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Climb Performance Testing of the PA-28-180 Utilizing Saw-Tooth Method and Level Acceleration Method

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Climb Performance Testing of the PA-28-180 Utilizing Saw-Tooth Method and Level Acceleration Method

by

Paige Elizabeth Christensen

A thesis submitted to the College of Engineering and Science of Florida Institute of Technology in partial fulfillment of the requirements for the degree of

> Master of Science in Flight Test Engineering

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We the undersigned committee hereby approve the attached thesis, "Climb Performance Testing of the PA-28-180 Utilizing Saw-Tooth Method and Level Acceleration Method" by

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Abstract

Title: Climb Performance Testing of the PA-28-180: Utilizing Saw-Tooth Method and Level Acceleration Method

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This thesis details the test methods, data reduction methods, and flight data analysis conducted in the climb performance evaluation of the PA-28-180 "Piper Arrow." The main objective of this thesis was to quantitatively compare the climb performance requirements detailed in the former regulation, CAR 3.85a to the current regulation, 14 CFR 23.2120, utilizing Saw-Tooth steady climb method conducted at various airspeeds. The report also evaluates the Level Acceleration method at reciprocating headings, and qualitatively assesses the usefulness of testing multiple headings on a general aviation aircraft at multiple altitudes.

The Saw-Tooth Climb and Level Acceleration flights were conducted on January 10^{th} , 2023 out of Saginaw County H.W. Browne Airport (KHYX) in Saginaw, Michigan. The Piper Arrow was not equipped with any specialized flight test instrumentation. The data was recorded from the Garmin G5 PFD/HSI dual avionics suite and the mechanical gauges on the standard instrumentation panel. The data analysis was completed in accordance with the Pilot Operating Handbook (POH) [4] and the reduction methods detailed in FTE 5701 Airplane Performance Flight Testing Laboratory Manual [3].

The flight analysis concluded the PA-28-180 (N3911T) met the climb performance minimum requirements of CAR 3.85a at 75 kts, 80 kts, 85 kts, and 90 kts; however, the 14 CFR 23.2120 requirements were only satisfied at 80 kts. The Level Acceleration data analyzed at 3,500 ft, 4,500 ft, and 5,500 ft showed minimal variability between the 20 degree heading and 200-degree heading accelerations.

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Lastly, I would like to thank my pilots Aaron Martin and Glen Christensen for making themselves and the Piper Arrow available to me to complete this thesis.

Dedication

This thesis is dedicated to my family. Thank you for instilling in me a strong work ethic, inquisitiveness, and fortitude. I stand on the shoulders of giants – you all inspire me to continue striving towards greatness.

Chapter 1: Introduction

Thesis Objectives

The main objective of this thesis was to demonstrate the climb performance of the PA-28-180 "Piper Arrow." The flight analysis quantitatively compared the climb performance requirements of CAR 3.85a and 14 CFR 23.2120. CAR 3.85a was the Part 23 regulation at the time the Piper Arrow was certified, whereas 14 CFR 23.2120 is the current regulation for Part 23 aircraft less with a maximum takeoff weight than 19,000 lbs. The Saw-Tooth Method was performed at four airspeeds to compare the minimum rate of climb requirement, per CAR 3.85a, and the minimum climb gradient, per 14 CFR 23.2120. The Level Acceleration method was conducted at reciprocating headings at three altitudes to evaluate how collecting data at reciprocating headings impacted the rate of climb determination.

Test Item Description

A 1967 Piper Cherokee "Arrow" (PA-28-180), tail number N3911T, was utilized as the test aircraft for the evaluation of this thesis. All flights took off out of Saginaw County H.W. Browne Airport (KHYX) in Michigan. The Piper Arrow, shown in Figure 1, is a single engine airplane with a controllable-pitch propeller and retractable landing gear. For the purposes of this thesis, all flights were conducted with a flaps up configuration, landing gear retracted, and the propeller set for optimal climb performance. The Piper Arrow airworthiness standards and regulations are defined under FAA 14 CFR Part 23. The PA-28-180 has a rated horsepower of 180 and a rated speed of 2700 RPM. The specified gross weight is 2500 lbs, with a 50-gal fuel capacity [4]. Additional information regarding the test aircraft can be found in Appendix A.

Figure 1: Piper Arrow PA-28 (N3911T)

Flight Tests Instrumentation

The Piper Arrow is equipped with a Garmin G5 PFD/HSI dual avionics suite, as shown in Figure 2. The Garmin displayed altitude, airspeed, altimeter setting, ground speed, and heading. The G5 was utilized as the primary instrumentation for the experiments; however, all data recorded from the G5 was also recorded on the mechanical instrument gauges for comparison. The power setting, manifold pressure, outside atmospheric temperature, and fuel quantity were recorded from the mechanical instrument gauges on the main panel, shown in Figure 2.

Figure 2: Piper Arrow Instrument Panel

Test Location and Conditions

All test flights for this thesis were conducted out of Saginaw County H.W. Browne Airport (KHYX) in Saginaw, Michigan, as shown in Figure 3. The test maneuvers were performed in VFR conditions East of the airport. The "Saw-Tooth Method" flight card was flown on January 10^{th} , 2023 at 12:15 PM EST (17:15 ZULU). A total of four climb and descents were conducted at four different airspeeds to determine a rate of climb. The "Level Acceleration" flight card was flown on January 10^{th} , 2023 at 12:33 PM EST (17:33 ZULU). The pilot performed a series of accelerations from near stalling speed to maximum airspeed in level flight at three different constant altitudes. At takeoff, the outside air temperature was 38 degrees Fahrenheit, the sea level pressure was 29.99 in Hg, and winds were 110, 6 kts.

Figure 3: Flight Test Path [8]

Test Configuration and Loading

For the duration of both test flights, the landing gear was retracted, and the flaps were in a clean (up) configuration. The airplane was loaded with a pilot in the left seat and a test engineer in the right seat with no additional internal ballast. At takeoff, the airplane was loaded with 34 gallons of Avgas. The aircraft configuration resulted in an estimated weight at engine start of 1,979 lbs, and a CG of 87.33 in aft of datum, which corresponded to a heavy-aft configuration.

Flight Test Procedure

The following flight test procedures were developed for the purpose of comparing the evaluation of climb performance using best rate of climb speeds and best angle of climb speeds, as well as, analyzing the impact of level flight accelerations with and without reciprocating headings.

