Rendezvous and Proximity Operations in Cis-Lunar Space

Khalid Jebari
Rendezvous and Proximity Operations in Cis-Lunar Space

by

Khalid Jebari

Bachelor of Science
Mechanical Engineering
Wentworth Institute of Technology
2019

A thesis
submitted to the College of Engineering and Science
at Florida Institute of Technology
in partial fulfillment of the requirements
for the degree of

Master of Science
in
Aerospace Engineering

Melbourne, Florida
May, 2023
We the undersigned committee hereby approve the attached thesis

Rendezvous and Proximity Operations in Cis-Lunar Space by Khalid Jebari

Madhur Tiwari, Ph.D.
Assistant Professor
Aerospace, Physics and Space Sciences
Major Advisor

Xianqi Li, Ph.D.
Assistant Professor
Mathematical Sciences

Eric D. Swenson, Ph.D.
Associate Professor
Aerospace, Physics and Space Sciences

David Fleming, Ph.D.
Associate professor and Department Head
Aerospace, Physics and Space Sciences
Abstract

Title:
Rendezvous and Proximity Operations in Cis-Lunar Space

Author:
Khalid Jebari

Major Advisor:
Madhur Tiwari, Ph.D.

As interest in Moon exploration grows and efforts to establish an orbiting outpost intensify, accurate modeling of spacecraft dynamics in cis-lunar space is becoming increasingly important. Contrary to satellites in Low Earth Orbit (LEO) where it takes around 5 ms to communicate back and forth with a ground station, in the Moon’s orbit it can take up to 2.4 seconds to do so. This delay in communication can make the difference between a successful docking and a catastrophic collision for a remotely controlled satellite. Moreover, due to the unstable nature of trajectories in cis-lunar space, it is necessary to design spacecraft that can make the frequent and needed maneuvers to stay on track. The communication delay and unstable trajectories are exactly why autonomous navigation is critical for proximity operations and rendezvous and docking missions in cis-lunar space.

In this paper, the relative equations of motion are derived, linearized, and a simulation is performed to compare state estimation results obtained from using the linearized dy-
namics equations of motion along with a Kalman filter and the nonlinear equations of
motion along with an Unscented Kalman filter. After it was shown that the linearized
model was sufficient for state estimation in the presence of noisy measurements, an
LQR (Linear Quadratic Regulator) controller was added to optimally control a space-
craft and successfully dock with another. The contribution of this work is twofold: to
compare the results obtained from the linearized model and the nonlinear model as
well as to simulate an optimal docking maneuver in cis-lunar space using the linearized
equations of motion in the presence of measurement noise.
# Table of Contents

Abstract ................................................................. iii

List of Figures .......................................................... vii

Abbreviations ........................................................... ix

Acknowledgments ........................................................ x

1 Introduction ............................................................ 1

2 Theory ................................................................. 3
   2.1 Three-body Dynamics .............................................. 3
      2.1.1 Single Spacecraft .......................................... 3
      2.1.2 Relative Motion ............................................. 9
      2.1.3 Linearization ............................................... 13
   2.2 State Estimation .................................................. 15
      2.2.1 Linear Model (Kalman Filter) ............................... 15
      2.2.2 Non-linear Model (Unscented Kalman Filter) ............. 17
   2.3 Controller Design ................................................ 19

3 Results .............................................................. 21
   3.1 State Estimation ................................................. 21
      3.1.1 KMF ...................................................... 22
List of Figures

2.1 Three-body model in the barycentric frame. .............................................. 4
2.2 Relative three-body model in the barycentric frame. .............................. 10

