7-10-2004

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The Astrophysical Journal, 609:1076–1081, 2004 July 10

INTERPLANETARY TRANSPORT MECHANISMS OF SOLAR ENERGETIC PARTICLES

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Received 2003 December 31; accepted 2004 March 19

ABSTRACT

Numerical simulations of charged particle trajectories in a Parker field with model turbulence are used to study the transport mechanisms of solar energetic particles (SEPs): magnetic focusing, pitch-angle diffusive transport due to magnetic fluctuations, and adiabatic cooling. The results of the simulations are compared with analytical formulae for the transport effects. We find that adiabatic cooling is significant for SEPs traveling from the Sun to 1 AU. The simulation results agree much better with the anisotropic adiabatic cooling effect theory than with the isotropic one. Also, we show that the simulation results agree well with the theory of magnetic focusing with pitch-angle scattering by magnetic fluctuations. In addition, in order for quasilinear theory to be valid, the average particle's Larmor radius must be much smaller than the correlation scale of the two-dimensional component of turbulence.

Subject headings: interplanetary medium — MHD — solar wind — Sun: particle emission

1. INTRODUCTION

The transport of cosmic rays in heliospheric magnetic fields is often described by a diffusion equation with an isotropic distribution first formulated by Parker (1965). The diffusion equation is a good approximation of particle transport if scattering by magnetic turbulence is frequent enough that the particle distribution is nearly isotropic. However, since solar energetic particles (SEPs) experience the effect of magnetic focusing by the nonuniform heliospheric magnetic field, we have to rely on the focused transport equation for anisotropic distributions (Parker 1963; Roelof 1969; Ng & Reames 1994). Since it is impossible to solve the equation analytically, previous authors have instead obtained numerical solutions (e.g., Ng & Wong 1979; Schlüter 1985; Ruffolo 1991). This equation, where adiabatic cooling is completely neglected, is good only for a short time period near the onset of an SEP event. In addition to the magnetic focusing effect, in fact, SEPs also experience adiabatic cooling effects due to differential solar wind convection. A formula for adiabatic cooling of particles with isotropic distributions is widely used (e.g., Parker 1965; Dorman 1965).

Ruffolo (1995) presented analytic formulae for adiabatic cooling and other solar wind effects as a function of pitch angle, which then were inserted into a transport equation. He also solved the transport equation with numerical simulations to examine the interplanetary transport of solar cosmic rays. However, the derivation of the formulae may be problematic, because the averaging of the formulae over phase angle is done before the transformation from the fixed frame to the solar wind frame.

Standard quasilinear scattering theory (QLT; Jokipii 1966) provides a framework for understanding pitch-angle scattering (parallel diffusion). In order to address the fact that the observed mean free paths are larger than QLT results, Bieber et al. (1994) suggested that turbulence in space might be a composite of slab and two-dimensional geometries with an energy ratio of 20:80, assuming that in QLT the two-dimensional component of turbulence contributes little to parallel diffusion. Qin et al. (2002b; see also Bieber 2003) found that nonlinear effects associated with a strong turbulence level might cause the two-dimensional component of turbulence to contribute to parallel diffusion.

In this paper we use numerical simulations to calculate charged particle trajectories in the Parker field with composite model turbulence to study the magnetic focusing effects, effects of pitch-angle diffusive transport due to magnetic fluctuations, and adiabatic cooling effects on SEPs. We give formulae explicitly for magnetic focusing with scattering by magnetic turbulence and adiabatic cooling effects for anisotropic energetic particles. In the formula for time evolution of the ensemble average of the pitch-angle cosine, we use QLT to describe a particle's pitch-angle diffusive scattering (parallel diffusion). To the authors' knowledge, this is the first time that the validity of the formulae of magnetic focusing and adiabatic cooling effects has been directly tested with test particle orbit-tracing simulations by solving the Newton-Lorentz equation. The results of our work can be used in numerical simulations (e.g., Ruffolo 1995) solving the transport equation with assumptions of the formulae of the particles' focusing and/or adiabatic cooling effects. We show that the simulation results agree much better with the anisotropic adiabatic cooling effect theory than with the isotropic one. In addition, we show that the simulation results agree well with the magnetic focusing effect with diffusive pitch-angle scattering by magnetic fluctuations. However, if there are strong nonlinear effects, e.g., if the average particle's Larmor radius is comparable to the correlation scale of the two-dimensional component of turbulence, then the two-dimensional component contribution of parallel diffusion cannot be ignored.

