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# Propagation Speed Of Runaway Electron Avalanches

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[1] Simulations of relativistic runaway breakdown (RRB) are performed as functions of both time and space, resulting in explicit calculations of *e*-folding lengths ($\lambda$) and times ($\tau$). The ratio of $\lambda$ to $\tau$ agrees well with the speed of the avalanche, which ranges from $2.61 \times 10^8$ m s$^{-1}$ to $2.72 \times 10^8$ m s$^{-1}$. Thus, using the speed of light, *c*, for the ratio of $\lambda$ to $\tau$ can cause a 10% error when estimating $\lambda$ from $\tau$. A 10% error in $\lambda$ will cause a factor of three error in the predicted number of runaway electrons for every ten estimated *e*-foldings. In addition, previous models that predict peak radiated electric fields from RRB have used avalanche speeds of 0.987*c* and higher. Using a propagation speed of 0.89*c* causes a dramatic change in the predicted beaming pattern of electromagnetic radiation caused by RRB in these models. **Citation:** Coleman, L. M., and J. R. Dwyer (2006), Propagation speed of runaway electron avalanches, *Geophys. Res. Lett.*, *33*, L11810, doi:10.1029/2006GL025863.

## 1. Introduction

[2] Relativistic Runaway Breakdown (RRB) may play a key role in many unexplained phenomena in atmospheric electricity, such as lightning initiation [e.g., *Gurevich et al.*, 1992; *Dwyer*, 2005], "narrow bipolar events" [e.g., *Smith et al.*, 2001; *Gurevich et al.*, 2004], and "terrestrial gamma-ray flashes" [e.g., *Fishman et. al.*, 1994; *Dwyer and Smith*, 2005]. RRB is caused when the ambient electric field in a medium creates a large enough force on an energetic electron to overcome the drag forces caused by the interactions of the electron with the medium. Eventually, such a "runaway electron" will knock loose a second electron from an atom in the medium and both electrons will become runaway electrons. As a result, the number of electrons increases exponentially with time and distance, and the RRB creates an "avalanche." The avalanche has a characteristic *e*-folding time, $\tau$, which is how much time passes on average before the number of runaway electrons increases by a factor of *e*. In addition, the avalanche has a characteristic *e*-folding distance, $\lambda$, which is how far the avalanche propagates on average before the number of runaway electrons increases by a factor of *e*. These two quantities depend on the density and composition of the medium and on the ambient electric field as discussed later in this work.

[3] Monte Carlo models have become a commonly used tool to investigate the properties of RRB, e.g., *Dwyer* [2003], *Babich et al.* [2005], and *Inan and Lehtinen* [2005]. Typically, Monte Carlo simulations have been used to calculate $\tau$. However, in order to apply the findings of the Monte Carlo models to thunderstorm situations, one must know $\lambda$. *Babich et al.* [2005] used their Monte Carlo simulation of RRB, ELIZA, to find $\tau$ and then multiplied $\tau$ by the speed of light (*c*) to estimate $\lambda$ for electric fields ranging from 305 kV m$^{-1}$ to 2.6 MV m$^{-1}$. When they compared their results to those of *Dwyer* [2003], they found differences ranging from 2.2% to 43% between the Dwyer model and ELIZA, depending on the electric field.

[4] This comparison seems to show that there is some significant disagreement between the two models. However, it is not clear that the true ratio between $\lambda$ and $\tau$ is, in fact, the speed of light. For this reason the Dwyer model is used in this work to calculate $\lambda$ and $\tau$ explicitly and find the ratio between the two.

[5] The Dwyer model is also used to calculate the speed of the average position of the electron avalanche. This result, found to be very close to the ratio of $\lambda$ to $\tau$, is important for the search for empirical evidence for RRB; a slower avalanche propagation speed means that beaming of the electromagnetic signal from RRB should be smaller in magnitude and more isotropic than previously expected, e.g. *Roussel-Dupré and Gurevich* [1996], *Gurevich et al.* [2002], and *Tierney et al.* [2005].