Determination of Climb Performance using Saw-Tooth Method

The following data was recorded between an altitude band of 3,600 ft and 4,600 ft at 75 kts, 80 kts, 85 kts, and 90 kts:

- 1. Time
- 2. Altitude
- 3. Airspeed
- 4. Vertical Climb Rate
- 5. Outside Air Temperature
- 6. Fuel Weight
- 7. Engine RPM
- 8. Engine Manifold Pressure

Determination of Climb Performance using Level Acceleration Method

The following data was recorded at 3,500 ft, 4,500 ft, and 5,500 ft, at alternating headings of 20 degrees to 120 degrees:

- 1. Time
- 2. Altitude
- 3. Airspeed
- 4. Outside Air Temperature
- 5. Fuel Weight
- 6. Engine RPM
- 7. Engine Manifold Pressure
- 8. Vh

Limitations to Scope

Testing was conducted on a single day, and there were not significant variations in outside air temperature, atmospheric pressure, center of gravity or weight configurations. As a result, the comparisons between factors impacting climb performance were limited. The most significant limitation to scope was the use of

an aircraft lacking flight test instrumentation. The Piper Arrow (N3911T) was made available for the purpose of this thesis, but as a general aviation aircraft its primary instrumentation was a Garmin G5. The indicated values for airspeed and altitude were recorded directly from the Garmin display and assumed to have zero instrument corrections for the intent of this report.

Chapter 2: Theory

Climb Performance

In physics, change of motion requires an exchange between potential energy and kinetic energy. For an aircraft, the potential energy is described by aircraft position (altitude) and the kinetic energy is described by aircraft velocity (airspeed). An aircraft demonstrates positive climb performance by gaining potential energy (increasing altitude). To climb, an aircraft uses excess power by converting kinetic energy (airspeed) to potential energy (altitude). [1]

The airspeed a pilot maintains during a climb impacts the climb performance. Aircraft manufacturers typically publish a maximum rate of climb, Vy, and a maximum angle of climb, Vx, in the pilot operating handbook. The maximum rate of climb, Vy, is the airspeed and angle of attack that equates to the maximum excess power. The rate of climb is a comparison of altitude gained relative to time. The maximum angle of climb, Vx, is the airspeed and angle of inclination that equates to maximum excess thrust. The angle of climb is a comparison of altitude gained relative to distance traveled. Maximum angle of climb would typically be used to avoid tall obstacles after takeoff. Maximum rate of climb conditions allow reaching a desired altitude in less time that flying at a maximum angle of climb for a given airspeed. The relationship between the corresponding airspeeds is shown in Figure 4. [3]

Figure 4: Max ROC and Max AOC Relationship [2]

The airspeeds for maximum rate of climb (ROC) and maximum angle of climb (AOC) vary with altitude. For a propeller aircraft, the maximum ROC occurs at the airspeed and angle of attack combination that is close to the maximum lift over drag ratio; therefore, an increase in altitude will slightly decrease the airspeed required to reach maximum ROC. Inversely, the maximum AOC occurs at an airspeed and angle of attack combination that is less than the maximum lift over drag ratio. An increase in altitude will increase the airspeed required to reach best AOC. The point at which the ROC and AOC airspeed is equal is called the "absolute ceiling." This is the point in which the aircraft cannot produce any excess power. A pilot would recognize they are nearing an absolute ceiling altitude when they are no longer able to climb greater than 100 ft/min, which is referred to the "service ceiling" altitude, as shown in Figure 5. An increase in weight, altitude, or drag (flaps or gear extended) all decrease the excess thrust and excess power, resulting in a lower service ceiling. [3]

Figure 5: Absolute and Service Ceiling [2]

Energy Equation

There are two mathematical approaches to evaluating climb performance: the vector approach and the energy equation approach. This thesis was evaluated using the energy equation approach. The total energy of the aircraft is the summation of the potential energy and kinetic energy, shown in the equation below. [1]

$$
E = mgh + \frac{1}{2}mv^2
$$

If the energy of the airplane remains constant, a pilot can trade airspeed for altitude to climb. By adding energy (or excess power) and aircraft can climb at a constant airspeed. The equation for excess power is defined as the change in energy over time. [3]

$$
P = \frac{dE}{dt} = mg\frac{dh}{dt} + mv\frac{dv}{dt}
$$

When airspeed is held constant, the change in velocity over time component of the excess power equation goes to zero and simplifies the equations. The equation below is described as the specific excess power and is the mathematical foundation for evaluating climb performance using the Saw-Tooth Method or the Level Acceleration Method. [3]

$$
P_s = \frac{P}{mg} = \frac{dh}{dt} + \frac{v}{g} \times \frac{dv}{dt}
$$

Saw-Tooth Method Climb Performance

Saw-Tooth Method is performed at a constant airspeed; the specific power equation simplifies to only include the change in altitude over time component, $P_s = \frac{dh}{dt}$. Climbs are conducted at different airspeeds, with varying altitudes, while flying reciprocating headings to mitigate wind shear effects. The method is typically used in climb performance evaluation for low-speed aircraft. The four primary reduction methods to evaluate Saw-Tooth climbs are the instrument and weight corrected relationship between power and climb rate ("PIW vs. CIW") method, density altitude method, equivalent altitude method, and dimensionless method. The PIW vs. CIW method is applicable to both constant-speed propellers and fixed-pitch propellers. The Piper Arrow is a constant-speed propeller aircraft, so the PIW vs. CIW method was used to evaluate the Saw-Tooth method in this paper. [1]

Level Accelerations Climb Performance

The Level Acceleration Method is conducted at a climb power setting and airspeed that is 1.1 times greater than the stalling speed. The altitude is held constant; therefore, the specific excess power equation simplifies to only the velocity components. After leveling off, the aircraft accelerated to Vh, the maximum speed in level flight with maximum continuous power. The acceleration is repeated at multiple altitude and airspeed combinations. Level acceleration method is typically used to determine parameters in the aircraft's drag polar for high performance military aircraft. [1]

FAA Regulation

The PA-28 ("Piper Arrow") was certified under CAR 3.85a climb performance regulation, as shown in Figure 7. The FAA has since updated the requirements for climb performance per 14 CFR 23.2120, shown in Figure 6. CAR 3.85a was written for the certification of aircraft with a maximum weight of 6,000 lbs. Per the regulation, the climb performance must be evaluated with the landing gear extended, wing flaps in takeoff position, and a takeoff power setting with speed not exceeding maximum continuous power. CAR 3.85a, required a steady rate of climb at maximum takeoff weight of 300 ft/min or 10 times the stall speed with flaps up; whichever is greater. The current regulation, 14 CFR 23.2120, requires a climb gradient of 8.3% for landplanes, conducted at maximum continuous power. The 14 CFR 23.2120 requirements are specific to aircraft with a maximum takeoff weight of 19,000 lbs and a maximum seating capacity between 2 to 6 passengers. The 14 CFR 23.2120 does not specify a required aircraft configuration for certification testing beyond specifying all available engines must be operating and the aircraft must be in a climb configuration. The current regulation, in comparison to CAR 3.85a, is much broader and leaves more opportunity for interpretation.

§ 23.2120 Climb requirements.

