7-2021

Scaling Flight Test Data Between a Scaled Aircraft and a Cessna 172 for Use in Trajectory Energy Model Validation

Cody William Nettleton

Scaling Flight Test Data Between a Scaled Aircraft and a Cessna 172 for Use in Trajectory Energy Model Validation

by

Cody William Nettleton

A thesis submitted to the College of Engineering and Science of
Florida Institute of Technology
in partial fulfillment of the requirements
for the degree of

Master of Science
in
Flight Test Engineering

Melbourne, Florida
July, 2021

We the undersigned committee hereby approve the attached thesis,
"Scaling Flight Test Data Between a Scaled Aircraft and a Cessna 172 for Use in Trajectory Energy Model Validation."
by
Cody William Nettleton

Brian Kish, Ph.D.
Associate Professor
Aerospace, Physics and Space Sciences
Major Advisor

Markus Wilde, Ph.D.
Associate Professor
Aerospace, Physics and Space Sciences
Committee Member

Isaac Silver, Ph.D.
Graduate Faculty
College of Aeronautics
Outside Member

David Fleming, Ph.D.
Professor and Department Head
Aerospace, Physics and Space Sciences

Abstract

Title: Scaling Flight Test Data Between a Scaled Aircraft and a Cessna 172 for Use in Trajectory Energy Model Validation

Author: Cody William Nettleton

Advisor: Dr. Brian Kish, Ph.D.

Currently in the general aviation sector, a new type of aircraft is rising to prominence. It is known as Urban Air Mobility (UAM) and takes many shapes and sizes. The goal of UAM is to produce aircraft that can be used to travel into and out of the city in an efficient manner through the sky. This may cut down on many people's commutes and lower the amount of ground traffic in cities. One core component of UAM research is to determine the fuel consumption during phases of flight such as conventional takeoff and landing, as well as cruise travel in a fixed wing configuration. Currently, energy technology such as batteries is lacking for full scale aircraft. While waiting for battery technology to improve, it is critical to develop a subscale model for the use of testing. This model will allow for testing to be accomplished while safety and technology reach an acceptable level for full scale flight. The purpose of this paper is to layout a model for scaling an RC aircraft to a general aviation vehicle. To meet this goal a Cessna 172 and AJ Slick 540 were used as the representative aircraft. Using the Froude Number and Reynolds Number approach the accuracy of using the AJ Slick 540 can be verified. From there a theoretical aircraft is designed using NASA's modeling aircraft techniques. This technique produced an aircraft that was similar to the AJ Slick 540. This paper verifies the concept that a trajectory can be scaled, and through using a properly scaled aircraft, produce scaled fuel usage.

Table of Contents

List of Figures

List of Tables

Nomenclature

σ = Standard Deviation, $\sqrt{\frac{\sum(x_i-\bar{x})^2}{N}}$

x_i = Sample from Set

$\bar{x}$ = Sample Mean, $\frac{\sum x_i}{N}$

N = Set Size

$\sigma_{\bar{x}}$ = Standard Error, $\frac{\sigma}{\sqrt{N}}$

δ = Percent Error, $\frac{\sigma_{\bar{x}}}{\bar{x}} * 100$, $\left|\frac{v_a-v_e}{v_e}\right| * 100$

$v_{a,e}$ = Actual Value, Expected Value

N_{FR} = Froude Number, $\frac{V^2}{lg}$

V = Velocity, (ft/sec)

l = Characteristic Dimension, (ft)

g = Acceleration of gravity, (ft/sec^2)

Re = Reynolds Number, $\left(\frac{\rho u l}{\mu}\right)$

ρ = Density of Fluid, (slugs/ft^3)

u = Linear Velocity, (ft/sec)

μ = Absolute Viscosity, (lb.sec/ft^2)

Acknowledgement

This project would not be possible without the support from the Federal Aviation Administration. Their funding and industry expertise has been pivotal in the research initiative for Urban Air Mobility at Florida Tech. They have coordinated conference calls with industry specialists for the benefit of this research.

Additionally, I would like to thank the pilots that have worked with me to acquire data for this project. Dr. Isaac Silver provided a Cessna 172 and flight time that was critical to the project. Additionally, John van Workum from the Aeromodelling Park provided the AJ Slick 540 and an open course in which to fly.

Finally, I would like to thank the students who helped make this all possible, Tahir Kanchwala, Hema Lata and Emils Senkans. I would like to extend a special thank you to Nolan Hopkins who helped with almost all the RC test flights.

Dr. Brian Kish, Dr. Markus Wilde, and Dr. Ralph Kimberlin all served as mentors and advisors throughout the research process. None of this would be possible without them.

Dedication

I would like to dedicate this paper to my mother. She has supported my decisions throughout my academic career and has pushed me to finish both my undergraduate and graduate degrees. I would not be where I am currently without her motivating me to do my best and strive for success in the aerospace industry.

Chapter 1
Motivation and Objectives

Motivation

The field of Urban Air Mobility (UAM) is currently a rapidly growing field in aviation. UAM aims at moving traffic in urban areas into the third dimension to ease congestion on other modes of transportation. The purpose of this sector is to produce vehicles that can traverse the urban landscape in an effective way that both saves money as well as being effective at its task. Many industries are approaching this aviation sector for different reasons, such as air lifting patients for hospitals, rooftop taxis, and package delivery. Each industry is approaching the problem differently and have produced a myriad of designs and prototypes. Many of these vehicles are electric Vertical Take Off and Landing (eVTOL) vehicles that are unique. While some vehicles are still closely modeled after conventional aircraft, many of the proposed styles include a tilt rotor configuration similar to the V22 Osprey. This configuration can be seen in Figure 1, images 1 and 3. Finally, a third configuration in which the lifting rotors are separate from the forward motion rotors is common. This configuration can be seen in Figure 1, images 2 and 4.

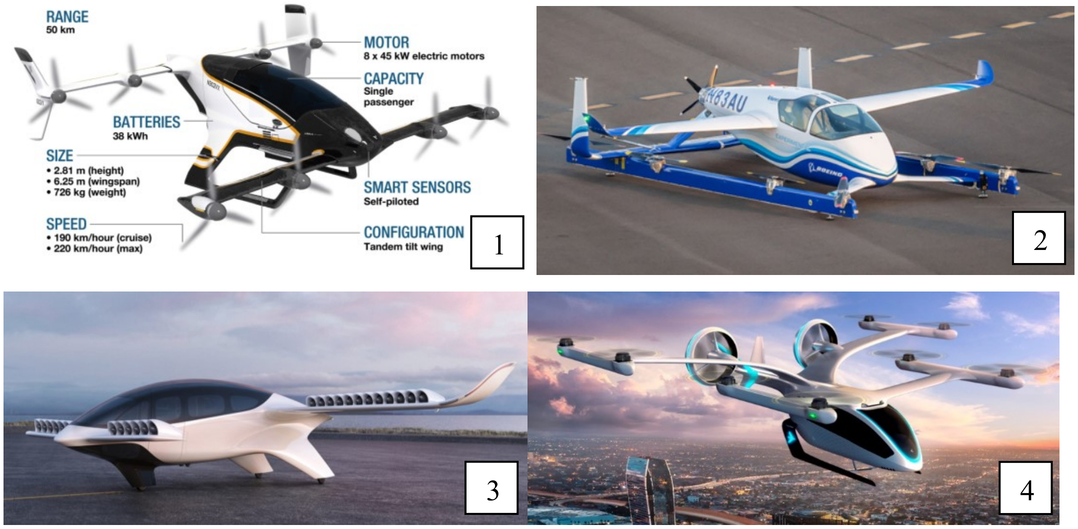

Figure 1 – 1: Airbus A^3 Vahana [1], 2: Aurora PAV [2], 3: Lilium Jet [3],

4: EmbraerX [4]

The FAA has been tasked with the certification of all of the vehicles in this sector and has produced a research program in the hopes to find an efficient way to certify these diverse aircraft. The motivation for this paper comes from a single problem within the growing industry of UAM. This paper will be focusing on the conventional takeoff and landing (CTOL) configuration. The study looks to identify the energy demands of the cruise segment as well as the takeoff and landing segments while in the conventional wing configuration. This is a main focus as many of the UAM routes will have travel outside of the urban areas to land at traditional airports. These segments will ideally be flown in CTOL configuration. To this end, a model for the cruise and airport interaction must be created and validated. The purpose of this study is to relate gas powered conventional aircraft to a scale model using scaling factors. Using this model, a scaled version of the aircraft could be produced. This will greatly lower the cost of testing by using scale model vehicles to determine baseline certification guidelines, that can be later tested and confirmed in larger vehicles. Greater detail of the objectives of this paper can be found in the following section.