2. FORMALISM

2.1. *Adiabatic Cooling*

It is usually assumed that the cosmic-ray adiabatic cooling effect could be expressed with the formula (Parker 1965; Dorman 1965)

$$\frac{dp}{dt} = -\frac{p}{3}\nabla\cdot\boldsymbol{V}^{\mathrm{sw}}, \tag{1}$$

where $\boldsymbol{V}^{\mathrm{sw}}$ is the solar wind velocity. This formula is valid for cosmic-ray particles with an isotropic pitch-angle distribution.

However, we can show that for cosmic-ray particles with anisotropic pitch-angle distributions the formula should be (see Appendix A)

$$\frac{dp}{dt} = -p\left[\frac{1-\mu^2}{2}\left(\frac{\partial V_x^{\rm sw}}{\partial x} + \frac{\partial V_y^{\rm sw}}{\partial y}\right) + \mu^2 \frac{\partial V_z^{\rm sw}}{\partial z}\right]. \quad (2)$$

Here p is the particle's momentum averaged over the gyrophase (we suppress the bar used in Appendix A). Note that we set the z-axis as the direction of the local averaged magnetic field. Consequently, μ and p are measured in the solar wind frame. This formula agrees with the wave-frame approach of Skilling (1971) because the Alfvén speed $v_{\rm A}$ is much smaller than the solar wind speed $V_{\rm sw}$ and yields equation (1) for particles with isotropic pitch-angle distributions.

2.2. *Time Evolution of the Pitch-Angle Cosine*

In addition to adiabatic cooling, charged particles experience magnetic focusing effects due to differential average magnetic fields and pitch-angle scattering due to magnetic fluctuations. The time derivative of the pitch-angle cosine, μ, due to focusing effects can be written as (Roelof 1969)

$$\frac{d\mu}{dt} = g_{\rm foc} = -\frac{v(1-\mu^2)}{2B}\frac{\partial B}{\partial z}. \quad (3)$$

Note that in the solar wind frame, there are additional smaller terms due to differential solar wind convection (Skilling 1971). Here we ignore them for energetic particles ($V^{\rm sw}/v \ll 1$). The time derivative of the ensemble average of the pitch-angle cosine, $\langle\mu\rangle$, due to diffusive transport in magnetic fluctuations can be written as (see Appendix B)

$$\frac{d\langle\mu\rangle}{dt} = G_{\rm sca} = -\int d^3\boldsymbol{r}\left[\int_{-1}^{1} D_{\mu\mu}\frac{\partial f(\mu,\boldsymbol{r},t)}{\partial\mu}d\mu - \mu D_{\mu\mu}\frac{\partial f(\mu,\boldsymbol{r},t)}{\partial\mu}\bigg|_{\mu=-1}^{\mu=1}\right]. \quad (4)$$

Here $D_{\mu\mu}$ is the pitch-angle diffusion coefficient. If the turbulence level b/B is not very large, where b is the total magnetic fluctuations, and the particle Larmor radius $r_{\rm L} \equiv v/\Omega$ is much smaller than the perpendicular correlation scale of the two-dimensional component of the turbulence, we can use QLT to describe $D_{\mu\mu}$. In QLT, if the magnitude of the particle's parallel resonant wavenumber $k_\parallel \equiv \Omega/(|\mu|v)$ is smaller than the maximum populated parallel wavenumber, we have the formula (Mace et al. 2000)

$$D_{\mu\mu} = \frac{\pi^{1/2}}{2}\frac{\Gamma(\nu)}{\Gamma(\nu-1/2)}\left(\frac{q}{\gamma mc}\right)^2 \times \langle b_{\rm slab}^2\rangle \frac{\lambda}{v}\frac{(1-\mu^2)}{|\mu|}(1+k_\parallel^2\lambda^2)^{-\nu}\bigg|_{k_\parallel \equiv \Omega/(\mu v)}. \quad (5)$$

Otherwise,

$$D_{\mu\mu} = 0. \quad (6)$$

Consequently, $G_{\rm sca} = 0$ if the resonant wavenumber is larger than the maximum populated parallel wavenumber. For example, if the turbulence level b/B and the slab turbulence correlation scale λ are taken to be constant, then when particles are very near the Sun where their resonant wavenumber is larger than the maximum populated wavenumber, they will not sample the turbulence, so there is no time derivative of μ due to scattering by magnetic fluctuations. In general, to combine the magnetic focusing and fluctuations effects, we have

$$\frac{d\langle\mu\rangle}{dt} = \langle g_{\rm foc}\rangle + G_{\rm sca}. \quad (7)$$

In this paper we use numerical simulations to examine the validity of these formulae.