The design must comply with the following minimum climb performance out of ground effect:

- (a) With all engines operating and in the initial climb configuration(s) -
	- (1) For levels 1 and 2 low-speed airplanes, a climb gradient of 8.3 percent for landplanes and 6.7 percent for seaplanes and amphibians; and
	- (2) For levels 1 and 2 high-speed airplanes, all level 3 airplanes, and level 4 single-engines a climb gradient after takeoff of 4 percent.
- (b) After a critical loss of thrust on multiengine airplanes -
	- (1) For levels 1 and 2 low-speed airplanes that do not meet single-engine crashworthiness requirements, a climb gradient of 1.5 percent at a pressure altitude of 5,000 feet (1,524 meters) in the cruise configuration(s);
	- (2) For levels 1 and 2 high-speed airplanes, and level 3 low-speed airplanes, a 1 percent climb gradient at 400 feet (122 meters) above the takeoff surface with the landing gear retracted and flaps in the takeoff configuration(s); and
	- (3) For level 3 high-speed airplanes and all level 4 airplanes, a 2 percent climb gradient at 400 feet (122 meters) above the takeoff surface with the landing gear retracted and flaps in the approach configuration(s).
- (c) For a balked landing, a climb gradient of 3 percent without creating undue pilot workload with the landing gear extended and flaps in the landing configuration(s).

Figure 6: 23.2120 Climb Requirement

§ 3.85a Climb requirements; airplanes of 6,000 lbs. or less. Airplanes having a maximum certificated take-off weight of 6,000 lbs. or less shall comply with the requirements of this section.

(a) Climb: take-off climb condition. The steady rate of climb at sea level shall not be less than 10 V_{r_1} or 300 feet per

minute, whichever is the greater, with: (1) Take-off power, (2) landing gear extended, (3) wing flaps in take-off position, (4) cowl flaps in the position used in cooling tests specified in \$\$3.581 through 3.596.

Figure 7: CAR 3.85a Climb Requirement

Chapter 3: Data Reduction

Weight and Balance

The test flights were conducted at a center of gravity of 87.33 inches aft of datum. The aircraft weight and balance were calculated using the following equations and values:

Total weight = \sum [front seat + back seat (N/A) + fuel + forward baggage (N/A) + aft baggage (N/A) + empty airplane]

 $Total weight = \sum [395.0 lbs + 204.3 lbs + 1380 lbs] = 1979 lbs$

 $Center~of~ Gravity = \frac{Total~Moment}{Total~Weight}$

 $Center\ of\ Gravity = \frac{1.216 * 10^5\ lbs * in}{1979\ lbs} = 87.33\ in$

	Weight (lbs)	Arm (in)	Moment (lbs-in)
Basic Empty Weight	1380	88.15	$1.216E + 0.5$
First Row	395.0	80.50	3.180E+04
Second Row	0.000	118.1	0.000
Baggage Area	0.000	142.8	0.000
Fuel	204.3	95.00	$1.941E + 04$
Gross Weight (lbs)	1979		
CG (in)	87.33		

Table 1: Weight and Balance

The PA-28-180 forward and aft CG limits are 81.0 inches and 91.0 inches, respectively, as shown in Figure 8 [5]. The test cards were flown within the CG limit of the Piper Arrow.

Saw-Tooth Climb Performance

A series of three saw-tooth climbs were conducted at 75 kts, 80 kts, 85 kts, and 90 kts between a pressure altitude band of 3,600 ft and 4,600 ft. Each climb was conducted at a power setting of 2300 RPM and a manifold pressure of 24 in Hg. The data collected during the Saw-Tooth climbs was used to calculate rate of climb, instrument and weight corrected climb rate, and instrumented and weight corrected power. The Saw-Tooth method is performed with a constant airspeed, so the specific excess power, P_s , equation assumes $\frac{dv}{dt} = 0$.

 $P_s = \frac{dh}{dt} + \frac{v}{g} \times \frac{dv}{dt} = \frac{dh}{dt} + 0 = \frac{dh}{dt}$

Altitude versus elapsed time was plotted for each airspeed tested. An example is shown in Figure 8. The trendlines, quadratic equations, were used to solve for the time at the midpoint altitude for each climb. The derivative of the trendline equations were calculated using this time to determine the rate of climb in feet per second, which was converted to feet per minute, as seen in Table 2.

Figure 8: Altitude vs. Time at 75 kts

The rate of climb values in Table 2 needed to be corrected for non-standard temperature at a given altitude. The standard temperature, T_s , was calculated at each altitude during the 1,000-foot climb. The tests were conducted on a colderthan-standard day, with test temperatures are below 15 degrees Celsius.

$$
T_s = -\frac{2 \cdot H_i}{1000} + 15 \, \text{°C}
$$

$$
T_s = -\frac{2 \cdot 3600 \, ft}{1000} + 15 \, ^\circ \text{C} = 7.8 \, ^\circ \text{C}
$$

The temperature ratio is calculated using the outside air temperature, OAT, for a given test point and the temperature for a standard day at sea level, 15 degrees

Celsius. The test corrected rate of climb, ROC_{TC} , was calculated by multiplying the observed rate of climb values by the temperature ratio, θ .

$$
\theta = \frac{273.15 + OAT}{273.15 + 15}
$$

$$
\theta = \frac{273.1 + (-2) \,^{\circ}\text{C}}{288.15} = 0.9410
$$

$$
ROC_{TC} = ROC * \theta
$$

$$
ROC_{TC} = 728.7 \frac{\text{ft}}{\text{min}} * 0.9410 = 703.2 \, ft/min
$$

In addition to the temperature ratio, the pressure ratio, δ , and density ratio, σ , were calculated to determine the instrument and weight corrected climb, CIW, and power, PIW. The density ratio, as shown in the equations below, is a function of the temperature ratio and pressure ratio at each test point. The pressure ratio is a function of the pressure altitude at each test point, H_i . The density ratio is computed by dividing the pressure ratio by the temperature ratio.

$$
\delta = [1 - (6.87535 \cdot 10^{-6}) \cdot H_i]^{5.2561}
$$

$$
\delta = [1 - (6.87535 \cdot 10^{-6}) \cdot 3600 \, ft]^{5.2561} = 0.8766
$$

$$
\sigma=\frac{\delta}{\theta}
$$

 $\sigma = \frac{0.8766}{0.9410} = 0.9315$

The CIW is a function of the test corrected rate of climb, density ratio, test weight, and standard weight of the aircraft, which for the Piper Arrow is 2,500 lbs. Each climb was conducted over a few minutes total, so there was not a significant change in test weight because minimal fuel was burned.

$$
CIW = \frac{ROC_{TC}\sqrt{\sigma}}{\sqrt{\frac{W_T}{W_S}}}
$$

$$
CIW = \frac{703.24 \frac{ft}{min} \cdot \sqrt{0.9315}}{\sqrt{\frac{1979 \text{ lbs}}{2500 \text{ lbs}}}} = 762.9 \text{ ft/min}
$$

The PIW is a function of test brake horsepower, density ratio, test weight, and standard weight. The equation below was used to calculate the test brake horsepower; where standard brake horsepower, BHP_s , was determined using the Lycoming IO-360-B1E Engine Chart in the Lycoming Operator's Manual, shown in Figure 9. The MAP, engine RPM, and indicated altitude were used to determine the standard brake horsepower.