Objectives

The objectives of this thesis can be narrowed down to four major goals. The first is to conduct flight tests with a combustion powered, fixed wing, FAR Part 23 Aircraft on a reference profile to determine the energy and power demands. To meet this goal, a Cessna 172 was flown along a GAMA Pub. 16 flight profile. The first and final test of the flight tests were flown as a destination trip (Figure 2) while all other tests were flown as a Local Training Flight (Figure 3). More detail in the flight test and analysis of this goal can be found in later sections.

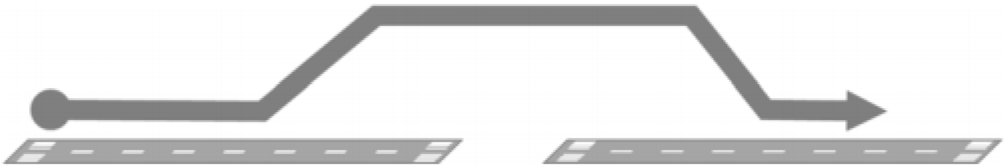

Figure 2 – Destination Flight Profile [5]

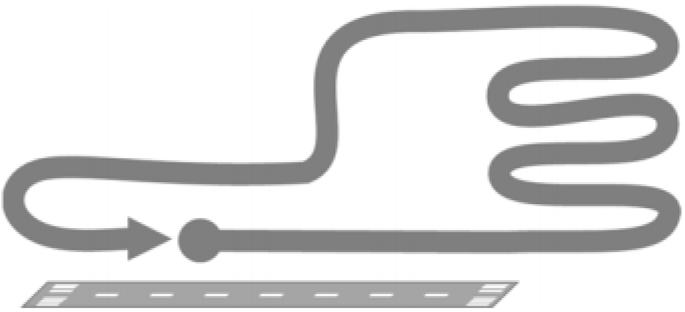

Figure 3 – Local Training Flight Profile [5]

Secondly, flight tests with a gasoline powered RC aircraft that has a CTOL configuration will be flown. This goal was accomplished using an AJ Slick 540. The RC Aircraft features a CTOL configuration and similar geometry to the Cessna 172. It fits the objective description. More detail in the flight test and analysis of this goal can be found in later sections. Third, scaling laws for trajectory power and energy will be developed. This will be accomplished using known scaling methods, which were then applied to the scale of an RC aircraft. Finally, the scaling laws will be investigated for use in the validation of trajectory power and energy models in the aircraft certification process.

Chapter 2
Test Articles and Locations

The purpose of this section is to give an overview of the two test articles, the Cessna 172 and the 103” AJ Slick 540 RC aircraft. Additionally, this section covers the area in which the test flights took place and any special conditions that were encountered due to the test location.

FAR Part 23 Aircraft - Cessna 172

To accomplish the first goal of this paper, a Cessna 172 was selected. It was selected as it is one of the most popular general aviation aircraft in the world. Its availability as well as its applicability to the study were the core reasons it was selected. The Cessna 172 Skyhawk general aviation aircraft with a four-seat cockpit (Figure 4). Relevant size parameters for the aircraft can be found in Table 1 below.

Table 1 – Cessna 172 Size Measurements [6]

Parameter	Measurement
Tip to Tail Length (feet)	27
Wingspan (feet)	36
Aspect Ratio	7.45
Mean Aerodynamic Chord (feet)	4.83
Max Takeoff Weight (pounds)	2550
Empty Weight (pounds)	1393
Fuel Quantity (gallons)	43
Fuel Type	100LL
Fuel Energy Density (BTU/Gal)	113341.09
Maximum Power (hp)	180
Power to Weight Ratio	0.1292

Figure 4 – Cessna 172

Another important feature of the test aircraft was the avionics suite that the pilot had installed. The Garmin system on board allowed for the logging of important values such as altitude, airspeed, and GPS position. This data could be exported to an SD card for later data analysis. Additionally, on board, the aircraft featured a fuel gauge that allowed for the instantaneous fuel rate to be tracked. This allowed for tracking of fuel rates for individual flight phases. This information was used for the calculation of energy spent for each phase later in this paper.

RC Aircraft - 103" AJ Slick 540

To meet the second objective of this paper, the AJ Slick 540 was used. This aircraft was chosen for its CTOL configuration as well as its dimensional similarity to the Cessna 172. One core difference in its shape is that it features a mid-wing instead of a high wing configuration. It is gas powered RC aircraft that measures about 8 feet in length. It features a two-stroke motor and weighs around 30 pounds. The gasoline used was a two-stroke gas mixture featuring a 40:1 gas to oil ratio. A picture of the aircraft (Figure 5) as well as a table of important size characteristics can be found below.

Figure 5 –Photograph of the AJ Slick 540

Table 2 – AJ Slick 540 Size Measurements [7]

Parameter	Measurement
Tip to Tail Length (feet)	8.33
Wingspan (feet)	8.58
Aspect Ratio	6.46
Mean Aerodynamic Chord (feet)	1.48
Max Takeoff Weight (pounds)	29.18
Empty Weight (pounds)	28
Fuel Quantity (gallons)	0.1875
Fuel Type	2 Stroke Gasoline 40:1 Mixture
Fuel Energy Density (BTU/Gal)	92029.09
Maximum Power (hp)	9.76
Power to Weight Ratio	0.3486

Test Areas

Cessna 172 Test Area – Valkaria Airport

Most of the flight testing took place on and around Valkaria Airport in Grant-Valkaria Florida. This airspace was selected for two major reasons. The first being that the Valkaria Airport is not towered. This means that no clearance to land, taxi and take off is required. This not only simplifies the procedure but also lowers the amount of time interfacing with the airport, lowering test time. Secondly the airport is used much less than Melbourne airport, minimizing risks and inconveniences during testing due to air traffic congestion. As seen in Figure 2, the test began and ended as a destination flight between Valkaria and Melbourne Airports. The remainder of the testing consisted of a series of touch and go maneuvers where the cruise portion of the flight took place over the Indian river. This portion of the testing could be modeled as a Local Training Flight (Figure 3). A general map of the area can be found in Figure 6 in the form of a sectional map.

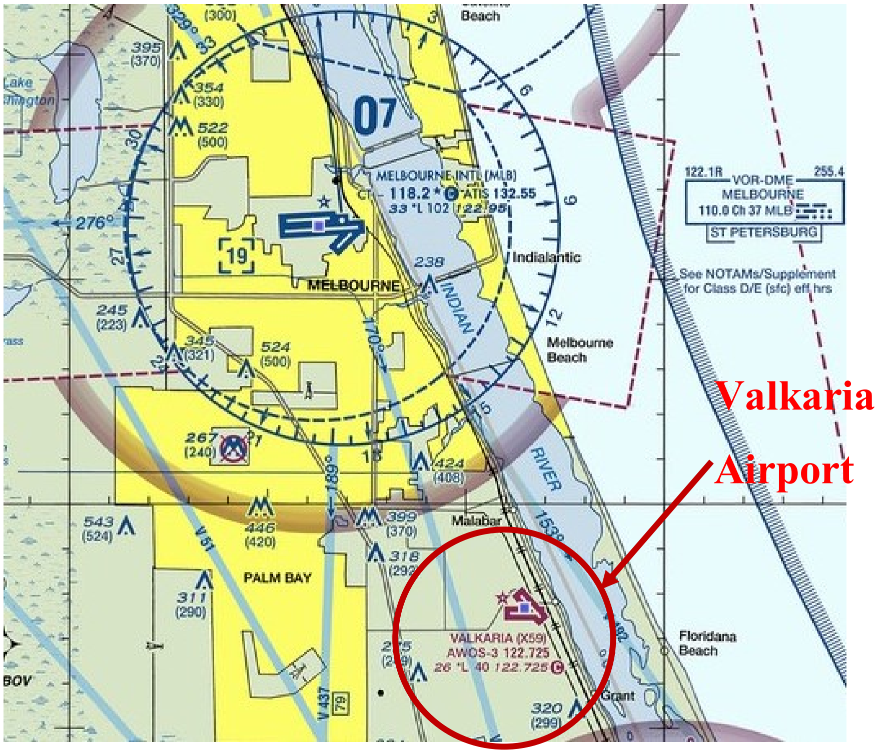

Figure 6 – Valkaria Airport Area Sectional Map [8]

A more detailed map of where the actual flying took place can be found below. This map was produced by taking the GPS produced during the flight by the Garmin avionics suite. Using these data points, a KML file was produced using a web client [9]. This KML file could be imported into Google Earth to display the position of the aircraft at any given point during the flight. A view of this 3D display can be found in Figure 7 below.

