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ABSTRACT

We used morphological filters to approximate wavelet scaling functions for multiresolution processing of an image. Because some spatial light modulators (SLMs) can only display binary data, wavelet processing of binary images is inhibited. Therefore, we considered an alternative way - morphological processing - to generate a wavelet representation that consists entirely of binary elements. The effects of these filters are dependent on the input signal and cannot be generalized. Therefore, we used a statistical approach to approximate the scaling functions or various wavelets using morphological filters.

Keywords: multiresolution processing, morphology, optical correlator, wavelet transform,

1.0 INTRODUCTION

Using a multiresolution representation, a hierarchical framework for processing an image can be achieved.[1] In terms of template matching, knowledge about an object's shape or location is made more precise on successively higher resolution versions of an image. Such an approach has been widely studied and has been shown to be useful for pattern recognition. [2-3]

Each level of a multiresolution representation contains a different size image. Because the resolution of an optical correlator is usually fixed, we can process only a constant size image. Therefore, we can often only process a portion of an image as its resolution is increased. Using this technique an optical correlator has been used to search a pyramid representation of an image.[4] In this method, input images were preprocessed with morphological filters to approximate low-pass filters to generate a binary image pyramid. As the resolution of the image decreased, objects appeared smaller which significantly decreased the signal-to-noise ratio (SNR). In addition, the data in these pyramid structures were correlated. As pointed out in Ref. 1 it is not clear whether a similarity between the image details at different resolutions is due to a property of the image itself or to the redundancy of the representation.

We approximated wavelet filters with morphological filters to operate on binary imagery. The multiresolution description of an image is described in the next section followed by a discussion of our approach.

2.0 MULTIRESOLUTION PROCESSING

2.1 Wavelet representation

In the orthogonal wavelet representation data at different resolutions are independent. The wavelet representation allows a multiresolution representation to be constructed based on the difference of information available at two successive resolutions.[1] We generally discussed the wavelet representation in terms of one-dimension, but all of our equations can be easily generalized to two dimensions.

Using the wavelet representation the approximation of a signal $s(x)$ at the resolution 2^j is referred to as $A_{2^j}\{s(x)\}$ where A_{2^j} is a projection operator that approximates the function $s(x)$ and $j \leq 0$. The signal $A_1\{s(x)\} = s(x)$, the original signal at the highest resolution, and $A_{1/2}\{s(x)\}$, $A_{1/4}\{s(x)\}$, etc. are lower resolution versions of $s(x)$. The operator A_{2^j} is an orthogonal projection on a vector space, and $A_{2^j}\{s(x)\}$ is not modified if we approximate it again at resolution 2^j. Furthermore, the approximation of a signal at a resolution 2^j contains all the necessary information to compute the signal at resolution 2^{j-1}. Finally, the approximation operation is similar at all resolutions.

The projection of $s(x)$ onto a vector space can be achieved with a convolution operation. The approximation of a signal $s(x)$ at resolution 2^j can be viewed as a convolution between the signal $s(x)$ and a scaling function $\phi(x)$ followed by a uniform sampling at the rate of 2^j. The Fourier Transform of the scaling function $\Phi(\omega)$, is related to the Fourier Transform of the wavelet $\Psi(\omega)$ by [1]

$$\Psi(\omega) = \Phi(\omega - \pi). \tag{1}$$

The approximation of a signal $s(x)$ at resolution 2^j was written as

$$A_{2^j}\{s(x)\} = \int s(x')\,\phi_{2^j}(x' - 2^{-j}x)\,dx', \tag{2}$$

where ϕ_{2^j} represents the scaling function at the resolution 2^j. In two dimensions Eq. (2) can be written as

$$A_{2^j}\{s(x,y)\} = \iint s(x',y')\,\phi_{2^j}(x' - 2^{-j}x)\,\phi_{2^j}(y' - 2^{-j}y)\,dx'dy', \tag{3}$$

if a separable scaling function is used. The multiresolution approximation is completely characterized by the scaling function. It is possible to chose scaling functions with good localization properties in both the frequency and input domains.

Each resolution of our representation contained a different size image. Because we processed only a constant size image with an optical correlator we used only a portion of an image when processing it as the resolution increased. In our analysis we assumed that at all resolutions complete objects are considered.

To implement cross-correlation using the wavelet representation, an input and reference signal corresponding to the same resolution are used. The input signal is cross-correlated with a similarly processed reference. If a correlation peak is produced at a resolution of 2^j that exceeds a specified threshold, then $A_{2^{j+1}}\{s(x)\}$ is used in the region near the correlation peak.

2.2 Examples of scaling functions

Many functions can serve as wavelets; however, for multiresolution processing wavelets with a multimodal frequency response or one that contains sharp peaks are usually not used. Generally, wavelets that act as smooth bandpass filters are usually used so they represents a signal's energy at a particular scale.