Figure 9: Lycoming IO-360-B1E Engine Chart [9]

$$
BHP_T = BHP_s \sqrt{\frac{273 + T_s}{273 + OAT}}
$$

$$
BHP_T = 145.475 \sqrt{\frac{273 + 7.8 \text{ °C}}{273 + (-2) \text{ °C}}} = 148.1
$$

$$
P_{IW} = \frac{BHP_T\sqrt{\sigma}}{\left(\frac{W_T}{W_S}\right)^{\frac{3}{2}}}
$$

$$
P_{IW} = \frac{148.1\sqrt{0.9410}}{\left(\frac{1979 \text{ lbs}}{2500 \text{ lbs}}\right)^{\frac{3}{2}}} = 321.8 \text{ HP}
$$

The climb gradient was calculated using the temperature-corrected rate of climb, ROC_{TC} , and the indicated ground speed, recorded from Garmin G5 instrument panel. A sample calculation is provided below.

 $\emph{Climb Gradient (}\%) = \frac{ROC_{TC}(fpm)}{Ground Speed(fpm)}$

 $Climb\;Gradient\; (\%) = \frac{703.2\; (fpm)}{8911\; (fpm)} = 7.891\%$

Level Acceleration Climb Performance

Six level accelerations were conducted at alternating headings of 20 degrees and 200 degrees at 3,500 ft, 4,500 ft, and 5,500 ft. Each acceleration was conducted at a power setting of 2300 RPM and a manifold pressure of 24 in Hg. The data collected was used to determine the best rate of climb, the best rate of climb airspeed, V_Y , and the best angle of climb airspeed, V_X . Three sets of plots were produced at each altitude: rate of climb versus airspeed at a 20-degree heading, rate of climb versus airspeed at a 200-degree heading, and the average rate of climb versus the average airspeed between the data collected at both the 20-degree and 200-degree headings. Much like the Saw-Tooth method, the calculations for the level acceleration are derived from the specific excess power equation; however, level acceleration assumes the altitude is held constant. Therefore, $\frac{dh}{dt} = 0$.

$$
P_s = \frac{v}{g} \times \frac{dv}{dt}
$$

The equations for temperature ratio, density ratio, and pressure ratio were used to calculate true velocity. The indicated velocity was recorded from the avionics suite during the test. The calculations assume the indicated velocity is equivalent to the calibrated velocity.

$$
\delta = [1 - (6.87535 \cdot 10^{-6}) \cdot 3500 \, ft]^{5.2561} = 1.000
$$

$$
\theta = \frac{273.1 + (-2)^{\circ}\text{C}}{288.15} = 0.941
$$

$$
\sigma = \frac{0.9410}{1.000} = 1.063
$$

$$
V_T = \frac{V_C}{\sqrt{\sigma}}
$$

$$
V_T = \frac{V_C}{\sqrt{\sigma}} = \frac{116.0 \, ft/s}{\sqrt{1.063}} = 66.93 \, ft/s
$$

Airspeed versus elapsed time was plotted for each altitude tested. An example is shown in Figure 10. The trendlines were obtained to find equations $\frac{dv}{dt}$; where v is the elapsed time at a given test point.

Figure 10: Airspeed vs. Elapsed Time at 3500 ft

$$
\frac{dv}{dt} = 2 * 0.0163 * (3 s) + 2.605 = 2.508 \text{ ft/s}^2
$$

The change in velocity over time is a variable in the equation to determine the weight-corrected excess thrust horsepower, $(FHP_{excess})_{WC}$, at each point in the accelerations.

$$
FHP_{excess} = \left(\frac{W_T}{g}\right) \left(\frac{dv}{dt}\right) V_T \frac{1 \, HP}{550 \frac{ft \cdot lb}{s}}; where \, g = 32.2 \frac{ft}{s^2}
$$

$$
FHP_{excess} = \left(\frac{1955 \text{ lbs}}{32.2 \frac{\text{ft}}{\text{s}^2}}\right) \left(2.605 \frac{\text{ft}}{\text{s}^2}\right) \left(66.93 \frac{\text{ft}}{\text{s}}\right) \frac{1 \text{ HP}}{550 \frac{\text{ft} \cdot \text{lb}}{\text{s}}} = 19.25 \text{ HP}
$$

$$
(FHP_{excess})_{WC} = \frac{FHP_{excess}}{\left(\frac{W_T}{W_S}\right)^{\frac{3}{2}}}; where W_s = 2,500 lbs
$$

$$
(FHP_{excess})_{WC} = \frac{19.25 \text{ HP}}{\left(\frac{1955 \text{ lbs}}{2500 \text{ lbs}}\right)^{\frac{3}{2}}} = 27.84 \text{ HP}
$$

The values for weight-corrected excess thrust were inputted into the rate of climb equation below to determine the computed ROC for each point in the level acceleration.

$$
ROC = \frac{(FHP_{excess})_{WC}}{W_s} \times \frac{550 \frac{ft \cdot lb}{s}}{1 HP} \times \frac{60 s}{1 min}
$$

$$
ROC = \frac{27.84 \text{ HP}}{2500 \text{ lbs}} \times \frac{550 \frac{ft \cdot lb}{s}}{1 \text{ HP}} \times \frac{60 \text{ s}}{1 \text{ min}} = 367.4 \text{ ft/min}
$$

Chapter 4: Flight Data Analysis

Saw-Tooth Climb Performance

The first Saw-Tooth climb was conducted at 75 kts between an altitude band of 3,600 ft and 4,600 ft, for a total duration of 82 seconds. The relationship between altitude and elapsed time is depicted in Figure 11. The quadratic trendline equation had a coefficient of determination of 0.9980. The root of the equation was determined by setting "y" to 4,000 the approximate midpoint of the data. This resulted in a root of 31.72 seconds, as shown in Table 4. The root of 31.72 seconds was used to compute the rate of climb using the derivative of the trendline equation, $0.0076x+11.90$. The rate of climb at 75 kts equated to 728.7 ft/min.

Figure 11: Altitude vs. Time at 75 kts

The second climb was conducted at 80 kts between the 1,000 ft altitude band, for a total duration of 76 seconds. The coefficient of determination for the quadratic trendline was 0.9991, as shown in Figure 12. The root of the trendline equation assumed a value of 4,000 for "y", resulting in a 28.53 second root. This was
plugged into the derivate equation, $-0.0172x+13.86$. The rate of climb at 80 kts was 861.1 ft/min. The maximum rate of climb airspeed, reported in the pilot operating handbook for the Piper Arrow was 100 mph (86.9 kts); therefore, it is expected the rate of climb at 80 kts is higher than the rate of climb at 75 kts since the maximum rate of climb should be at the test airspeed closest to Vy.