## 2. Monte Carlo Model

[6] The Monte Carlo model used for this work is identical to that used by *Dwyer* [2003]. It is a 4-D Monte Carlo simulation of runaway breakdown in air which includes energy losses through ionization, atomic excitation, bremsstrahlung photon production, and Møller scattering. In addition, this Monte Carlo fully models elastic Coulomb scattering using a shielded-Coulomb potential.

[7] The position and interactions of a single runaway electron are calculated in the Monte Carlo using an adaptive step size Runge-Kutte method to propagate the electron through a region with forces from electric and (when applicable) magnetic fields. In addition a "frictional" force is calculated from a modified Bethe-Bloch equation [*Bethe and Ashkin*, 1953; *Lehtinen et al.*, 1999]. After each step is calculated based on the kinematics of the electron, the final position and momentum are then modified based upon the interactions listed above.

## 3. Calculating $\lambda$ and $\tau$

[8] Avalanches were simulated at 30 different field strengths with 5 runs at each field strength to calculate $\lambda$ and $\tau$. The field strengths were chosen at every 100 kV m$^{-1}$ from 300 kV m$^{-1}$ to 2.2 MV m$^{-1}$ and every 200 kV m$^{-1}$ from 2.4 MV m$^{-1}$ to 3.2 MV m$^{-1}$. (In this work, all reported field strengths are normalized to sea level equivalents.) Each avalanche was started with 20 electrons propagating downward from the same point in space with

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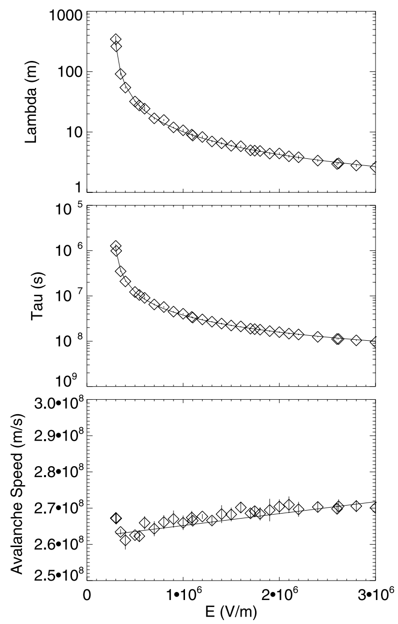


**Figure 1.** Monte Carlo Estimates of λ and τ. This figure shows the estimates of λ and τ calculated during this study. (top) λ vs. *E*. The diamonds indicate the mean value of λ as calculated by 5 runs of the Monte Carlo at each field magnitude. The error bars indicate the standard deviation between the runs at each field magnitude. The solid line shows the fit of equation (2) for λ as a function of *E*. (middle) τ vs. *E*. The diamonds represent the mean value of τ and the error bars represent the standard deviation. The solid line shows the best fit for τ as function of *E*. (bottom) Speed of the average position of the electrons with energy greater than 500 keV. The diamonds represent the mean speed calculated for each run, with the error bars representing the standard deviation for each field magnitude. The solid line shows a linear regression line fit to the ratio of λ to τ from the data values in the first plot divided by the data values in the second plot.

energies distributed randomly over an exponential distribution with the mean kinetic energy at 7.2 MeV. In addition, the values were calculated for the five field strengths tested by *Babich et al.* [2005].

[9] Each avalanche was allowed to run for a time equal to $5c\lambda_{\text{est}}$, where $c$ is the speed of light and $\lambda_{\text{est}}$ is given by the empirical equation for λ as a function of field strength in kV $m^{-1}$ found in the work by *Dwyer* [2003]:

$$\lambda_{est} = \frac{7200 \text{ kV}}{\left(E - 275 \text{ kV m}^{-1}\right)}. \quad (1)$$

[10] As explained in detail by *Dwyer* [2004], this empirical equation indicates that the runaway electrons have an average kinetic energy of 7.2 MeV. We note that the avalanches could not be run for too many *e*-foldings, or else feedback mechanisms would have changed the results of the calculations [*Babich et al.*, 2005; *Dwyer*, 2003, 2005].