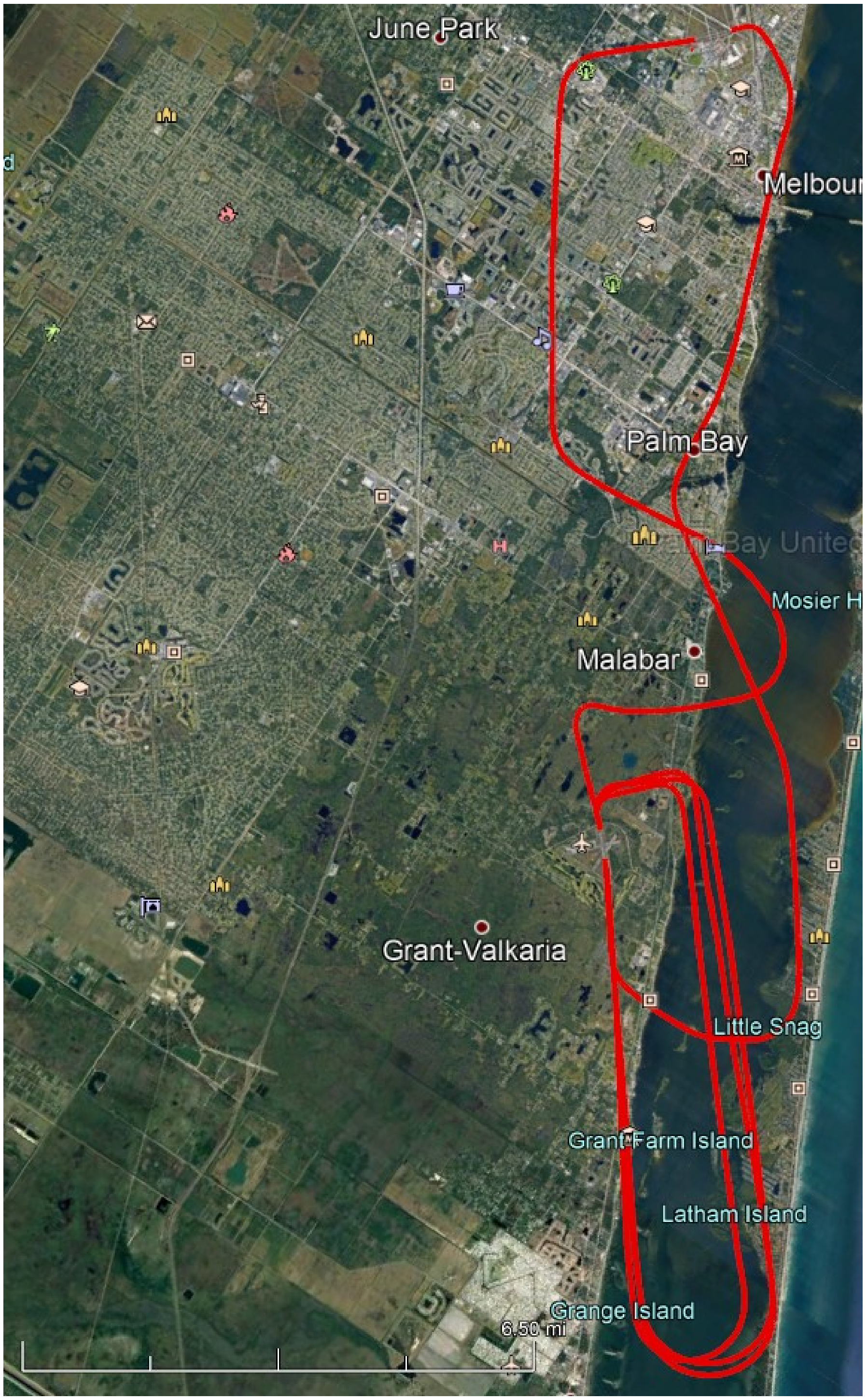

Figure 7 – GPS Tracking of Cessna 172 Flight

AJ Slick 540 Test Area – Space Coast Aeromodelling Park

The Space Coast Aeromodelling Park is an RC air course that is located within the property of the Brevard County Landfill (Figure 8). On each end of the air course there is a pole that marked the turnaround point. These poles are marked with a red X. When the point was reached, flag bearers near the poles signaled the pilot to turn around. Due to visibility constraints, the aircraft flew within line of sight.

Figure 8 – Satellite Image of the Aeromodelling Park

Chapter 3
Test Methodologies

The purpose of this section is to outline the testing methodology used during the flight tests of the two aircraft. Each aircraft methodology section is split into the flight profile and the data collection.

Cessna 172 Test Flight Methodology

Flight Profile

The Cessna 172 was to be flown a profile of five legs that each constituted as their own test points. Test points 1 and 5 would be similar to a Destination Flight Profile (Figure 2). This profile was chosen as the aircraft was stationed at Melbourne Airport, but would be conducting most of this flight test in the Valkaria Airport region. The aircraft was set to steadily climb to 2000 feet and then descend into the Valkaria Airport. The reverse of this procedure was conducted as the final flight test to head home to Melbourne Airport.

For the remaining 3 test points, a Local Training Flight profile was conducted (Figure 3). For these flights, the aircraft would taxi to the runway, take off and climb to an altitude of 2000 ft while covering 5 nautical miles. At this time, the aircraft would turn around over the Indialantic River and begin its descent. Much like the climb, this descent would last 5 nautical miles. The aircraft would then land and immediately begin to taxi to start the next test point.

Data Collection

The data was collected using a Garmin system. The system recorded many fundamental measurements such as altitude, airspeed, and GPS position. Additionally, there was a gauge that was not tied into the flight computer that measured both the remaining fuel as well as the fuel flow rate. This flow rate proved useful for power analysis at different stages of the flight. Fuel flow rate was recorded by hand at the start of flight phases (Table 3). The time of phase start was also annotated. The Garmin data was exported using an on-board SD card.

Table 3 – Cessna 172 Fuel Flow Data

Profile	Phase	Phase Duration (s)	Fuel Flow (gal/hr.)	OAT (˚F)
1	Takeoff/Climb	230	15.5	76
	Cruise	95	5.6	76
	Descent	325	2.8	76
2	Takeoff/Climb	200	14.6	76
	Cruise	100	7.5	76
	Descent			76
3	Takeoff/Climb	230	15.2	77
	Cruise	70	8	77
	Descent	355	4	77
4	Takeoff/Climb	300	14.2	77
	Cruise			77
	Descent	365	4	77
5	Takeoff/Climb	170	15.2	76
	Cruise			76
	Descent			76

Fuel flow data matches well when compared against literature data in the Cessna 172 Pilot handbook where a standard cruise consumes 6-8 gallons per hour of fuel. Additionally, when climbing to 2000 feet a pilot should expect to lose 0.6 gallons of fuel, which is consistent with the fuel flows displayed above.

AJ Slick 540 Methodology

Flight Profile

The general profile for this aircraft was to run laps of the field until the required number of laps had been run. Once the aircraft was started, the pilot aimed to minimize taxi and take off time. The aircraft got to 100 feet as quickly as possible as it headed to one end of the field. Flag poles were stationed at each end of the course and marked the turnaround points

(Figure 8). When the aircraft passed the flagpole, a crew member stationed at the flagpole would wave their signal flag for the pilot to turn around and head to the other flagpole. Each test point consisted of an increasing number of laps of 7, 13, and 19 passes, respectively. A lap was defined as the length of the field from flag to flag. This was recorded as a lap when the aircraft passed the pilot on the runway. The aircraft was then promptly landed.

Data Collection

When the aircraft is turned on, some fuel exits the system through the exhaust. This is a normal occurrence for this type of engine. However, this loss should not be included in the energy expenditure analysis. Due to this, for each test point the aircraft was fully fueled. A catch basin was placed under the exhaust to catch any ejected fuel. This fuel was then added back to the fuel storage and measurement device. The starting fuel in the fill bottle was then measured. The pilot taxied and took off promptly. Flag bearers stood at the ends of the flight area and signaled for the pilot when to turn. The pilot flew an increasing amount of laps each test to provide a variation in results. Following landing, the aircraft was fueled to maximum. The change in fuel from the entire flight course was calculated and tabulated. Time was taken as a total flight time as well as by lap basis to investigate variation in laps. The results of this flight test can be found in Table 4 below.

Table 4 – AJ Slick 540 Test Results

Test Number	Fuel Spent (gal)	Time (s)	Laps	Fuel Flow (gal/hr.)	Time Per Lap (s)	Speed (mph)
1	0.00321	127	7	0.09093	18.14	64.49
2	0.00321	259	13	0.04463	19.92	58.72
3	0.00336	370	19	0.03274	19.47	60.08
Averages				0.05610	19.18	61.10
Standard Deviation				0.03075	0.93	3.01
Percent Error				31.64%	2.79%	2.85%

The error in the table above was calculated using the standard deviation and standard error approach. The standard deviation of the set was taken (Equation 1). This allowed for the determination of the standard error and percent error of the average (Equations 2 and 3).