We considered wavelets that have been well-studied and shown to be useful. We chose wavelets with similar frequency responses but differed in the broadness of their frequency response. Specifically, we considered the Haar wavelet and some wavelets discovered by Daubechies.[5] All of these wavelets could be described by filter coefficients used in a moving average filter. We considered Daubechies wavelets that had 4, 6, and 12 filter coefficients and referred to them as Daub4, Daub6, and Daub12.

We compared the scaling functions in the frequency domain of different wavelets in Fig. 1. The figure shows that the scaling function associated with the Haar wavelet has the broadest response while the scaling function associated with the Daub12 wavelet had the narrowest.

3.0 BINARY APPROXIMATION TO SCALING FUNCTIONS

3.1 Morphological processing

Generally, low-pass filtering a binary sequence produces values other than one or zero. Because some SLMs can only display binary data, wavelet processing binary images is inhibited. Therefore, we considered an alternative way - morphological processing - to generate a wavelet representation that consists entirely of binary elements. We used morphological operations that perform smoothing of the contour of an object. The output of morphological operations are binary so that the resulting representation can be displayed on a binary SLM without distortion.

In morphology, a sequence is transformed with patterns of predefined shapes called structuring elements. A structuring element is essentially the region of support in the vicinity of a pixel where morphological convolution may take place.

Morphological operations are nonlinear and cannot be represented by a transfer function. The effects of these filters are dependent on the input signal and cannot be generalized. Therefore, we used a statistical approach to approximate the scaling functions or various wavelets using morphological filters.

3.2 Approximation to scaling functions

To approximate scaling functions associated with the wavelets discussed earlier we performed experiments on the 256 x 256 binary image in Fig. 2. We filtered this image with a particular scaling function, then downsampled the result by a factor of 2. We also filtered Fig. 2 with various morphological functions using a 3 x 3 structuring element and correlated the result with, the result obtained when the scaling function was used. By equating energies in the images before correlation, we measured the approximation of the scaling function by the morphological filter by the cross-correlation value. We used opening and closing filters of various iteration and connectivities in our approximation and retained the function that had the largest cross-correlation value. The results are shown in Table 1.In Table 1 n refers to the number iterations of the morphological operation and c refers to the connectivity.

TABLE 1. Approximations to scaling functions

Wavelet scaling function	Best morphological filter for Fig. 2	Correlation coefficient
Haar	closing, n=1, c=4	0.938
Daub4	closing, n=1, c=4	0.966
Daub6	closing, n=1, c=4-8	0.859
Daub12	closing, n=9, c=4-8	0.638

The results showed that for the scaling functions selected, a closing function with the indicated parameters was the best match in terms of mean-squared error. In addition, the approximations were not as good for scaling functions with steep frequency cutoff characteristics.

4.0 CONCLUSION

We used morphological filters to approximate wavelet scaling functions for multiresolution processing of an image. We found that by using a closing operation with the appropriate connectivity and number of iterations, we could approximate wavelet scaling functions for binary processing. The approximations were not as good for scaling functions with steep frequency cutoff characteristics. In addition, the effects of these morphological filters are dependent on the input signal and cannot be generalized.

5.0 REFERENCES

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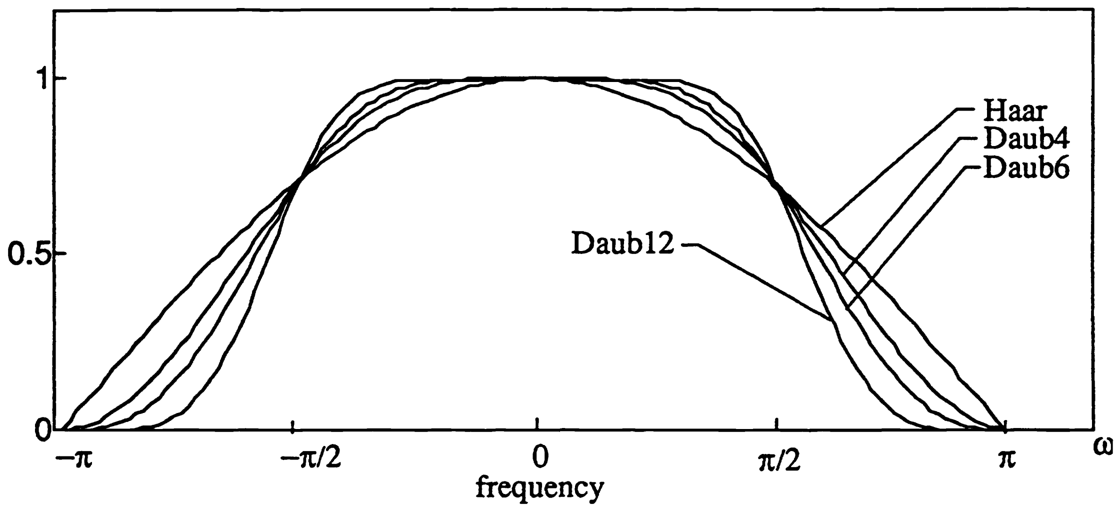

FIGURE 1. Frequency response of scaling functions associated with different wavelets.

FIGURE 2. 256 x 256 binary image used in experiments