Figure 12: Altitude vs. Time at 80 kts

The climb conducted at 85 kts was expected to yield the shortest duration to climb the 1,000 ft and result in the highest rate of climb since it was the closest climb airspeed to the POH Vy. The total duration of the 85 kts climb was 78 seconds, 2 seconds more than the 80 kts climb. The coefficient of determine for the quadratic trendline was approximately 0.9989, as shown in Figure 13. Using the 4,000 value for "y" the quadratic root was 30.83 seconds, resulting in a rate of climb of 765.2 ft/min. This data was in consistent with the expectation that the highest rate of climb amongst the four tested airspeeds of 75 kts, 80 kts, 85 kts, and 90 kts would occur at 85 kts.

Figure 13: Altitude vs. Time at 85 kts

The climb conducted at 90 kts took 85 seconds to complete, the longest climb time of the four tested airspeeds. The coefficient of determination for the quadratic trendline was 0.9994 and produced a root of 32.93 seconds. Using the derivative of the trendline, the rate of climb was 727.6 ft/min. In comparison to the other tested airspeeds, 90 kts resulted in the least efficient climb rate. The relationship between altitude and elapsed time at 90 kts is shown below in Figure 14.

Figure 14: Altitude vs. Time at 90 kts

Table 4 shows the trendline equation, derivative of the trendline equation, root value of "t", and rate of climb for 75 kts, 80 kts, 85 kts, and 90 kts. Using the theoretical understanding for best rate of climb airspeed, Vy, the data should have suggested the highest rate of climb value was at the airspeed closest to 100 mph (86.9 kts). However, the table shows the highest rate of climb occurred at 80 kts and gradually decreased with an increase in airspeed. The flights were conducted in a clean configuration and 2300 RPM climb power setting, but they were not evaluated against multiple altitude bands. To reach a more accurate conclusion on which airspeed produces the best rate of climb, each airspeed should be conducted at different altitudes. This data will be compared to the data collected in the level acceleration method.

Table 4 also shows the maximum rate of climb calculated for each airspeed. Based on the climb performance regulations under CAR 3.85, each airspeed satisfied the Part 23 climb performance regulation at the time of certification of the PA-28. The regulation states an aircraft must have a climb rate of 300 ft/min or 10 times the stall speed with flaps up, whichever is greater. In the case of the PA-28, 10 times

the stall speed would be 630 ft/min. The maximum rate of climbs ranged from 727.6 ft/min to 801.2 ft/min, surpassing the minimum climb rate required under CAR 3.85. Since the Saw-Tooth climbs were conducted at 2300 RPM, rather than the rated 2700 RPM, the results are conservatively passing the FAA regulation. If the same flights were conducted at maximum continuous power, the data should show increased climb performance. The same steady-climb data was evaluated against the current FAA climb performance regulation, 14 CFR 23.2120, below.

Table 4: Altitude vs. Time Equations

Speed (kts)	Trendline Eq	Derivation Eq	t (sec)	$ROC(f_t/s)$	ROC (ft/min)
75	$0.0038t^{2}+11.90t+3618$	$0.0076t+11.90$	31.72	12.14	728.7
80	$-0.0086t^2+13.86t+3597$	$-0.0172t+13.86$	29.57	13.35	801.2
85	$-0.0028t^{2}+12.93t+3604$	$-0.0056t+12.93$	30.83	12.75	765.2
90	$-0.0126t^{2}+12.96t+3587$	$-0.0252t+12.96$	32.93	12.13	727.6

The current regulation, 14 CFR 23.2120, requires a minimum climb gradient of 8.3% for landplanes. The climb gradient was computed using the temperaturecorrected rate of climb and the indicated ground speed at each test point. The data in Tables 5-8 illustrates the airspeeds that did (green) and did not (red) satisfy the requirement.

At 127 ft/s (75 kts), shown in Table 5, only 2 out of the 25 test points met the 8.3% climb gradient metric. Since the PA-28 documented value for best angle of climb airspeed is approximately 83 kts, it is not unlikely that an airspeed below Vx would produce inadequate climb performance.

Altitude (ft)	Indicated Airspeed (ft/s)	Ground Speed $({\rm ft/s})$	ROC_TC (ft/min)	Climb Gradient (%)
3600	127	148.5	703.2	7.89%
3640	127	146.8	703.4	7.98%
3680	127	143.5	703.6	8.17%
3740	127	141.8	703.9	8.28%
3780	127	140.1	704.1	8.38%
3820	127	140.1	701.7	8.35%
3860	127	141.8	701.9	8.25%
3880	127	143.5	702.0	8.16%
3940	127	145.2	702.3	8.06%
3980	127	145.2	702.5	8.07%
4020	127	143.5	702.7	8.16%
4080	127	143.5	703.0	8.17%
4120	127	148.5	703.2	7.89%
4160	127	146.8	703.4	7.98%
4200	127	146.8	701.0	7.96%
4240	127	146.8	701.2	7.96%
4300	127	146.8	701.5	7.96%
4320	127	146.8	701.6	7.96%
4380	127	146.8	701.9	7.97%
4440	127	143.5	702.3	8.16%
4500	127	145.2	702.6	8.07%
4540	127	146.8	702.8	7.98%
4560	127	148.5	702.9	7.89%
4580	127	151.9	703.0	7.71%
4600	127	151.9	703.1	7.71%

Table 5: Climb Gradient Data for 75 kts (127 ft/s)

Table 6, shows the data recorded at 135 ft/s (80 kts). At 80 kts, every test point exceeded a climb gradient of 8.3%. The minimum climb gradient at 80 kts was 8.81% at 4440 ft. The maximum climb gradient was 9.24% at 3840 ft.