3. NUMERICAL SIMULATIONS

To study various SEP acceleration and transport processes in the heliosphere, such as the properties of the magnetic focusing effect, adiabatic cooling, and diffusive transport in the pitch-angle distribution, we calculate particle trajectories in the Parker field with model turbulence.

We use numerical methods similar to those of Mace et al. (2000) and Qin et al. (2002a, 2002b). The particles with mass m and velocity $\boldsymbol{v}'$ in a fixed frame obey

$$m\frac{d\boldsymbol{v}'(t)}{dt} = \frac{q(\boldsymbol{v}'(t) - \boldsymbol{V}^{\rm sw}(\boldsymbol{X}))}{c} \times \boldsymbol{B}(\boldsymbol{X}), \quad (8)$$

where we set the constant radial solar wind velocity $\boldsymbol{V}^{\rm sw}(\boldsymbol{X}) = V^{\rm sw}\hat{\boldsymbol{r}}$ with $V^{\rm sw} = 400$ km s^{-1}, $\hat{\boldsymbol{r}}$ being the solar radial direction, and $\boldsymbol{X}$ being the global position. The model magnetic field $\boldsymbol{B}(\boldsymbol{X}) = \boldsymbol{B}_p(\boldsymbol{X}) + \boldsymbol{b}(\boldsymbol{X})$ consists of a Parker field $\boldsymbol{B}_p$ and a composite transverse magnetic fluctuation model $\boldsymbol{b} = (b_x(x,y,z), b_y(x,y,z), 0)$ consisting of a two-dimensional part $\boldsymbol{b}^{\rm 2D}(x,y)$ and "slab" part $\boldsymbol{b}^{\rm slab}(z)$. Note that (x,y,z) is a local Cartesian system in the solar wind frame with $\hat{\boldsymbol{z}} \parallel \boldsymbol{B}_p(\boldsymbol{X})$. We suppose that the background magnetic field $\boldsymbol{B}_p$ is a constant locally when compared to the magnetic turbulence term $\boldsymbol{b}$ but a function of position $\boldsymbol{X}$ globally. In this paper we use B to represent the local background magnetic field B_p. We set the Parker field so that the background magnetic field at 1 AU is 5 nT. In the simulations discussed below we fix the ratio of the turbulence energies $E^{\rm slab} \equiv \langle b_{\rm slab}^2\rangle$ and $E^{\rm 2D} \equiv \langle b_{\rm 2D}^2\rangle$ to $E^{\rm slab}:E^{\rm 2D} = 20:80$ while also controlling the turbulence level, i.e., the ratio of turbulence amplitude to the local Parker field strength b/B. Both slab and two-dimensional spectra are constant at wavenumbers k much less than the correlation scales (turnover scales) and become $k^{-5/3}$ at high k. The spectral turnover scales in the z- and x-directions have fixed values for the entire solar distance range, $\lambda = 0.02$ AU and $\lambda_x = d\lambda$, respectively, where d is the ratio between λ_x and λ. We can control the strength of the transverse complexity by changing the value of d. Note that for simplicity we choose a constant turbulence level b/B and constant turbulence correlation scales λ and λ_x. In this kind of simulation, as the average particle's radial distance increases with time, the averaged Larmor radius $r_{\rm L}$ increases; therefore, the average magnitude of the particles' parallel resonant wavenumber $k_\parallel = 1/(|\mu|r_{\rm L})$ decreases, and the ratio $r_{\rm L}/\lambda_x$ increases. The particle trajectory equation (8) is integrated with a fourth-order adaptive step Runge-Kutta method with relative error control set to 10^{-9}.