3.1 Comparison between measured, estimated (KMF), and true position (x-axis) ........................................................................... 23
3.2 Comparison between measured, estimated (KMF), and true position (y-axis) ........................................................................... 24
3.3 Comparison between measured, estimated (KMF), and true position (z-axis) ........................................................................... 24
3.4 Comparison between measured, estimated (KMF), and true velocity (x-axis) ........................................................................... 25
3.5 Comparison between measured, estimated (KMF), and true velocity (y-axis) ........................................................................... 25
3.6 Comparison between measured, estimated (KMF), and true velocity (z-axis) ........................................................................... 26
3.7 Comparison between measured, estimated (UKF), and true position (x-axis) ........................................................................... 27
3.8 Comparison between measured, estimated (UKF), and true position (y-axis) ........................................................................... 28
3.9 Comparison between measured, estimated (UKF), and true position (z-axis) ........................................................................... 28
3.10 Comparison between measured, estimated (UKF), and true velocity (x-axis) .......................................................... 29
3.11 Comparison between measured, estimated (UKF), and true velocity (y-axis) .......................................................... 29
3.12 Comparison between measured, estimated (UKF), and true velocity (z-axis) .......................................................... 30
3.13 Comparison between position percentage errors from KMF and UKF (x-axis) .......................................................... 31
3.14 Comparison between position percentage errors from KMF and UKF (y-axis) .......................................................... 31
3.15 Comparison between position percentage errors from KMF and UKF (z-axis) .......................................................... 32
3.16 Comparison between velocity percentage errors from KMF and UKF (x-axis) .......................................................... 32
3.17 Comparison between velocity percentage errors from KMF and UKF (y-axis) .......................................................... 33
3.18 Comparison between velocity percentage errors from KMF and UKF (z-axis) .......................................................... 33
3.19 Plot of relative position (x-axis). ............................................. 36
3.20 Plot of relative position (y-axis). ............................................. 36
3.21 Plot of relative position (z-axis). ............................................. 37
3.22 Plot of relative velocity (x-axis). ............................................. 37
3.23 Plot of relative velocity (y-axis). ............................................. 38
3.24 Plot of relative velocity (z-axis). ............................................. 38
3.25 Plot of the control input in the three axes. ............................ 39
**List of Symbols, Nomenclature or Abbreviations**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td>GEO</td>
<td>Geostationary Earth Orbit</td>
</tr>
<tr>
<td>CR3BP</td>
<td>Circular Restricted Three-Body problem</td>
</tr>
<tr>
<td>ER3BP</td>
<td>Elliptic Restricted Three-Body problem</td>
</tr>
<tr>
<td>LVLH</td>
<td>Local Vertical Local Horizontal</td>
</tr>
<tr>
<td>KMF</td>
<td>Kalman Filter</td>
</tr>
<tr>
<td>UKF</td>
<td>Unscented Kalman Filter</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>$n$</td>
<td>Moon Mean Motion</td>
</tr>
<tr>
<td>$|\cdot|$</td>
<td>Euclidean Norm</td>
</tr>
<tr>
<td>$I$</td>
<td>Identity Matrix</td>
</tr>
<tr>
<td>$\tilde{r}$</td>
<td>Nondimensional Quantity</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Nondimensional Time Quantity</td>
</tr>
<tr>
<td>$\Omega_{B/I}$</td>
<td>Skew-symmetric matrix</td>
</tr>
<tr>
<td>$\mathbf{R}$</td>
<td>Vector</td>
</tr>
<tr>
<td>$R$</td>
<td>Norm of Vector</td>
</tr>
<tr>
<td>$[\mathbf{R}]_I$</td>
<td>Vector expressed in Inertial Frame</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Relative State Vector</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular Velocity</td>
</tr>
<tr>
<td>$G$</td>
<td>Gravitational Constant</td>
</tr>
</tbody>
</table>
Acknowledgements

I would like to express my deepest gratitude to my parents and siblings for their unwavering support throughout my academic journey. Without their financial and moral support, this accomplishment would not have been possible.

I would also like to extend my sincere thanks to my thesis advisor, Dr. Tiwari, and his friend Dr. Zuehlke for their guidance throughout the research process. Their expertise and insights were instrumental in shaping the direction of my work.

Lastly, I would like to acknowledge the support of my friends and colleagues who have encouraged me and supported me in many ways throughout this journey.
Chapter 1

Introduction

Until now, space missions have predominantly taken place in Low Earth Orbit (LEO) or Geostationary Orbit (GEO) [6]. Relative equations of motion like the Hill-Clohessy-Wiltshire [3] and Tschauner-Hempel [11] have been tested and validated over the years in proximity operations in circular and elliptical orbits around Earth. These equations are only valid for a two-body system where the spacecraft is influenced by one planet’s gravity alone. In cis-lunar space, spacecraft are not only affected by the Moon’s gravity but by the Earth’s gravity as well, hence the name three-body problem.

Recently, there has been a growing interest in lunar exploration in order to pave the way for the ultimate goal of landing humans on Mars. There are even plans of establishing an outpost that will orbit around the Moon and serve as a research lab and station for astronauts [13]. These efforts will require the development of autonomous navigation algorithms to dock with the outpost and perform the frequent maneuvers needed to stay on track in the highly unstable cis-lunar trajectories.