Altitude (ft)	Indicated Airspeed (ft/s)	Ground Speed (ft/s)	ROC TC $\textbf{(ft/min)}$	Climb Gradient (%)
3600	135	151.9	831.1	9.12%
3620	135	153.6	831.2	9.02%
3660	135	155.3	831.5	8.92%
3800	135	150.2	832.3	9.23%
3840	135	150.2	832.5	9.24%
3880	135	153.6	829.7	9.00%
3900	135	153.6	829.8	9.00%
3940	135	153.6	830.0	9.01%
3960	135	153.6	830.2	9.01%
4000	135	153.6	830.4	9.01%
4040	135	153.6	830.6	9.01%
4080	135	153.6	830.9	9.02%
4140	135	153.6	831.2	9.02%
4200	135	157.0	831.6	8.83%
4240	135	151.9	828.7	9.09%
4300	135	151.9	829.1	9.10%
4340	135	153.6	829.3	9.00%
4360	135	155.3	829.5	8.90%
4400	135	155.3	829.7	8.91%
4440	135	157.0	829.9	8.81%
4520	135	155.3	830.4	8.91%
4560	135	155.3	830.6	8.92%
4600	135	155.3	830.9	8.92%

Table 6: Climb Gradient Data for 80 kts (135 ft/s)

The data collected for 143 ft/s (85 kts) resulted in climb gradients less than 8.3%, as shown in Table 7. Since the Piper Arrow v-speed for best angle of climb is 83 kts, it was expected the values for 80 kts and 85 kts to be similar; however, the data showed the maximum climb gradient was only 7.76%. The minimum climb gradient, 7.36% was greater than the minimum climb gradient recorded at 75 kts. This observation is likely due to the 2300 RPM power setting. More power was required for the climb at 85 kts than the climb at 80 kts. The results are not representative of the climb performance of the PA-28-180 since they were not conducted at maximum continuous power. For instance, if the climb gradient values at 85 kts were increased by 15%, which would assume the relationship between engine RPM and climb gradient were linear, the range would be 8.46%- 8.93% and pass the 14 CAR 23.2120 regulation.

Altitude (ft)	Indicated Airspeed (ft/s)	Ground Speed (ft/s)	ROC TC (f t/min)	Climb Gradient (%)
3600	143	158.7	738.5	7.76%
3660	143	158.7	738.8	7.76%
3680	143	160.3	738.9	7.68%
3760	143	162.0	739.4	7.61%
3800	143	162.0	739.6	7.61%
3860	143	162.0	737.2	7.58%
3900	143	160.3	737.4	7.66%
4000	143	160.3	737.9	7.67%
4020	143	160.3	738.0	7.67%
4080	143	162.0	738.3	7.59%
4120	143	160.3	738.5	7.68%
4160	143	160.3	738.7	7.68%
4200	143	160.3	739.0	7.68%
4240	143	160.3	739.2	7.68%
4360	143	162.0	737.1	7.58%
4400	143	163.7	737.3	7.51%
4420	143	165.4	737.4	7.43%
4440	143	167.1	737.5	7.36%
4500	143	167.1	737.8	7.36%
4520	143	167.1	737.9	7.36%
4580	143	167.1	738.2	7.36%
4600	143	167.1	738.3	7.36%

Table 7: Climb Gradient Data for 85 kts (143 ft/s)

Table 8 displays the data collected at 152 ft/s (90 kts). The data at 90 kts proved to have the worst climb performance of the four tested airspeeds. The maximum climb gradient was 6.88%, substantially lower than the lower airspeeds.

Table 8: Climb Gradient Data for 90 kts (152 ft/s)

Table 9 shows the averaged climb gradient and rate of climb at each test airspeed. As seen in Table 6, the greatest climb gradient was observed at 80 kts. Although the climb data for 75 kts had two test points that were greater than 8.3%, overall, the airspeed would not satisfy the requirements in 14 CFR 23.2120.

Table 9: Averaged Climb Gradient

Indicated Airspeed (kts)	Ground Speed (ft/s)	ROC TC (f t/min)	Climb Gradient (%)
75	127	702.6	8.04%
80	135	830.5	9.00%
85	143	738.2	7.57%
90	152	701.7	6.78%

Table 10 shows the instrument and weight corrected climb rate, CIW, and instrument and weight corrected power, PIW. Two altitudes were selected, one lower altitude within the test band and one higher altitude within the test band, to tabulate and plot the CIW and PIW. The trendline was extended to show the CIW for the PA-28-180 at 180 HP. An increase in PIW should produce an increase in CIW. All four airspeeds yielded the expected trend. As the excessive power increased there was more available power to climb at a higher rate. The plots for each PIW vs. CIW relationship are shown in Figures 15-18.

Table 10: CIW and PIW Calculations

The Piper Arrow is equipped with a 180 HP engine. The Pilot Operating Handbook states a rate of climb of 875 ft/min, as shown in Appendix A. At a value of 180 HP for PIW, the data should show approximately 875 ft/min for CIW. Figure 15 shows the PIW versus CIW relationship for 75 kts. The CIW at a value of 180 HP for PIW equates to 707 ft/min. The plot shows that during the Saw-Tooth climb at 75 kts, the results yielded only 80% of the expected performance per the POH. The outside air temperature during testing was lower than a standard day and the test weight was less than the standard weight of the aircraft. As a result, the PIW values were greater than 180 HP. To assess what the climb rate, CIW, at the 180 HP, a linear trendline was forecasted. It should be noted that Figures 15-18 are included as

generic trendlines and should not be used to determine climb characteristics of the Piper Arrow given the limited data set.

Figure 15: PIW vs. CIW at 75 kts

At 80 kts, as shown in Figure 16, a PIW value of 180 HP yielded a CIW of approximately 833 ft/min. The data at 80 kts resulted in the highest climb rate of the four airspeed and had the closest performance to the POH documented climb rate for the PA-28-180.

Figure 16: PIW vs. CIW at 80 kts

The PIW and CIW relationships for 85 kts and 90 kts are shown in Figure 17 and Figure 18, respectively. The climb performance at 180 HP is significantly lower than the POH value of 875 ft/min. The deviation can be explained by the Saw-Tooth climbs being conducted at 2300 RPM power setting rather than maximum continuous power. To make a true comparison of the PIW versus CIW relationship, the data should reflect the maximum power setting to maximize the climb performance.

Figure 17: PIW vs. CIW at 85 kts

Figure 18: PIW vs. CIW at 90 kts

Figures 19-22 show the relationship between pressure altitude and rate of climb. As altitude increases, the rate of climb should decrease because the air density is decreasing, resulting in diminished engine performance. The plotted points are

from Table 10. The greatest ROC values within the pressure altitude band are seen at the two airspeeds closest to the best rate of climb airspeed, as expected. Although theory indicates rate of climb would decrease with pressure altitude, Figures 19-22 have a limited data set and show a forecasted trendline.

Figure 19: Hp vs. ROC at 75 kts

Figure 20: Hp vs. ROC at 80 kts

Figure 21: Hp vs. ROC at 85 kts

Figure 22: Hp vs. ROC at 90 kts

The results showed the decreased power setting negatively impacted the climb performance observed during the Saw-Tooth test. The rate of climb is a function of the excess power produced from the engine. Since the climbs were conducted at 2300 RPM, there was 15% less power available than the rated engine performance of the PA-28-180 (2700 RPM). Assuming the power required to conduct the climb remained constant, multiplying the specific power by an increase of power available factor of 15%, would give an approximation of what the rate of climb would have been if performed at maximum continuous power.

 $P_s = P_{av} - P_{req}$; assume P_{req} remains constant.