For the slab components we use a periodic box of size $L_z = 100\lambda$ and $N_z = 2^{18} = 262144$ grid points. For the two-dimensional component we use a periodic box of size

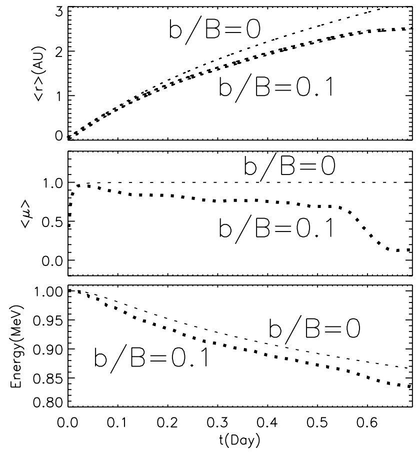

FIG. 1.—Numerical calculations of test particle trajectories in a Parker field with constant radial solar wind. Thick and thin dashed lines represent magnetic fields with and without turbulence, respectively. Particles' ensemble averaged values are plotted in the three panels. *Top panel*: radial distance vs. time. *Middle panel*: pitch-angle cosine μ vs. time. *Bottom panel*: energy vs. time.

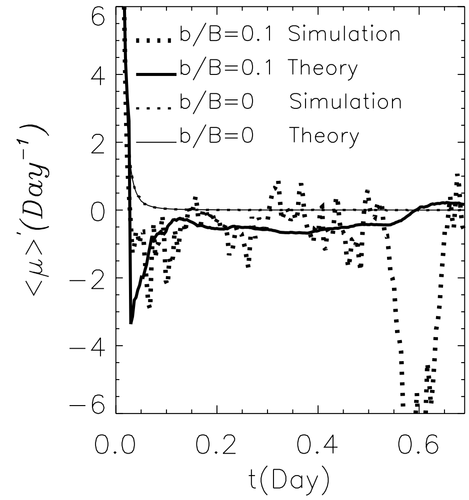

FIG. 2.—Time derivative of particles' ensemble-averaged pitch-angle cosine, $\langle\mu\rangle$, from the same numerical simulations (*dotted lines*) as in Fig. 1. For reference, solid lines represent the analytical formulae of time derivatives of $\langle\mu\rangle$ due to the magnetic focusing effect, both without turbulence (*thin line*) and with turbulence, assuming QLT for $D_{\mu\mu}$ (*thick line*).

$L_x \times L_y = 10\lambda \times 10\lambda$ and $N_x \times N_y = 1024 \times 1024$ points. Note that we can use smaller box sizes and grid numbers for both the slab and two-dimensional components than those in Mace et al. (2000) and Qin et al. (2002a, 2002b), because here in the timescale to calculate time derivatives the test particle's displacement is much smaller than the periodic box size. In the slab component we take the maximum excited wavenumber $k_z^{\max} = (N_z/2)/L_z = 4.3 \times 10^{-3}$ km^{-1}, which is a typical wavenumber for the solar wind magnetic turbulence power spectrum to turn over from inertial to dissipation range (e.g., see Dröge 2003).

4. NUMERICAL RESULTS

By computing the time derivative of both the test particles' pitch-angle cosine, μ, and the magnitude of the momentum in the solar wind frame, we can examine the magnetic focusing effects, the effects of pitch-angle diffusive transport due to magnetic fluctuations, and adiabatic cooling effects on SEPs. In this study we have made one major numerical simulation with model turbulence $b/B = 0.1$ and ratio $d = \lambda_x/\lambda = 0.1$. The results are shown in Figures 1, 2, and 4. In addition, in order to discuss the contribution of the two-dimensional component of turbulence to the parallel diffusion, we also have run a simulation similar to the major one but with a much larger correlation scale of the two-dimensional component turbulence, i.e., $d = \lambda_x/\lambda = 1$, as shown in Figure 3. For reference purposes, we have performed another almost identical simulation without turbulence, which is shown in all figures. For all three simulations, in the beginning we release 20,000 protons originated at a solar distance $r_0 = 0.05$ AU and latitude 30° with a ring-beam velocity distribution with pitch-angle cosine $\mu_0 = 0.2$ and energy $E_0 = 1$ MeV. For consistency, in all figures, thick and thin lines indicate results from the simulation with and without turbulence, respectively. The dotted lines indicate results from the simulations, and dashed lines indicate results from theoretical calculations using the formulae in § 2.