[11] The number of runaway electrons as a function of time, N(t), was found at intervals of $0.005c\lambda_{\text{est}}$ and used to calculate τ with a linear fit of ln(N(t)) to the data from $t = 2.5c\lambda_{\text{est}}$ to $t = 5c\lambda_{\text{est}}$. The simulation was allowed to run for a few *e*-foldings before a fit was attempted so that the simulation could reach a steady state.

[12] The number of runaway electrons as a function of altitude, N(z), was calculated every $0.005\lambda_{\text{est}}$, and a line was then fit to ln(N(z)) from $z = 2.5\lambda_{\text{est}}$ to $z = 3.75\lambda_{\text{est}}$ to find λ. Again, the simulation was allowed to run for a few *e*-foldings so that it would reach a steady state. Also, it was not possible to fit the exponential growth for the full range of *z*, as only the most energetic electrons traveled the full distance of $5\lambda_{\text{est}}$.

[13] After λ and τ were calculated for each run of the Monte Carlo, the average values and standard deviations of λ and τ were calculated for each field strength and used to fit an equation with the same form as equation (1) to λ and τ. The results are shown in Figure 1 and can be approximated with the following empirical equations, where the ranges quoted indicate the statistical variations in the empirical fit:

$$\lambda = \frac{7300 \pm 60 \text{ kV}}{\left(E - 276 \pm 4 \text{ kV m}^{-1}\right)} \quad (2)$$

$$\tau = \frac{27.3 \pm 0.1 \text{ kV}\mu\text{s m}^{-1}}{\left(E - 277 \pm 2 \text{ kV m}^{-1}\right)} \quad (3)$$

[14] Equations (2) and (3) apply to RRB at sea level. In order to find λ and τ as a functions of atom number density (n) and ultimately of altitude (z), the following relationships should be used, where the z subscripts indicate the value at altitude and the 0 subscripts indicate the equivalent sea-level value:

$$\lambda_z(E_z, n_z) = \lambda_0(E_0)\frac{n_0}{n_z} = \frac{7300 \text{ kV}}{\left(E_z - 276\frac{n_z}{n_o}\text{kV}\right)} \quad (4)$$

$$\tau_z(E_z, n_z) = \tau_0(E_0)\frac{n_0}{n_z} = \frac{27.3 \text{ kV}\mu\text{s m}^{-1}}{\left(E_z - 277\frac{n_z}{n_0}\text{kV}\right)}. \quad (5)$$

[15] These formulas for λ and τ only apply at the field range studied here: greater than 300 kV $m^{-1}$. Table 1 shows a comparison for the five field values tested by *Babich et al.* [2005] between the estimated results for λ and τ from equations (1) and (2), the values actually calculated, and the values reported by *Babich et al*. At smaller electric field magnitudes, it was found that Coulomb scattering plays a much larger role in the avalanche process. For fields from the threshold field of 286 kV $m^{-1}$ to 300 kV $m^{-1}$, the following empirical fit was found to be a better description of λ and τ:

$$\lambda = \frac{5100 \pm 100 \text{ kV}}{\left(E - 285 \pm 10 \text{ kV m}^{-1}\right)} \quad (6)$$

$$\tau = \frac{19.1 \pm 0.3 \text{ kV } \mu\text{s m}^{-1}}{\left(E - 285 \pm 1 \text{ kV m}^{-1}\right)} \quad (7)$$

**Table 1.** A Comparison Between the Results of This Study and the Results of ELIZA [*Babich et al.,* 2005][a]

| $E$, kV m$^{-1}$ | $\tau_{est}$, ns | $\tau$, ns | $\tau_B$, ns | $\lambda_{est}$, m | $\lambda$, m | $\lambda_v$, m | $\lambda_B$, m |
|---|---|---|---|---|---|---|---|
| 305 | 975 | 988 ± 50 | 1236 | 251 | 262 ± 17 | 328 | 371 |
| 545 | 102 | 105 ± 3 | 110 | 27.1 | 27.6 ± 0.8 | 29.4 | 33 |
| 1090 | 33.6 | 34.3 ± 1.0 | 34.3 | 8.97 | 9.07 ± 0.36 | 9.16 | 10.3 |
| 1744 | 18.6 | 18.5 ± 0.6 | 17.8 | 4.97 | 4.91 ± 0.24 | 4.75 | 5.3 |
| 2616 | 11.7 | 11.2 ± 0.3 | 10.45 | 3.12 | 3.03 ± 0.10 | 2.79 | 3.14 |