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{N}} \tag{1}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} \tag{2}$$

$$\delta = \frac{\sigma_{\bar{x}}}{\bar{x}} * 100 \tag{3}$$

The average lap duration and speed featured acceptable errors. The error in fuel flow is acceptable for the use of this paper but should be investigated further using additional flight tests to verify the result.

Chapter 4
Flight Test Data Reduction

Cessna 172 Reduction Methods

The data for the Cessna 172 came as a bulk data dump from an SD card. Figure 9 shows altitude versus time. This graph showed a saw tooth pattern.

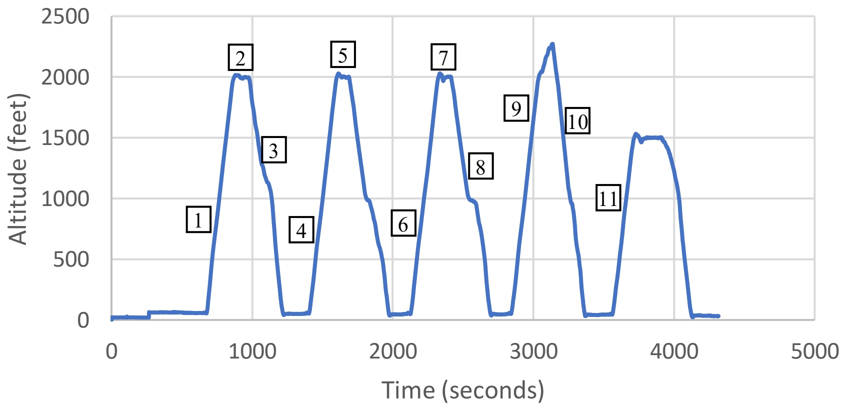

Figure 9 – Cessna 172 Altitude vs. Time Graph

Indicator	Fuel Flow (gal/hr.)
1	15.5
2	5.6
3	2.8
4	14.6
5	7.5
6	15.2
7	8
8	4
9	14.2
10	4
11	15.2

Taking each test point as a single tooth in the pattern, the data could be split into 5 test flights. During the flight, the fuel flow per phase was recorded by hand on an external gauge in the aircraft. This experimental data matches the expected data in the Cessna 172 Pilots Handbook. Using this information, the energy spent per phase could be identified using known energy densities of 100L Avgas, 31.59 MJ/L [14].

RC Aircraft Reduction Methods

The data recorded for this aircraft was fuel expenditure per test flight and the duration of each flight as well as the number of laps flown. The RC aircraft did not feature any onboard data recording software, so this data was modeled as a singular phase. Using the fuel, time and number of laps, the fuel per lap and time per lap could be calculated. Using this information, the distance flown could be scaled to any scaling factor. Additional information required for the model creation was the size and weight of the aircraft.

Model Definition

Using NASA's Modeling Flight scaling guide, an aircraft can be scaled using a scaling factor and an exponential value [10]. The primary scaling factor chosen in this paper is the wingspan of the aircraft, as that is what was used in the guide. Found below are the recommended scaling parameters from the Modeling Flight scaling guide.

Table 5 – Scaling Factor Guide [10]

Property	Scaling Factor
Wingspan	N
Length	N
Wing Area	N^2
Aspect Ratio	1
Chord Length	N
Empty Weight	N^3
Max Takeoff Weight	N^3
Max Power	$N^{3.5}$
Total Fuel Capacity	N^3
Reynolds Number	$N^{1.5}$

Using the Cessna 172 and AJ Slick 540 sizing tabulated in the test article description section, a scaling factor for the wingspan could be determined. This value was found to be 4.19. Using this value, a theoretical scaled aircraft of the Cessna 172 was produced. Using the percent error equation, the error between the AJ Slick 540 and the model aircraft could be determined. Additionally, using the scaling factor for fuel usage, the energy expenditure for the test flights could be found.

Using dimensional analysis, additional parameters can be used as scaling factors. Additional scaling factors were applied to find the best fit for this model. They can all be found in the appendix below. The scaling factors must be modified to format the model for that parameter. The required modifications are in Table 6. The scaling factor is defined as the ratio of the properties, with the subscript denoting which property it is part of. This scaling factor is then modified and then replaces the scaling factor in the above model.

Table 6 – Weight Based Scaling Factors

Property	Scaling Factor
Wingspan	$N=N_{ws}$
Length	$N=N_{L}$
Wing Area	$N=N_{wa}^{1/2}$
Aspect Ratio	1
Chord Length	$N=N_{cl}$
Empty Weight	$N=N_{ew}^{1/3}$
Max Takeoff Weight	$N=N_{mw}^{1/3}$
Max Power	$N=N_{mp}^{1/3.5}$
Total Fuel Capacity	$N=N_{mf}^{1/3}$
Reynolds Number	$N=N_{re}^{1/1.5}$

Chapter 5
Flight Data Analysis

Comparing the Aircraft

The cornerstone of this paper is the implication that the two aircraft being tested can be related to one another. This relation can be tested by using a method known as the Froude Number comparison. The Froude Number is an expression of the inertial and gravitational effects on an aircraft. If the Froude Number is identical, it is said that the aircraft have geometric similitude. The equation for the Froude Number can be found below (Equation 4).

$$N_{FR} = \frac{V^2}{lg} \tag{4}$$

When calculating the Froude Number for both the aircraft tested, the maximum velocity of the aircraft was used. It is assumed that the maximum velocity of the AJ Slick 540 was the fastest speed achieved during the test flights. The results of the Froude Number Calculation can be found tabulated below.

Table 7 – Froude Number Analysis

Aircraft	Froude Number	Percent Error
Cessna 172	25561.35	
AJ Slick 540	30105.91	16.33%

The percent error was determined using the standard percent error formula (Equation 5).

$$\delta = \left|\frac{v_a - v_e}{v_e}\right| * 100 \tag{5}$$

Considering that the AJ Slick 540 was not intentionally made to model the Cessna, this error is acceptable. In the development of a new prototype, the model would be expected to have an identical Froude Number during the design phase. This difference in Froude Number helps explain deviations in other data types throughout the analysis section.

Additionally, a Reynolds Number analysis was conducted to investigate the inertial differences in the aircraft. The Reynolds number is a nondimensional value used to compare the inertial and viscous forces on an aircraft. In the creation of a model aircraft, similitude should be present in the Reynolds numbers. This ensures that the aircraft is inertially similar. Displayed below is the equation for the Reynolds Number (Equation 6). It is a simple ratio between the density, speed, and characteristic length of the aircraft to the kinematic viscosity of the air.

$$Re = \frac{\rho u L}{\mu} \tag{6}$$

The Reynolds numbers for both aircraft tested are tabulated below (Table 8). The Reynolds Number must be scaled based on the model in use. As an example, the NASA Modeling Aircraft recommendation for scaling Reynolds Numbers using the wingspan can be found in Table 8.

Table 8 – Reynolds Number Analysis

Aircraft	Reynolds Number	Percent Error
Cessna 172	1189504755	
AJ Slick 540	157511128	
Scaled Aircraft	203961720.7	22.77%

Cessna 172 Analysis

The first chart displayed in this section shows the average fuel flow during each flight phase during each flight test of the Cessna 172 (Figure 10).

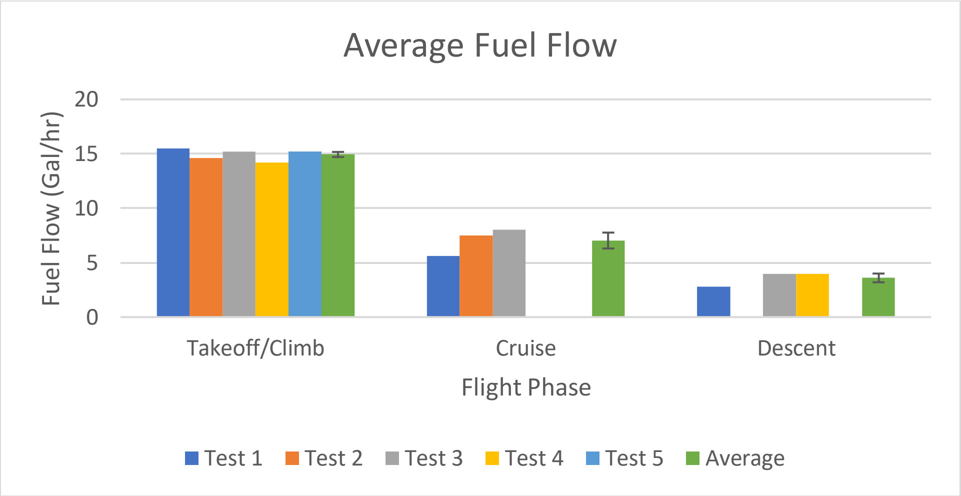

Figure 10 – Average Fuel Flow for Cessna 172 Flight Tests

Data loss occurred during two of the cruise and landing portions of the flight testing. This was primarily due error by data recorder and hand recorded notes. However due to the similarity between tests, the average fuel flow was used in place of the missing values. This is justified by the accuracy of the autopilot on board the aircraft. The flight profile as described in the prior sections was followed almost exactly for each flight test. This was due to the onboard autopilot handling most of the pathing related to the flight. Below, charts of the altitude (Figure 11) and airspeed (Figure 12) can be found for test point comparison.