Figure 1 shows results for the two numerical simulations (the major one and the reference one without turbulence). The top panel shows the average solar distance for the two simulations. We can see that it takes a longer time for particles to travel the same radial distance with turbulence because of particles scattering in turbulent fluctuations. The middle panel shows the particles' average pitch-angle cosine, $\langle\mu\rangle$. We can see that in a very short timescale, before the particle's average solar distance r exceeds 0.3 AU, $\langle\mu\rangle$ increases to nearly 1 for both simulations because of the strong focusing effect, but after that, particles in the two simulations behave differently. For the one without turbulence, $\langle\mu\rangle$ stays at the constant value of 1, but for the one with turbulence, $\langle\mu\rangle$ decreases, showing a strong scattering effect due to turbulence. Note that after $t = 5.5$ days, the decreasing of $\langle\mu\rangle$ becomes more rapid and in half a day it stabilizes approximately at 0. The bottom panel shows the average energy in the solar wind frame for the simulations. For both simulations, there is energy loss from the adiabatic cooling effect.

Figure 2 shows the time derivative of the particle's mean pitch-angle cosine, $\langle\mu\rangle$, from the two simulations shown in Figure 1. We can see in the very short time range when particles are very near the Sun, the time derivative of the particle's average $\langle\mu\rangle$ from both simulations (*dotted lines*) decreases rapidly from a very large value to almost 0.

For the simulation without turbulence, after $\langle\mu\rangle$ reaches its maximum value, 1, the time derivative of $\langle\mu\rangle$ remains at 0. For the one with turbulence, if we do not consider higher frequency fluctuations in the time profile of the derivative of $\langle\mu\rangle$, we can see that after $\langle\mu\rangle$ reaches its maximum value of about 1, it starts to decrease until reaching a negative minimum value. Thereafter, it increases to stabilize at a very small negative value on average. Note, however, that at a larger time scale, around $t = 0.6$ days, it decreases rapidly to about -6 per day but before long returns approximately to 0 after the value of $\langle\mu\rangle$ is saturated at 0 (see the middle panel of Fig. 1).

For reference, in Figure 2 we also plotted the results from the theoretical time derivative formula, both without turbulence (pure focusing effect; *thin solid line*) and with turbulence, assuming QLT for the pitch-angle diffusion coefficient $D_{\mu\mu}$ (combined focusing and diffusive scattering effects; *thick solid line*). We can see that the theoretical curve without turbulence agrees with the simulation result very well for the whole timescale, which demonstrates that without turbulence in the short timescale when particles are very near the Sun, a strong

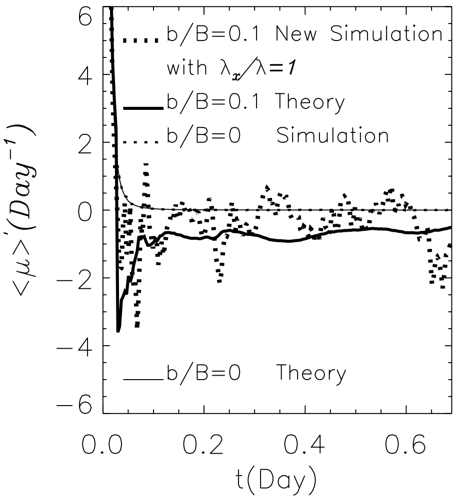

FIG. 3.—Same as Fig. 2, but for a different turbulence parameter $d = \lambda_x/\lambda = 1$.

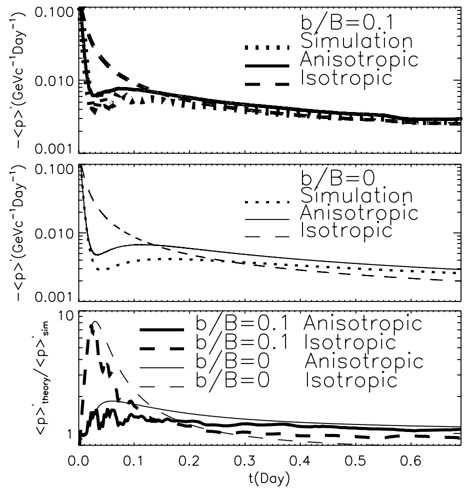

FIG. 4.—Change of momentum for the simulations shown in Fig. 1 and the theoretical results. Top and middle panels show negative time derivatives of test particles' ensemble average momentum magnitude with and without turbulence, respectively. Results from the numerical simulations, as well as isotropic and anisotropic formulae, are shown with different curves. The bottom panel shows the ratio of time derivatives of test-particle momentum obtained from theories to that from numerical simulations. Different line styles indicate which theory (anisotropic or isotropic) is used and whether there is magnetic turbulence in numerical simulations.

focusing effect puts energetic particles into a beamlike distribution and keeps them in that distribution thereafter.

