even though the objective function is same as previous. Formally, it is

$$\phi(\Sigma(d_0), C) = min\{\phi(\Sigma(d), C)\}. \tag{5.6}$$

and if same cost rate function $g(p)$ is used, then

$$\phi(\Sigma(d_0), C) = min\{\mathbb{E}[g(P_r)] : 0 < d < \text{1-km}\}. \tag{5.7}$$

The reasonable constraint can be ERP P_t:

$$P_t \le \gamma,$$

where γ is confidential capacity of ERP. Since this paper deals with idle model, we can check this model with the experimental model for the next research.

6. CONCLUSION

In this paper, we show how to combine the mathematical techniques for solving the microcell prediction model. It makes more easy to measure the physical length of blockage. So we can reduce time spending to get the physical data. Since this paper deals with the idle model, we just have the artificial data for the small experiment Comparison between the probabilistic prediction model and actual model using actual data is the further project of this research.

The probabilistic optimization (one parts of the stochastic optimization) technique is used. This technique shows an example how to use the performance factors with the optimization methods. In the example of section 5, the reasonable constraint is also included.

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