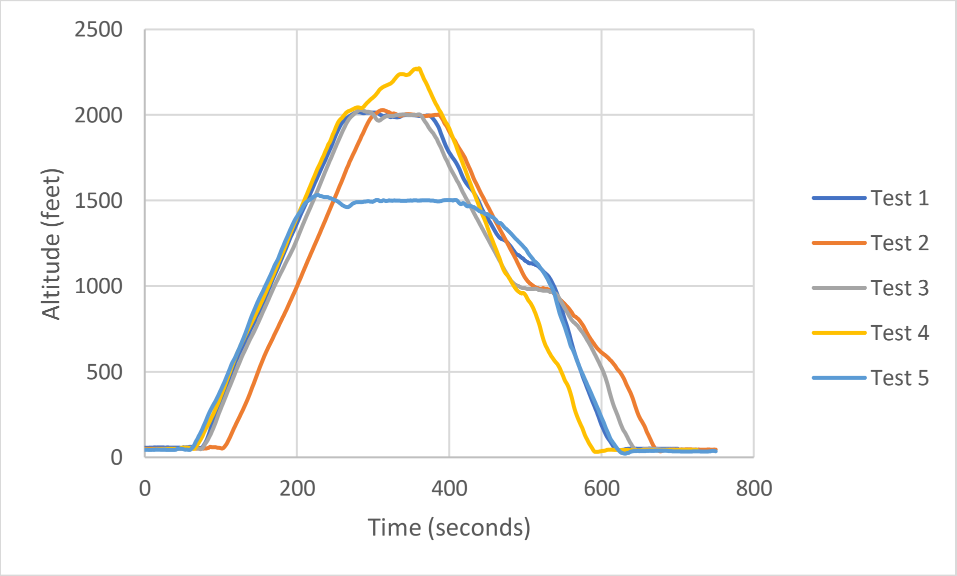

Figure 11 – Cessna 172 Altitude Profiles

As seen in Figure 11 above, test 5 had a much lower maximum altitude. This was due to test 5 being the return home flight from Valkaria to Melbourne airport. Despite this, test 5 still featured a similar airspeed profile as the other tests as seen in Figure 12 below.

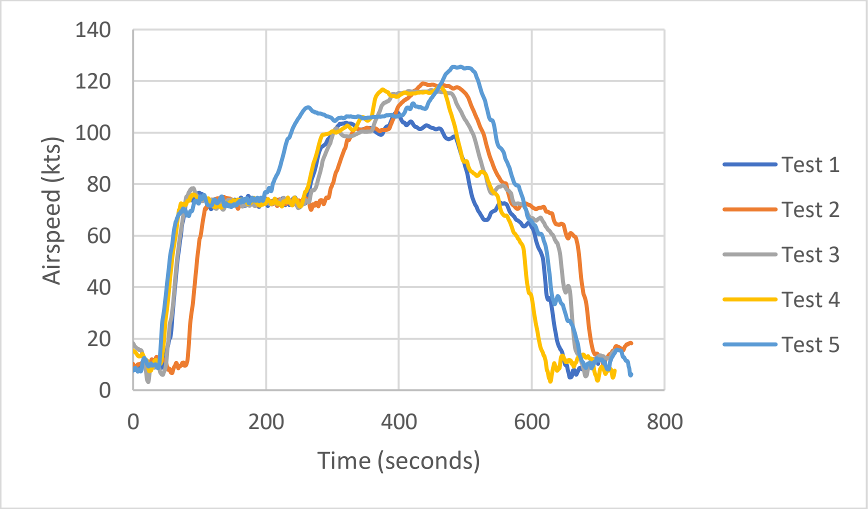

Figure 12 - Cessna 172 Airspeed Profiles

By assuming the fuel spent during a phase was entirely converted to energy without losses, the energy density of 100LL Avgas could be multiplied by the volume of the fuel expended in each phase. Taking phase duration into account, the maximum possible power of the fuel expended can be calculated. Using the Cessna 172 pilot's handbook, the actual engine power used during the maneuver can be found. Comparing these two values, a general sense of engine efficiency can be determined (Figure 13).

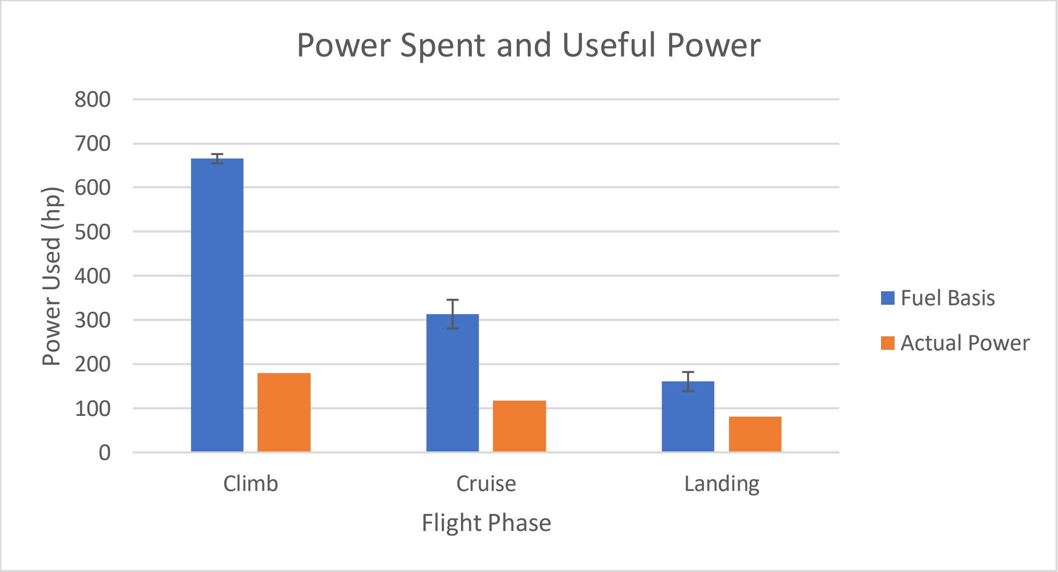

Figure 13 – Power Usage for Cessna 172 Flight

The energy spent during climb vastly out paces the power the engine can produce. This phase both requires the most energy and is by far the least efficient. Landing proved the most efficient phase of the three.

Issues Encountered

Most issues occurred due to loss of data in flight and uncontrollable flight occurrences. Loss of data occurred due to the short nature of the cruise period in flight. Despite this, the autopilot controlled the flight path, leading to consistent tests. The cruise fuel flow data is supported by the pilot handbook published values. Finally, one test was cut short due to aircraft impeding testing space. This can be seen in the map images earlier in this paper. The variation is not enough to cause concern.

AJ Slick 540 Analysis

Three test flights were recorded for the AJ Slick 540 flight test. During each test flight the fuel expenditure, time and laps flown were recorded. An altitude chart was made for the flight tests to better portray the flight profile flown (Figure 14).