However, with turbulence, the theoretical curve can be divided into two periods. The first period is for t from 0 to about 0.03 days, when the particles' parallel resonant wavenumber is larger than the maximum populated parallel wavenumber of the turbulence, i.e., there is no scattering due to magnetic fluctuations, and the time derivative of $\langle\mu\rangle$ is only due to the focusing effect. We can see that in this period the simulation and theory agree very well. We can call this the "focusing period." The second period is for t from about 0.03 days to larger values, when the particles' parallel resonant wavenumber is smaller than the maximum populated wavenumber of the model turbulence, i.e., there is scattering due to magnetic fluctuations. From Figure 2 we can see that starting from this period, the scattering effect suddenly kicks in and the theory agrees with the numerical simulation (after filtering off the higher frequency fluctuations in the time derivative of $\langle\mu\rangle$ profile for a relatively long timescale). We can call this the "focusing-scattering period." Note that there is a large discrepancy between the theory and simulation at large times, around $t \gtrsim 0.55$ days. We know that in this simulation as time t increases the ratio of $r_{\rm L}/\lambda_x$ also increases, and at the time $t \approx 0.55$ days $r_{\rm L}$ is comparable to λ_x, or $r_{\rm L} \approx \lambda_x/4$. Thus, at these times either the nonlinear effects are too large and the QLT does not work any more or we cannot ignore the two-dimensional component of turbulence contribution to parallel diffusion.

We have another similar simulation with turbulence that is almost identical to the major one, but with the only difference that $\lambda_x = \lambda = 0.02$ AU. From this new simulation we find that $r_{\rm L} \approx \lambda_x/40$ at $t \approx 0.55$ days, i.e., quasilinear theory is still valid. Figure 3 shows the time derivative of the particles' mean pitch-angle cosine, $\langle\mu\rangle$, from both simulations (the one with new turbulence and the reference one without turbulence). From Figure 3 we can see that the time derivative of $\langle\mu\rangle$ in the new simulation with turbulence (*thick dotted line*) behaves in a manner similar to that in the major simulation (Fig. 2, *thick dotted line*) on smaller timescales but without the big discrepancy between theory and simulation at times $t \gtrsim 0.55$.

Figure 4 shows the time derivative of the particles' mean magnitude of momentum $\langle p\rangle$ from the simulations in Figures 1 and 2. The top and middle panels show the time derivative of $\langle p\rangle$ with and without turbulence, respectively. Results from numerical simulations as well as theoretical results from analytic calculations with anisotropic and isotropic distributions are shown with different curves. The bottom panel shows the ratio of time derivatives of $\langle p\rangle$ obtained from theories to those from numerical simulations. Different lines indicate which theory (anisotropic or isotropic) is used and whether there is any magnetic turbulence in numerical simulations. From the figure we can see that at a small timescale, $t < 0.1$ days, theoretical results from the anisotropic formulae agree with simulations well, but there is a big discrepancy between that from the isotropic formulae and that from the simulations. However, for larger times, i.e., $t > 0.1$ days, the two theoretical results, anisotropic or isotropic, agree with the simulations well for the following reasons. For the simulation with turbulence in larger times, particles become very isotropic so the isotropic formula agrees with the simulation well, but for the simulation without turbulence at time $t > 0.1$ days, with all particles traveling along magnetic field lines, i.e., $\mu = 1$, the anisotropic formula should be used. However, since at that solar distance on average $\partial V_z/\partial z \approx \nabla \cdot \boldsymbol{V}^{\rm sw}/3$, the results from the isotropic formula agree with the simulation as well. Here we demonstrate that in general the anisotropic formula has to be used to describe the adiabatic cooling effect for SEPs.