 $P_s = 1.15 * P_{av}$

 $ROC = \frac{P_s}{W}$

The calculated rate of climb can then be plugged into the equation for the instrument and weight corrected climb rate.

$$
CIW = \frac{ROC_{TC}\sqrt{\sigma}}{\sqrt{\frac{W_T}{W_S}}}
$$

The brake horsepower value for each test point was redetermined using the Lycoming Engine Chart in Figure 9 with a 2700 RPM power setting. The updated brake horsepower was used to approximate what the instrument and weight correct power for each test step would have been if operating at a higher power setting. The values for brake horsepower were approximately 10 HP greater at 2700 RPM than at 2300 RPM.

$$
P_{IW} = \frac{BHP_T\sqrt{\sigma}}{\left(\frac{W_T}{W_s}\right)^{\frac{3}{2}}}
$$

The results of the approximation are tabulated in Table 11. Compared to the results in Table 10, the instrument and weight corrected climb rate, CIW, was 15% higher at an engine power setting of 2700 RPM. The instrument and weight corrected power, PIW, averaged 5.6% higher at the full power setting. Although the climb rates in Table 10 would have satisfied the FAA minimum rate of climb requirements, the values in Table 11 are more representative of how the aircraft would have performed with maximum power available. As the rate of climb increased by 15%, so did the climb gradient. Table 11 shows that with a higher power setting, additional airspeeds (75 kts, 80 kt, and 85 kts) would have satisfied the minimum 8.3% climb gradient per 14 CFR 23.2120.

Level Acceleration Performance

The level accelerations were conducted at 3,500 ft, 4,500 ft, and 5,500 ft at a heading of 20 degrees, exactly perpendicular to the winds reported by ATIS. The level accelerations were repeated at the same three altitudes at the reciprocating test heading, 200 degrees. The analysis for the level accelerations was conducted in three parts: 20 degree heading data individually, 200 degree heading data individually, and the averaged data between the two headings. Each level acceleration began at the airspeed 10% greater than the stall speed with flaps up, V_{s1} , and ended at maximum speed in level flight at maximum continuous power, V_h . To determine the V_{s1} for the PA-28, the pilot stalled the aircraft at 63 kts, resulting in a starting speed of 69 kts. The airspeed versus elapsed time at each altitude is plotted in Figures 24-26.

At 3,500 ft, the level accelerations at a 20-degree heading and a 200-degree heading lasted 77 seconds and 79 seconds, respectively. As shown in Figure 23, the indicated airspeeds at each heading were closely related. The V_h for the 20-degree heading was 132 kts and the V_h for the 200-degree heading was 131 kts. The derivatives of the quadratic equation trendlines were determined to find the $\frac{dv}{dt}$ component of the specific excess power equation. The derivative at 20-degree

heading at 3,500 ft was -0.0326x+2.605 and the derivative at the 200-degree heading was -0.0372x+2.789.

Figure 23: Airspeed vs. Elapsed Time at 3500 ft

The 20-degree heading at 4,500 ft had a total elapsed time of 72 seconds with a recorded V_h of 127 kts, as shown in Figure 24. The reciprocating heading, at 200degrees, lasted 75 seconds with V_h of 128 kts. The data taken at 3,500 ft, the airspeeds at each heading at 4,500 ft were closely related. The derivative of the trendlines at 20-degree heading and 200-degree heading were -0.0396x+2.720 and - 0.372x+2.629, respectively.

Figure 24: Airspeed vs. Elapsed Time at 4500 ft

Figure 25, shows the relationship between airspeed and elapsed time at 5,500 ft for the 20-degree and 200-degree headings. Compared to the data collected at lower airspeeds, there was a larger variation in indicated airspeed between the two headings. The trendline derivative for the 20-degree curve was -0.0396x+2.720, with a V_h of 124 kts. The trendline derivative for the 200-degree curve was -0.0372x+2.629, with a V_h of 123 kts.

Figure 25: Airspeed vs. Elapsed Time at 5500 ft

Table 12 shows the trendline equations and derivative equations for each altitude and heading combination. An increase in altitude should increase the power required, and therefore decrease the power available. A result of the increase in altitude is a decrease in the maximum speed in level flight at maximum continuous power, V_h . The tabulated data reflects this relationship between excess power and indicated airspeed. Each 1,000 ft increase in altitude resulted in a 3-5 kts decrease in V_h for both headings. The V_h are visually depicted in Figures 27-35 as the calibrated airspeed at the lowest rate of climb.

Table 12: Airspeed vs. Elapsed Time Data

Altitude (ft)	20 _{deg}			200 deg		
	Trendline Eq	Derivative Eq	Vh (kts)	Trendline Eq	Derivative Eq	Vh (kts)
3500	$-0.0163v^{2}+2.6054v+117.37$	$-0.0326*v+2.605$	132	$-0.0186v^2+2.7894v+115.85$	$-0.0372*_{V}+2.789$	131
4500	$-0.0198v^{2}+2.7203v+120.01$	$-0.0396*_{V}+2.720$	127	$-0.0186v^{2}+2.629v+120.82$	$-0.0372*$ v $+2.629$	128
5500	$-0.0198v^{2}+2.7203v+120.01$	$-0.0396*v+2.720$	124	$-0.0186v^{2}+2.629v+120.82$	$-0.0372*v+2.629$	123

The rate of climb was computed and plotted against the calibrated airspeed at 3,500 ft, 4,500 ft, and 5,500 ft at a 20-degree heading in a clean (no flaps) configuration.

The maximum rate of climb airspeed, V_y , was determined by finding the airspeed associated with the maximum rate of climb for each dataset. The maximum angle of climb airspeed, V_x , was determined by finding the airspeed on the curve fit that was tangential from the plot origin. Figure 26: ROC vs. Airspeed at 3500 ft with 20 deg HeadingFigure 26 shows the rate of climb and v-speeds at 3,500 ft flying at a 20-degree heading. At 3,500 ft (20 degree heading), V_v was 150 ft/s (9000 ft/min) at a rate of climb of 402.8 ft/min and V_x was 130 ft/s (7800 ft/min).

Figure 26: ROC vs. Airspeed at 3500 ft with 20 deg Heading

At 4,500 ft and a 20-degree heading, as shown in Figure 27, the maximum rate of climb was 413.6 ft/min at an airspeed, V_v , of 140 ft/s (8400 ft/min). The maximum angle of climb airspeed, V_x , was 132 ft/s (7920 ft/min).

Figure 27: ROC vs. Airspeed at 4500 ft with 20 deg Heading

At 5,500 ft and a 20-degree heading, as shown in Figure 28, the maximum rate of climb was 357.02 ft/min at an airspeed, V_y , of 137 ft/s (8220 ft/min). The maximum angle of climb airspeed, V_x , was 127 ft/s (7620 ft/min). The highest altitude produced the lowest rate of climb since the air was less dense at 5,500 ft, resulting in less available power being produced by the engine. As the altitude increased, the maximum rate of climb airspeed decreased. This relationship was shown in Figure 5, illustrating how V_y and V_x converge at the absolute ceiling.