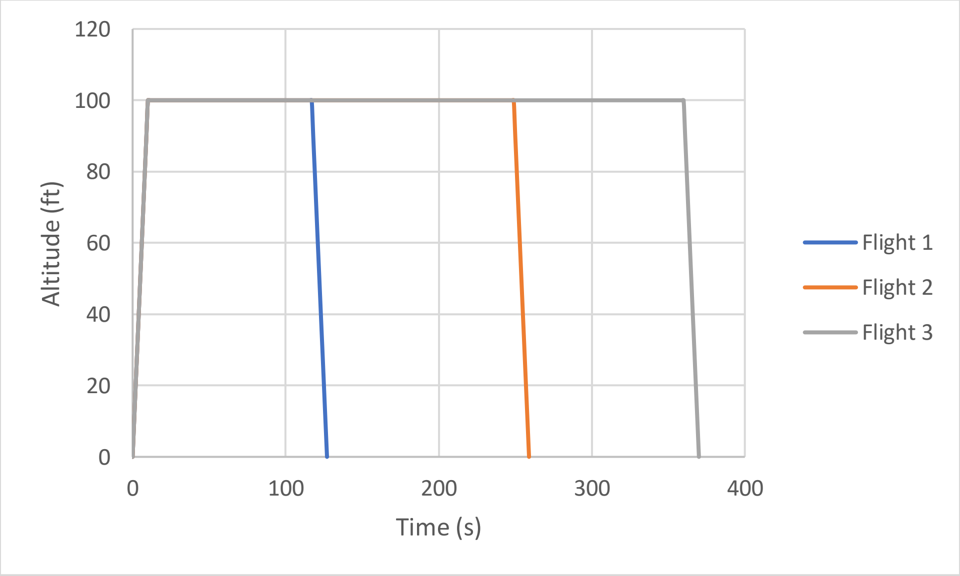

Figure 14 – Altitude Chart for AJ Slick 540 Flights

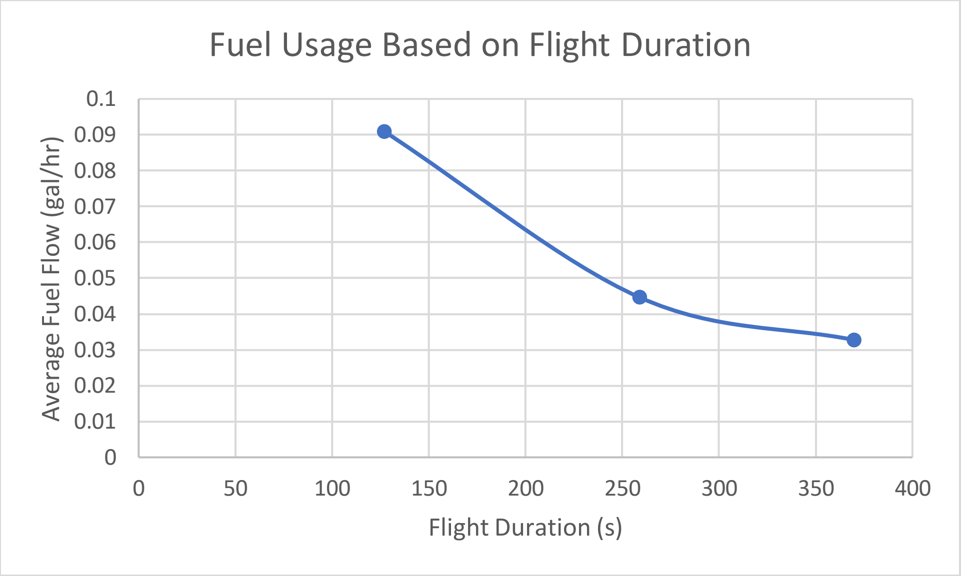

Figure 15 – Fuel Usage for AJ Slick 540 Flights

As the duration of the flight increased, the average fuel flow for the flight drastically decreased. This is potentially because of taxiing and climbing to altitude decreasing as flight duration increased.

Since a different number of laps were flown during each flight test, the fuel expenditure and time per lap were calculated. This allows for the estimation of the values of a test flight that can be reasonably scaled up or down to fit the model.

Issues Encountered

Many of the issues incurred with this test flight were due to the lack of communication prior to test day. This led to some improvisation at the airfield. Understaffing led to unique problems but was promptly solved by having test crew members assume multiple roles in the test flight. Despite this, consistent results came out of the testing. Other issues stem from the limitations of the RC aircraft. The aircraft featured no on-board avionics. The pilot kept the aircraft within line of sight for the duration of the flight. Additionally, energy analysis could not be completed for different phases of flight, and the entire flight was modeled as a cruise duration.

Model Analysis

Using the scaling factor guide in the section above, the dimensions for a theoretical aircraft can be produced. In the table below, the dimensions for the Cessna 172, AJ Slick 540 and theoretical aircraft can be found. Additionally, an error between the theoretical scaled aircraft and the AJ Slick 540 was produced to determine the authenticity of the model.

Table 9 – Scaled Aircraft in Terms of Wingspan

Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	8.58	0	N
Fuselage Length (ft.)	27	8.33	6.43	29.45	N
Wing Area (ft.^{2})	174	11.40	9.89	15.27	N^2
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.15	28.36	N^3
Empty Weight (lbs.)	1393	28	18.88	48.30	N^3
Max Takeoff Weight (lbs.)	2550	29.18	34.56	-15.57	N^3
Max Power(hp)	180	9.76	1.19	719.32	$N^{3.5}$
Max Power to Weight ratio	0.129	0.35	0.034	911.34	
Total Fuel Capacity (Gal.)	43	0.1875	0.5828	-67.83	N^3
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.092	-39.13	N^3
Cruise Power Req (hp)	313.26	2.029	2.073	-2.14	$N^{3.5}$
Reynolds Number	$1.19\text{x}10^9$	$1.58\text{x}10^8$	$1.38\text{x}10^8$	13.74	$N^{1.5}$

By scaling only the cruise power required, which is defined as the theoretical maximum amount of energy provided by the fuel burned, an error of 2.14% is achieved. A chart displaying each model scaling factor type can be found in Figure 16. The red line denotes the power burned in the flight of the AJ slick 540.

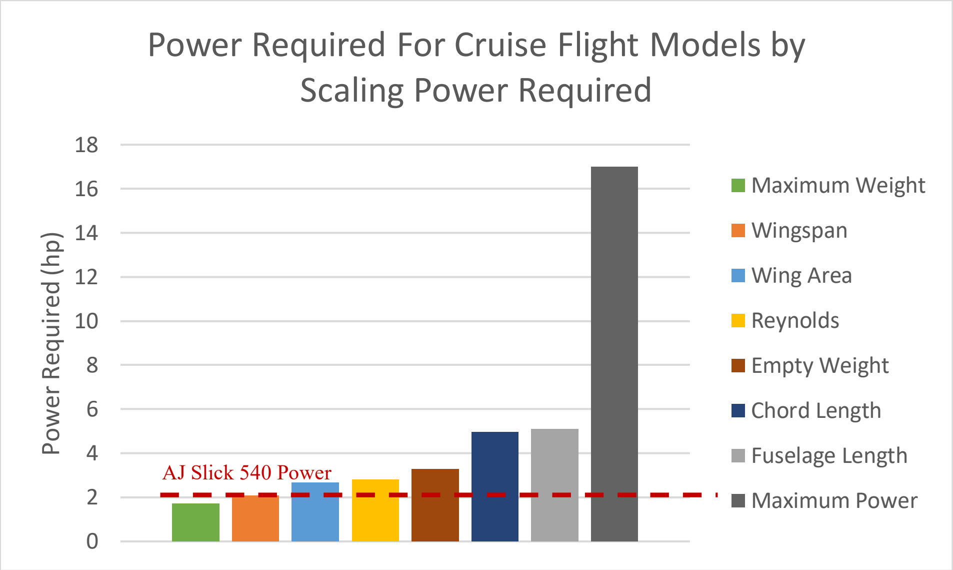

Figure 16 – Power Required for Cruise Based on Scaled Power

From this display, the maximum weight scaling factor and the wingspan scaling factor are the most reliable.

By approaching the model from a different direction by using a scaling factor for time flown and the fuel volume, a general model for energy expenditure could be determined for the model and be compared against the Cessna 172 flight test. The models theoretical flight phases and the Cessna 172's actual flight phases can be found below in Table 10.

Table 10 – Model Energy Expended

Aircraft	Phase	Phase Duration (s)	Fuel Used (gal)	Energy Density (BTU/gal)	Power Required (hp)
Cessna 172	Takeoff/Climb	226	0.94	113341.09	665.43
	Cruise	88.33	0.17		313.26
	Landing	323.75	0.32		160.34
AJ 540	Cruise	140.51	0.0022	92029.09	2.02
Model Aircraft	Climb	53.81	0.0030	92029.09	7.29
	Cruise	21.03	0.0006		3.43
	Landing	77.08	0.0010		1.76
Percent Error	Cruise				-40.91

By producing a model of this type a similar trend is apparent. The Maximum weight and wingspan models are the most accurate. The error for this model is significant and possibly unacceptable. The larger error in this most likely due to the splitting of the model into flight phases, a liberty that is not afforded to the AJ Slick 540 data. Once again, the red line on the chart displays the power in the fuel burned by the AJ Slick 540.