5. SUMMARY AND DISCUSSION

This paper is summarized as following. First, explicit analytical formulae are given to describe energetic particles' magnetic focusing effects with diffusive scattering and adiabatic cooling effects. We provide a formula of the time derivative of the ensemble-averaged pitch-angle cosine, μ, due to magnetic focusing effects and diffusive scattering, where the quasilinear theory of the pitch-angle diffusion coefficient $D_{\mu\mu}$ is used. We also show a formula of the time derivative of the average momentum magnitude p due to the adiabatic cooling effect, which is the same as that used by Skilling (1971).

Second, numerical simulations are provided to calculate the orbit trajectories of SEPs in the Parker field with model turbulence, which is generated with a composite model with 20% slab and 80% two-dimensional energy and a relatively weak turbulence level $b/B = 0.1$. We use weaker turbulence in order to avoid nonlinear effects so that we can use quasilinear theory for $D_{\mu\mu}$. Comparisons are made between numerical simulations and analytical results. We show that numerical simulations agree with the theory of magnetic focusing effect with diffusive scattering by turbulence very well. In addition, in the simulations there are significant cooling effects that agree much better with the formula of adiabatic cooling effects for particles in an anisotropic distribution than that of an isotropic distribution. This tells us that in order to study SEPs in shorter solar distances we have to consider adiabatic cooling effects in anisotropic distributions.

Third, from the simulations we can see that if the particle's Larmor radius is not much smaller than the correlation scale of the two-dimensional component of turbulence, the two-dimensional component contributes significantly to parallel diffusion. This is reasonable, because if the particle's Larmor radius is compatible with the two-dimensional correlation scale, for each gyrocycle a particle experiences large fluctuations of the field due to the two-dimensional component of turbulence, which causes quasilinear theory to be invalid, i.e., the contribution of the two-dimensional component of turbulence is not negligible. Furthermore, if the turbulence level is strong, the nonlinear effect might also cause the two-dimensional component to contribute to parallel scattering (Qin et al. 2002b; Bieber 2003). This means that it is necessary to do more research in nonlinear theory of energetic particle diffusion in turbulence. Alternatively, we could use $D_{\mu\mu}$ from numerical simulations instead of theoretical models.

Fourth, the study of turbulence correlation scales in the solar wind is important in the study of cosmic ray transport. Here, for simplicity, we use ad hoc models of magnetic turbulence that have a constant correlation scale λ and λ_x and a constant turbulence level b/B. In the future we will use the steady-state turbulence propagation equations provided by Zank et al. (1996) and Matthaeus et al. (1996) to investigate the SEPs. Parhi et al. (2003a, 2003b) studied the integration of a turbulence model with an ab initio modulation model. They found that the comparison between their results and the observational data with turbulence calculated from the governing equations is better than that from the ad hoc turbulence model.

The work is supported partly by NASA grants NAG5-11036, NAG5-13514, and NAG5-119211, and NSF SHINE grant 0203252.

APPENDIX A

ANISOTROPIC ADIABATIC COOLING EFFECT

It is assumed that the heliospheric magnetic field is frozen in the solar wind, so there is no electric field in the solar wind frame. Supposing that $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the particle's momentum in the solar wind frame and the fixed frame, respectively, and that $\boldsymbol{V}^{\rm sw}$ is the solar wind speed, we have

$$\boldsymbol{p} + m\boldsymbol{V}^{\rm sw} = \boldsymbol{p}'. \tag{A1}$$

In the nonrelativistic condition, the time derivative of the particle's momentum in the solar wind frame is written as

$$\frac{d\boldsymbol{p}}{dt} = -m\frac{d\boldsymbol{V}^{\rm sw}(\boldsymbol{x})}{dt} + \frac{d\boldsymbol{p}'}{dt} = -m\left(\frac{d\boldsymbol{x}}{dt}\cdot\nabla\right)\boldsymbol{V}^{\rm sw} + \boldsymbol{f}^{\rm L} = -(\boldsymbol{p}\cdot\nabla)\boldsymbol{V}^{\rm sw} + \boldsymbol{f}^{\rm L}, \tag{A2}$$

where $\boldsymbol{f}^{\rm L}$ is the Lorentz force. This equation shows that because of the differential solar wind convection, in the solar wind frame a particle "feels" a force $\boldsymbol{F}$ in addition to the Lorentz force $\boldsymbol{f}^{\rm L}$, where