[a]The first column shows E normalized to sea level. The second column, "$\tau_{est}$," shows the empirical estimate of $\tau$ found from equation (3). The third column, "$\tau$," shows the mean values of $\tau$ calculated using the Monte Carlo in this work, with the standard deviation for the five runs at each field value shown as errors. The third column, "$\tau_B$," shows the values of $\tau$ calculated by ELIZA. The fifth column, "$\lambda_{est}$," shows the empirical estimate of $\lambda$ found from equation (2), and the sixth column shows the mean values of $\lambda$ calculated using the Monte Carlo in this work, the sixth column, "$\lambda_v$," shows the values of $\lambda$ estimated from ELIZA using $\lambda = 0.89c\tau$. For contrast, the last (sixth) column, "$\lambda_B$," shows estimates for $\lambda$ as reported in *Babich et al.* [2005], where the relation $\lambda = c\ \tau$ was used.

[16] The estimate for $\lambda$ in equation (2) agrees within the statistical variation of the current study to the estimate in equation (1) from *Dwyer* [2003]. Both this study and *Dwyer* [2003] used the same Monte Carlo model with only minor revisions since 2003. However, the technique used in this study to measure $\lambda$ and that used in *Dwyer* [2003] were different. In *Dwyer* [2003], $\lambda$ was calculated using two planes for each field strength considered: one a sufficient distance from the start of the avalanche to insure steady state behavior and another several estimated avalanche lengths further from the initiation. These two values were then used to calculate $\lambda$. As discussed previously, data from several hundred points were used in each fit to find $\lambda$ and $\tau$ in this study. Thus, both studies have arrived at approximately the same values for $\lambda$ using different techniques.

[17] The ratio of $\lambda$ to $\tau$ based on equations (1) and (2) is approximately 0.89$c$. As can be seen in Figure 1, 0.89$c$ is also the average speed of the average position of the runaway electrons in the avalanche. (Figure 1 compares the ratio of $\lambda$ to $\tau$ and the speed of the average position of the runaway electrons by showing a fit to the ratio of $\lambda$ to $\tau$ from the Monte Carlo data plotted over the speed calculations from the Monte Carlo data.) Thus, while most of the runaway electrons that make up the avalanche end up moving very close to the speed of light, the actual avalanche is propagating at less than 90% of the speed of light.

## 4. Discussion

[18] As shown in Table 1, the Monte Carlo models reviewed here compare very favorably for their estimates of $\lambda$ when 0.89$c$ is used instead of $c$ as the ratio between $\lambda$ and $\tau$. A 10% difference in the ratio of $\lambda$ to $\tau$ can create a significant error in estimating the number of runaway electrons. Because the number of electrons increases exponentially with distance, a 10% error in the estimate of $\lambda$ can result in a factor of $e$ difference in the estimated number of runaway electrons every 10$\lambda$.

[19] The disagreement between the models at lower field strengths is most likely caused by the differences in how they apply Coulomb scattering processes in the Monte Carlo simulation. At low electric field magnitudes, Coulomb scattering plays a very important role in the propagation of the avalanche. However, at fields close to the runaway threshold field $\lambda$ is estimated to be well over 100 m; in fact it rapidly approaches 1 km. In thunderstorms, the region where large electric fields exist is expected to be a few kilometers at most. Thus, at fields close to the runaway threshold field, only a few $e$-folding lengths can be achieved in thunderstorms; it is not expected that RRB will play a role in storm electrification or lightning initiation at field strengths close to the threshold electric field in real thunderstorms.