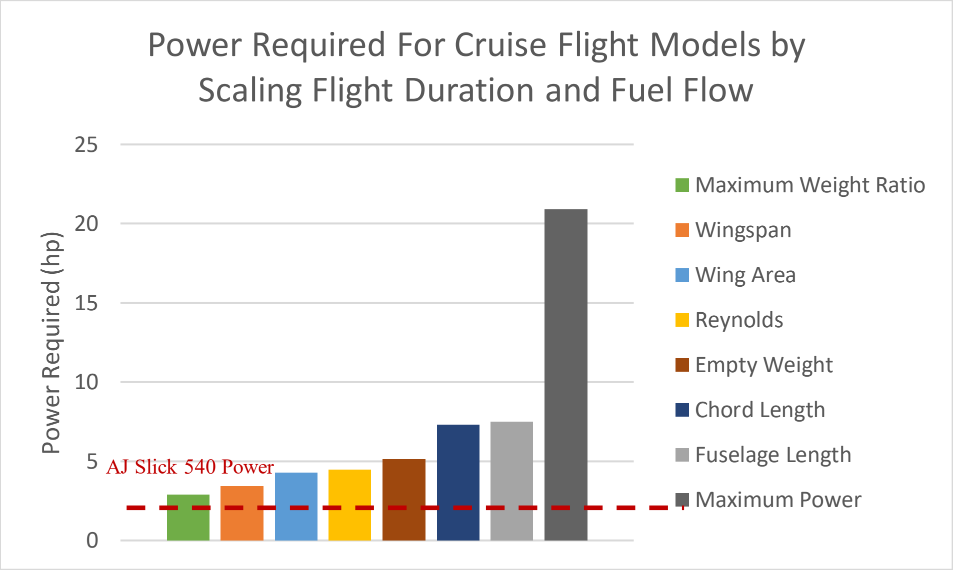

Figure 17 – Power Required for Cruise Flight Based on Scaled Flight Duration and Fuel Flow

Issues Encountered

One major issue involving many of the models occurs when scaling the power to weight ratio. The engine used for the AJ Slick 540 provided a 0.349 power to weight ratio when the expected tends to be much lower. This is most likely due to the large 120cc engine that was installed on the aircraft. Regularly available engines go down to as low as 35cc engines. Additional tests with this type of engine installed could prove useful. Despite this issue, the wingspan and maximum weight of the AJ Slick 540 still pair well with the model.

Chapter 6
Lessons Learned and Recommendations

One interesting lesson regarding the scaling law was the idea that a linear dimension such as the wingspan of the aircraft could be applied to the weight or power of the aircraft. Additionally, using the Froude Number and Reynolds Number analysis to perform an initial similarity check on the aircraft tested was interesting. The geometric and aerodynamic similarity between the Cessna 172 and AJ Slick 540 was very close.

Issues arose in some of the flight tests in which a clear communication disconnect occurred between the pilots and the data recorders. This led to confusions during data collection as well as the actual circuit being flown. This kind of communicative error could be removed by having test cards that have been reviewed by both the pilot and data recorders. Before each test, a comprehensive test plan should be written, reviewed, and discussed with all parties present. Additionally, a preflight debrief as well as actual practice runs before the flight could open time for discussions, questions and any problems that could occur during the flight be found before the test. Additionally, during the test, alongside the flight card, the test coordinator or pilot should call out data points in a clear and concise manner for the best data recording experience.

Going forward the Cessna 172 flight should be flown again with clear callouts regarding each flight phase. Additionally, more care should be given to the flight phase definitions such as the cruise phase which was tied into the climb and descent phases. Secondly, should an aircraft such as the AJ Slick 540 be flown again it should be flown with on board avionics. This will give better indications for both speed and altitude as well as possibly fuel consumption. AJ Slick 540 should also be equipped with a smaller engine and more fuel to better compare to the Cessna 172 in scale. The difference in fuel flow for the small engine would be of interest.

Chapter 7
Conclusion

Flight tests were conducted with the Cessna 172 to simulate the power and energy demands of a Part 23 aircraft on a GAMA Pub 16 profile. These flights were all done with an autopilot system to ensure similarity between test flights. Similarly, an AJ Slick 540 was flown to represent the flight of a scaled gas-powered RC aircraft. The aircraft featured Froude Number and Reynolds Number similarity. By choosing the wingspan as the scaling dimension of the aircraft, a model theoretical scaled aircraft was produced. This was done by using a method from a NASA Modeling Aircraft publication. The scaling model featured similar geometric properties to the AJ Slick 540 as well as similar energy expenditure. Large errors arose when considering the maximum fuel capacity as well as the power to weight ratio of the engine. A variety of other models could be created using a similar process, while scaling different properties, such as maximum takeoff weight. These models could be used to produce scale aircraft or verify the results of an energy analysis using a scaled aircraft. This would allow for the testing phase to begin on UAM style aircraft before the battery technology is available for full scale aircraft.

References

[1] “Vahana,” *Airbus* Available: https://www.airbus.com/innovation/zero-emission/urban-air-mobility/vahana.html.

[2] “PAV – PASSENGER AIR VEHICLE,” *Aurora Flight Sciences* Available: https://www.aurora.aero/pav-evtol-passenger-air-vehicle/.

[3] “Lilium Air Mobility,” *Lilium* Available: https://lilium.com/.

[4] *Portal Embraer* Available: https://embraerx.embraer.com/global/en/media.

[5] General Aviation Manufacturers Association, “GAMA PUBLICATION NO. 16 HYBRID & ELECTRIC PROPULSION PERFORMANCE MEASUREMENT” Available

[6] “1978 Cessna 172 Pilot's Operating Handbook” Available: https://takewingaviation.com/wpcontent/uploads/2017/02/N739EF_172N_POH.pdf.

[7] Pierce, D., Scharping, N., Cruz, N., Schmidt, R., and Dobies, M., “AJA AJS_103-00-103’ AJ Slick 540 - Generation 1.2,” *AJ Aircraft* Available: https://www.aj-aircraft.com/103-AJ-Slick-540--Generation-12_p_462.html.

[8] “Flight Planning / Aeronautical Charts,” *SkyVector* Available: https://skyvector.com/.

[9] “Convert your GPS data for use in Google Earth,” *GPS Visualizer* Available: https://www.gpsvisualizer.com/map_input?form=googleearth.

[10] Chambers, J. R., *Modeling flight: the role of dynamically scaled free-flight models in support of NASA's aerospace programs*, Washington, D.C.: National Aeronautics and Space Administration, 2010.

[11] Wolowicz, C. H., Bowman, J. S., and Gilbert, W. P., “Similitude Requirements and Scaling Relationships as Applied to Model Testing,” *NASA Scientific and Technical Information Branch*, Aug. 1979.

[12] Senkans, E., “Flight Tests of Trajectory Energy Management Systems Using a Vertical Takeoff and Landing Vehicle,”

[13] Hoffmeister, M., “Desert Aircraft ‘DA-120,’” *RC Flyer News* Available: http://www.rcflyernews.com/wp-content/uploads/2012/01/DA120_Engine_Test.pdf.

[14] “Avgas 100 ll,” *Warter Aviation* Available: https://www.warteraviation.com/fuels/avgas-100-ll/.

Appendix

Scaled to Wingspan					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	8.58	0	N
Fuselage Length (ft.)	27	8.33	6.43	29.45	N
Wing Area (ft.^{2})	174	11.40	9.89	15.27	N^2
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.15	28.36	N
Empty Weight (lbs.)	1393	28	18.88	48.30	N^3
Max Takeoff Weight (lbs.)	2550	29.18	34.56	-15.57	N^3
Max Power(hp)	180	9.76	1.19	719.32	$N^{3.5}$
Max Power to Weight ratio	0.129	0.349	0.034	911.34	
Total Fuel Capacity (Gal.)	43	0.1875	0.5828	-67.83	N^3
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.092	-39.13	N^3
Cruise Power Req (hp)	313.26	2.029	2.073	-2.14	$N^{3.5}$
Reynolds Number	1.19×10^9	1.58×10^8	1.38×10^8	13.74	$N^{1.5}$

Scaled to Fuselage Length					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	11.11	-22.75	N
Fuselage Length (ft.)	27	8.33	8.33	0	N
Wing Area (ft.^{2})	174	11.40	16.58	-31.21	N^2
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.49	-0.84	N
Empty Weight (lbs.)	1393	28	40.96	-31.63	N^3
Max Takeoff Weight (lbs.)	2550	29.18125	74.97	-61.078	N^3
Max Power(hp)	180	9.76	2.94	231.97	N$^{3.5}$
Max Power to Weight ratio	0.129	0.349	0.039	788.89	
Total Fuel Capacity (Gal.)	43	0.1875	1.26	-85.17	N^3
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.20	-71.94	N^3
Cruise Power Req (hp)	313.26	2.029	5.12	-60.35	N$^{3.5}$
Reynolds Number	1.19 x10^9	1.58x10^8	2.04x10^8	-22.77	N$^{1.5}$