$$F_j = -p_i\frac{\partial V_j^{\rm sw}}{\partial x_i}. \tag{A3}$$

Therefore, the time derivative of the magnitude of the particle momentum can be written as

$$\frac{dp}{dt} = \frac{\partial p}{\partial p_j}\frac{dp_j}{dt} = \frac{\partial p}{\partial p_j}\left(-p_i\frac{\partial V_j^{\rm sw}}{\partial x_i} + f_j^{\rm L}\right) = \frac{1}{p}\left(-p_j p_i\frac{\partial V_j^{\rm sw}}{\partial x_i} + p_j f_j^{\rm L}\right). \tag{A4}$$

Suppose that the particle gyrofrequency Ω is much larger than the magnitude of the differential solar wind convection $|\partial V_j^{\rm sw}/\partial x_i|$. Then we can average equation (A4) over the gyrophase to obtain

$$\frac{d\bar{p}}{dt} = -\bar{p}\left[\frac{1-\mu^2}{2}\left(\frac{\partial V_x^{\rm sw}}{\partial x} + \frac{\partial V_y^{\rm sw}}{\partial y}\right) + \mu^2\frac{\partial V_z^{\rm sw}}{\partial z}\right], \tag{A5}$$

where the z-component represents the direction of the local magnetic field. This is a formula for energetic particles with an anisotropic pitch-angle distribution, which is the same as the one used in the transport equation by Skilling (1971) with the wave-frame approach using the fact that the Alfvén speed, $v_{\rm A}$, is much smaller than the solar wind speed $V_{\rm sw}$. Furthermore, if the particles' pitch-angle distribution is isotropic, we can average the anisotropic formula (eq. [A5]) over the pitch-angle cosine μ to obtain the formula for particles with an isotropic distribution,

$$\frac{d\bar{p}}{dt} = -\frac{\bar{p}}{3}\nabla \cdot \boldsymbol{V}_{\rm sw}. \tag{A6}$$

APPENDIX B

TIME EVOLUTION OF THE PITCH-ANGLE COSINE BY DIFFUSIVE SCATTERING

Now let us examine the time derivative of the average energetic particle's pitch-angle cosine, $\langle\mu\rangle$, due to diffusive scattering by magnetic fluctuations $G_{\rm sca}$. We have

$$\frac{d\langle\mu\rangle}{dt} = G_{\rm sca}. \tag{B1}$$

Here, angle brackets, $\langle\cdots\rangle$, indicate the ensemble average. The time evolution of the particle's distribution function $f(\mu, \boldsymbol{r}, t)$ due to scattering can be written as

$$\frac{\partial f(\mu, \boldsymbol{r}, t)}{\partial t} = \frac{\partial}{\partial\mu} D_{\mu\mu} \frac{\partial f(\mu, \boldsymbol{r}, t)}{\partial\mu}, \tag{B2}$$

where $D_{\mu\mu}$ is the particle's pitch-angle diffusion coefficient, which is a function of position ($\boldsymbol{r}$) and μ. The time derivative of the ensemble-average pitch-angle cosine μ can be written as

$$\begin{aligned}\frac{d\langle\mu\rangle}{dt} = G_{\rm sca} &= \frac{d}{dt}\int d^3\boldsymbol{r}\int_{-1}^{1} f(\mu, \boldsymbol{r}, t)\mu\, d\mu = \int d^3\boldsymbol{r}\int_{-1}^{1}\frac{\partial f(\mu, \boldsymbol{r}, t)}{\partial t}\mu\, d\mu \\ &= \int d^3\boldsymbol{r}\int_{-1}^{1}\mu\frac{\partial}{\partial\mu}D_{\mu\mu}\frac{\partial f(\mu, \boldsymbol{r}, t)}{\partial\mu}\, d\mu = -\int d^3\boldsymbol{r}\left(\int_{-1}^{1} D_{\mu\mu}\frac{\partial f(\mu, \boldsymbol{r}, t)}{\partial\mu}\, d\mu - \mu D_{\mu\mu}\frac{\partial f(\mu, \boldsymbol{r}, t)}{\partial\mu}\bigg|_{\mu=-1}^{\mu=1}\right).\end{aligned} \tag{B3}$$

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