Figure 28: ROC vs. Airspeed at 5500 ft with 20 deg Heading

At 3,500 ft and a 200-degree heading, as shown in Figure 29, the maximum rate of climb was 415 ft/min at an airspeed, V_y , of 142 ft/s (8520 ft/min). The maximum angle of climb airspeed, V_x , was 132 ft/s (7920 ft/min).

Figure 29: ROC vs. Airspeed at 3500 ft with 200 deg Heading

At 4,500 ft and a 200-degree heading, as shown in Figure 30, the maximum rate of climb was 407.1 ft/min at an airspeed, V_y , of 147 ft/s (8820 ft/min). The maximum angle of climb airspeed, V_x , was 137 ft/s (8220 ft/min).

Figure 30: ROC vs. Airspeed at 4500 ft with 200 deg Heading

At 5,500 ft and a 200-degree heading, as shown in Figure 31, the maximum rate of climb was 404.4 ft/min at an airspeed, V_y , of 135 ft/s (8100 ft/min). The maximum angle of climb airspeed, V_x , was 116 ft/s (6960 ft/min).

Figure 31: ROC vs. Airspeed at 5500 ft with 200 deg Heading

The averaged heading data was determined by taking the average of each test point in the 20-degree and 200-degree headings for each altitude. For example, test point 1 for the 20-degree heading was averaged with test point 1 for the 200-degree heading, and so forth. The average rate of climb and calibrated airspeeds are shown in Figures 33-35 below.

Figure 32 is the rate of climb versus calibrated airspeed for the averaged heading data at 3,500 ft. The maximum rate of climb was 408.3 ft/min, occurring at a V_y of 143 ft/s (8557 ft/min). From the graph, the V_x was determined to be 132 ft/s (7920 ft/min).

Figure 32: ROC vs. Airspeed at 3500 ft with Averaged Heading

The maximum rate of climb at 4,500 ft with an averaged heading was 408.6 ft/min. The values for V_v and V_x were determined to be 138 ft/s (8304 ft/min) and 126 ft/s (7560 ft/min), respectively, using the plot in Figure 33. Compared to the 20-degree heading data at 4,500 ft, in Figure 27, and the 200-degree heading data at 4,500 ft, in Figure 30, the averaged heading resulted in a lower maximum rate of climb airspeed, V_v .

Figure 33: ROC vs. Airspeed at 4500 ft with 200 deg with Averaged Heading

Figure 34 is the rate of climb versus calibrated airspeed for the averaged heading data at 5,500 ft. The maximum rate of climb was 379.5 ft/min, occurring at a V_y of 138 ft/s (8253 ft/min). From the graph, the V_x was determined to be 131 ft/s (7860 ft/min).

Figure 34: ROC vs. Airspeed at 5500 ft with 200 deg with Averaged Heading

Table 13 is the tabulated data plotted in Figures 27-35. At each altitude, the 20 degree heading data, 200-degree heading data, and averaged heading data all satisfied the climb performance requirements at the time of certification under the CAR 3.85 regulation. The regulation states that the aircraft must climb at a minimum rate of 300 ft/min. As seen in the table, every test configuration exceeded a maximum rate of climb of 300 ft/min. In both the individual headings and the averaged heading, the data shows a decrease in rate of climb with an increase in altitude. This is expected due to the decrease in power available produced by the engine at higher altitudes due to the air being less dense.

Table 13: ROC vs. Airspeed Tabulated Data

Chapter 5: Conclusion

This thesis aimed to quantitatively compare the climb performance regulations detailed in CAR 3.85a and 14 CFR 23.2120. A series of Saw-Tooth steady climbs were conducted in a PA-28-180 Piper Arrow, at varying airspeeds. The data was reduced to analyze the experimental rate of climb at each airspeed and the climb gradient at each test point. The analysis showed each test airspeed met the requirements stated in CAR 3.85a; however, only the data collected at 80 kts consistently performed per 14 CFR 23. 2120.The second objective of the thesis was to qualitatively evaluate the use of reciprocating headings when conducting flights per the Level Acceleration method. The data supported the hypothesis that reciprocating headings do not have a significant impact on the results since the data is instrument and weight corrected at each test step. The results of this test may be used as justification to modify climb performance test methodologies. The additional resources and costs required to fly test points at reciprocating headings is unnecessary if testing with minimal wind conditions.

Chapter 6: Future Work

The data collected for the thesis was limited by aircraft availability. To further improve and expand on this topic, a flight test instrumented aircraft would be advantageous. Multiple flights at various weight configurations would further augment the flight analysis data.

References

[1] R. D. Kimberlin, *Flight Testing of Fixed-Wing Aircraft*, American Institute of Aeronautics and Astronautics, Inc., 2003

- [2] "Pilot's Handbook of Aeronautical Knowledge." *Pilot's Handbook of Aeronautical Knowledge | Federal Aviation Administration*, https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/phak. Accessed 11 Jan 2023.
- [3] B. Kish and R. D. Kimberlin, *FTE 5701 Airplane Performance Flight Testing Laboratory Manual*, Florida Institute of Technology, 2022.
- [4] "Cherokee "Arrow" Owner's Handbook," *Piper Aircraft*, Vero Beach, FL, 1967.

[5] "Approved Aeroplane Flight Manual for the Piper Model PA-28R-180," *Piper Aircraft*, Vero Beach, FL, 1967.

- [6] The National Archives of the United States. "14 CFR 23.2120." *Code of Federal Regulations,* https://www.ecfr.gov/current/title-14/chapter-I/subchapter-C/part-23/subpart-B/subject-group-ECFR0f27e01839859ea/section-23.2120. Accessed 5 Mar 2023.
- [7] The National Archives of the United States. "Federal Register." *GovInfo*, https://www.govinfo.gov/content/pkg/FR-1950-12-15/pdf/FR-1950-12- 15.pdf#page=1. Accessed 5 Mar 2023.
- [8] "N3911T Flight Tracking and History 10-Jan-2023 (Khyx KHYX)." *FlightAware*, https://flightaware.com/live/flight/N3911T/history/20230110/1713Z/KHYX/ KHYX. Accessed 11 Jan 2023.
- [9] "Operator's Manual Lycoming," *Lycoming,* Williamsport, PA, 2005.

Appendix A: PA-28-180 Specifications

Figure 35: PA-28-180 Specifications

Appendix B: Flight Cards

Figure 36: Saw-Tooth Method Flight Card

Figure 37: Level Acceleration Flight Card
Appendix C: Data Reduction

Saw-Tooth Climb

Level Acceleration