[20] At field strengths significantly larger than the threshold field (approximately 400 kV m$^{-1}$ normalized to sea level) over distances comparable to those seen in thunderstorms, *Dwyer* [2003] shows that runaway breakdown influences storm electrification by limiting the magnitude of the electric field attainable in air. At similar field magnitudes, *Dwyer* [2005] shows that runaway breakdown and the resultant feedback mechanisms can significantly augment the electric field created by the conventional charging mechanism of a thunderstorm.

[21] A speed of 0.89$c$ is slow compared with the average kinetic energy of the electrons, 7.2 MeV. However, the "speed" of the avalanche only considers the component of the electrons' velocities that is parallel to the direction of the avalanche. In the absence of a magnetic field, this is equivalent to the component of the electrons' velocities anti-parallel to the electric field driving the avalanche.

[22] In small electric fields, Coulomb scattering causes the electrons to "zig-zag." The electric field pushes the electrons to all travel in the same direction, but the Coulomb interactions cause the electrons to scatter at various angles. At smaller E magnitudes, a significant portion of the velocity of each electron is perpendicular to the electric field because of Coulomb scattering.

[23] In large electric fields, a different process causes the avalanche to propagate more slowly than $c$. When a new runaway electron is created through Møller scattering, it typically travels perpendicular to the old runaway electron that knocked it lose. Thus, newly created runaway electrons have little or no velocity in the direction of the avalanche at large fields. As the electric field gets larger, the ratio of $\lambda$ to the distance it takes for new electrons to accelerate to a significant velocity in the direction of the avalanche gets smaller, causing the average speed of the electrons in the direction of the avalanche to get lower.

[24] Also, larger magnitude electric fields allow lower energy electrons to run away. Thus, a larger number of lower energy electrons contribute to the avalanche and reduce the average speed of the avalanche when the electric field is very large.

[25] The speed of the avalanche has significant effects on the "beaming" of the electromagnetic radiation expected from RRB. Several studies [*Gurevich et al.*, 2002; *Roussel-Dupré and Gurevich*, 1996; and *Tierney et al.*, 2005] use the

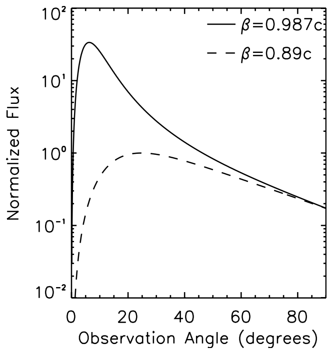


**Figure 2.** A comparison of the angular dependency of electromagnetic radiative flux as a function of observation angle for two avalanches traveling at different speeds: 0.987*c* (solid line) and 0.89*c* (dotted line). The results are shown logarithmically and normalized so that the maximum flux radiated by the 0.89c solution is equal to one.

one dimensional velocity of an avalanche to predict the radiation patterns from RRB. A one dimensional avalanche speed of 0.89*c* would greatly alter the beaming pattern in the electromagnetic radiation from RRB predicted by these previous models.

[26] To illustrate this point, Figure 2 shows the dependence of electromagnetic flux as a function of observation angle based on equation (5) of *Roussel-Dupré and Gurevich* [1996] and equation (11) of *Tierney et al.* [2005]. The maximum electromagnetic flux is plotted for two avalanche speeds: 0.987*c* and 0.89*c*, where 0.987*c* is the value used by *Tierney et al.* [2005] for the speed associated with an average kinetic energy of 7.2 MeV. The two data sets are normalized so that the maximum flux for the 0.89*c* avalanche is equal to unity. A thermal electron drift velocity of $5 \times 10^4$ m s$^{-1}$ is used, corresponding to a sea level electric field of 750 kV m$^{-1}$ [*Raether*, 1964].

[27] As shown in Figure 2, a slower speed for the avalanche means that less electromagnetic signal beaming is expected; relative to an avalanche moving at 0.987*c*, the electromagnetic flux from the slower avalanche is almost isotropic. Thus, a 10% difference in the speed of the electron avalanche will have implications for experimental work looking for such beaming; while some beaming almost certainly occurs as a result of RRB, it will not be as pronounced as expected in the studies mentioned in the previous paragraph.

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