Scaled to Wing Area					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	9.22	-6.86	$N^{1/2}$
Fuselage Length (ft.)	27	8.33	6.91	20.57	$N^{1/2}$
Wing Area (ft.^{2})	174	11.40	11.40	0	N
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.24	19.55	$N^{1/2}$
Empty Weight (lbs.)	1393	28	23.37	19.83	$N^{3/2}$
Max Takeoff Weight (lbs.)	2550	29.18	42.77	-31.78	$N^{3/2}$
Max Power(hp)	180	9.76	1.53	538.92	$N^{3.5/2}$
Max Power to Weight ratio	0.129	0.349	0.0357	876.04	
Total Fuel Capacity (Gal.)	43	0.1875	0.72	-74.00	$N^{3/2}$
Cruise Fuel Flow (Gal/Hr.)	6.8	0.056	0.11	-50.82	$N^{3/2}$
Cruise Power Req (hp)	313.26	2.029	2.66	-23.69	$N^{3.5/2}$
Reynolds Number	1.19 x10^{9}	1.58x10^{8}	1.54x10^{8}	2.24	$N^{1.5/2}$

Scaled to Chord Length					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	11.02	-22.09	N
Fuselage Length (ft.)	27	8.33	8.27	0.85	N
Wing Area (ft.^{2})	174	11.40	16.30	-30.03	N^2
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.48	0	N
Empty Weight (lbs.)	1393	28	39.93	-29.87	N^3
Max Takeoff Weight (lbs.)	2550	29.18	73.09	-60.07	N^3
Max Power(hp)	180	9.76	2.85	241.98	$N^{3.5}$
Max Power to Weight ratio	0.129	0.349	0.039	792.67	
Total Fuel Capacity (Gal.)	43	0.1875	1.23	-84.79	N^3
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.195	-71.22	N^3
Cruise Power Req (hp)	313.26	2.029	4.97	-59.16	$N^{3.5}$
Reynolds Number	1.19 x10^9	1.58x10^8	2.01x10^8	-21.79	$N^{1.5}$

Scaled to Empty Weight					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	9.79	-12.31	$N^{1/3}$
Fuselage Length (ft.)	27	8.33	7.34	13.51	$N^{1/3}$
Wing Area (ft.^{2})	174	11.40	12.86	-11.36	$N^{2/3}$
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.31	12.56	$N^{1/3}$
Empty Weight (lbs.)	1393	28	28	0	N
Max Takeoff Weight (lbs.)	2550	29.18	51.26	-43.07	N
Max Power(hp)	180	9.76	1.887	417.35	$N^{3.5/3}$
Max Power to Weight ratio	0.129	0.349	0.037	847.05	
Total Fuel Capacity (Gal.)	43	0.1875	0.86	-78.31	N
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.137	-58.96	N
Cruise Power Req (hp)	313.26	2.029	3.28	-38.21	$N^{3.5/3}$
Reynolds Number	1.19 x10^{9}	1.58x10^{8}	1.69x10^{8}	-6.60	$N^{1.5/3}$

Scaled to Maximum Weight					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	8.40	2.23	$N^{1/3}$
Fuselage Length (ft.)	27	8.33	6.30	32.33	$N^{1/3}$
Wing Area (ft.^{2})	174	11.40	9.47	20.46	$N^{2/3}$
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.13	31.21	$N^{1/3}$
Empty Weight (lbs.)	1393	28	17.67	58.43	N
Max Takeoff Weight (lbs.)	2550	29.18125	29.18125	0	N
Max Power(hp)	180	9.76	0.978	898.16	$N^{3.5/3}$
Max Power to Weight ratio	0.129	0.349	0.0335	940.27	
Total Fuel Capacity (Gal.)	43	0.1875	0.55	-65.63	N
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.086	-34.98	N
Cruise Power Req (hp)	313.26	2.029	1.92	5.70	$N^{3.5/3}$
Reynolds Number	$1.19 \text{ x}10^{9}$	$1.58\text{x}10^{8}$	$1.34\text{x}10^{8}$	17.56	$N^{1.5/3}$

Scaled to Maximum Power					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	16.19	-46.99	$N^{1/3.5}$
Fuselage Length (ft.)	27	8.33	12.14	-31.37	$N^{1/3.5}$
Wing Area (ft.^{2})	174	11.40	35.19	-67.60	$N^{2/3.5}$
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	2.17	-31.95	$N^{1/3.5}$
Empty Weight (lbs.)	1393	28	126.71	-77.90	$N^{3/3.5}$
Max Takeoff Weight (lbs.)	2550	29.18	209.68	-86.08	$N^{3/3.5}$
Max Power(hp)	180	9.76	9.76	0	N
Max Power to Weight ratio	0.129	0.349	0.046	648.87	
Total Fuel Capacity (Gal.)	43	0.1875	3.91	-95.21	$N^{3/3.5}$
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.619	-90.93	$N^{3/3.5}$
Cruise Power Req (hp)	313.26	2.029	19.11	-89.38	N
Reynolds Number	1.19×10^{9}	1.58×10^{8}	3.59×10^{8}	-56.10	$N^{1.5/3.5}$

Scaled to Maximum Fuel					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	5.88	45.94	$N^{1/3}$
Fuselage Length (ft.)	27	8.33	4.41	88.92	$N^{1/3}$
Wing Area (ft.^{2})	174	11.40	4.64	145.51	$N^{2/3}$
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	0.79	87.32	$N^{1/3}$
Empty Weight (lbs.)	1393	28	6.07	360.97	N
Max Takeoff Weight (lbs.)	2550	29.18	11.119	162.44	N
Max Power(hp)	180	9.76	0.317	2976.61	$N^{3.5/3}$
Max Power to Weight ratio	0.129	0.349	0.028	1121.76	
Total Fuel Capacity (Gal.)	43	0.1875	0.1875	0	N
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.030	89.19	N
Cruise Power Req (hp)	313.26	2.029	0.55	267.46	$N^{3.5/3}$
Reynolds Number	1.19×10^9	1.58×10^8	0.79×10^8	100.53	$N^{1.5/3}$

Scaled to Cruise Fuel Flow					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	7.27	18.00	$N^{1/3}$
Fuselage Length (ft.)	27	8.33	5.46	52.75	$N^{1/3}$
Wing Area (ft.^{2})	174	11.40	7.10	60.50	$N^{2/3}$
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	0.98	51.46	$N^{1/3}$
Empty Weight (lbs.)	1393	28	11.49	143.65	N
Max Takeoff Weight (lbs.)	2550	29.18	21.036	38.71	N
Max Power(hp)	180	9.76	0.67	1362.21	$N^{3.5/3}$
Max Power to Weight ratio	0.129	0.348	0.0317	998.58	
Total Fuel Capacity (Gal.)	43	0.1875	0.35	-47.14	N
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.056	0	N
Cruise Power Req (hp)	313.26	2.029	1.16	74.64	$N^{3.5/3}$
Reynolds Number	1.19 x10^{9}	1.58x10^{8}	1.08x10^{8}	45.79	$N^{1.5/3}$

Scaled to Reynolds Number					
Property	Cessna 172	RC Aircraft	Scaled Aircraft	Error	Dimensional Analysis
Wingspan (ft.)	36	8.58	9.35	-8.22	$N^{1/1.5}$
Fuselage Length (ft.)	27	8.33	7.01	18.80	$N^{1/1.5}$
Wing Area (ft.^{2})	174	11.40	11.74	-2.91	$N^{2/1.5}$
Aspect Ratio	7.45	6.46	7.45	-13.25	1
Chord Length (ft.)	4.83	1.48	1.26	17.80	$N^{1/1.5}$
Empty Weight (lbs.)	1393	28	24.43	14.63	$N^{3/1.5}$
Max Takeoff Weight (lbs.)	2550	29.18125	44.712	-34.736	$N^{3/1.5}$
Max Power(hp)	180	9.760	1.608	506.72	$N^{3.5/1.5}$
Max Power to Weight ratio	0.129	0.349	0.036	868.86	
Total Fuel Capacity (Gal.)	43	0.1875	0.75	-75.13	$N^{3/1.5}$
Cruise Fuel Flow (Gal/hr.)	6.8	0.056	0.119	-52.95	$N^{3/1.5}$
Cruise Power Req (hp)	313.26	2.029	2.80	-27.53	$N^{3.5/1.5}$
Reynolds Number	1.19 x10^9	1.58x10^8	1.58x10^8	0	N