Franzini et al. [4, 5] developed the relative equations of motion for a circular restricted three-body problem (CR3BP) in the Local Vertical Local Horizontal (LVLH) frame that was used in numerous papers to investigate rendezvous scenarios with a
passive spacecraft in orbit around the Moon [10, 1, 2, 7]. Zuehlke et al. [15] investigated the regions of application of the linear equations of motion in an elliptical restricted three-body problem (ER3BP) and Innocenti et al. [7] compared results of nonlinear CR3BP, linear CR3BP, and linear ER3BP equations.

Contrary to the previous works where state propagation results using nonlinear and linearized CR3BP equations were compared. The contribution of this work is twofold: Compare how accurate the results for states estimation are when the linearized equations of motion are used instead of the nonlinear equations of motion as well as to simulate a successful docking scenario with a spacecraft in cis-lunar space using the linearized equations along with a state estimator and an optimal controller.

The paper is organized as follows, in Chapter 1 a brief introduction is given. Chapter 2 covers the derivation of relative equations of motion and linearization followed by a discussion on the Kalman filter (KMF) and unscented Kalman filter (UKF) algorithms as well as the LQR controller. In Chapter 3, results from a simulation comparing the KMF and UKF state estimation results as well as a simulation of a docking scenario between two spacecraft are presented. Finally, conclusions are presented in Chapter 4.
Chapter 2

Theory

In this chapter, the derivation of equations of motion for the case of a CR3BP [10, 14] is presented. First, the equations of motion are derived for a single spacecraft and the relative equations of motion are derived next. Then two Kalman filter algorithms (KMF and UKF) that can be used for state estimation are discussed. Finally, an optimal controller (LQR) that will be used to control the deputy spacecraft to dock with the chief spacecraft is presented.

2.1 Three-body Dynamics

2.1.1 Single Spacecraft

To simplify the equations of motion they will be written with respect to the barycentric frame (see Figure 2.1) centered at the center of mass of the Moon and Earth. The barycentric frame is also assumed to be rotating with the Moon around Earth at an angular velocity $\omega_{B/I} = \omega_B/\hat{k}_B$.

When considering the CR3BP the following assumptions need to be made: The Moon’s
orbit around Earth is circular and the spacecraft’s mass is very negligible compared to the one of the Moon and Earth.  

Note that all vectors are represented by bold characters and an uppercase subscript corresponds to a vector written w.r.t. the inertial frame while a lowercase subscript corresponds to a vector written w.r.t. the barycentric frame.

To derive the equations that govern the motion of a spacecraft in a CR3BP Newton’s third law of motion is first applied.

\[ \sum \mathbf{F} = m \mathbf{a} \]  

(2.1)

The only forces evaluated in this model are the gravitational forces applied by each of the three bodies (Earth, Moon, and spacecraft).

Figure 2.1: Three-body model in the barycentric frame.
The gravitational force between two bodies is defined as:

$$ F = \frac{G m_1 m_2}{r^3} \mathbf{r} $$ \hspace{1cm} (2.2)

G is the gravitational constant, $m_1$ and $m_2$ are the masses of the bodies, $\mathbf{r}$ is the vector pointing from body 1 to body 2, and $r$ is the distance between the centers of their masses. After applying Newton’s third law to the spacecraft, the following is obtained:

$$ m_s \ddot{\mathbf{r}}_S = -\frac{G m_E m_s}{r_{ES}^3} \mathbf{r}_{ES} - \frac{G m_M m_S}{r_{MS}^3} \mathbf{r}_{MS} \hspace{1cm} (2.3) $$

$$ \ddot{\mathbf{r}}_S = -\frac{G m_E}{r_{ES}^3} \mathbf{r}_{ES} - \frac{G m_M}{r_{MS}^3} \mathbf{r}_{MS} \hspace{1cm} (2.4) $$

where, $\ddot{\mathbf{r}}_S$ is the acceleration of the spacecraft expressed in the inertial frame ($I$).

Same follows for the Earth and Moon:

$$ \ddot{\mathbf{r}}_E = \frac{G m_M}{r_{EM}^3} \mathbf{r}_{EM} + \frac{G m_S}{r_{ES}^3} \mathbf{r}_{ES} \hspace{1cm} (2.5) $$

$$ \ddot{\mathbf{r}}_M = -\frac{G m_E}{r_{EM}^3} \mathbf{r}_{EM} + \frac{G m_S}{r_{MS}^3} \mathbf{r}_{MS} \hspace{1cm} (2.6) $$

with $\mathbf{r}_{ES} = \mathbf{R}_S - \mathbf{R}_E$, $\mathbf{r}_{MS} = \mathbf{R}_S - \mathbf{R}_M$, and $\mathbf{r}_{EM} = \mathbf{R}_M - \mathbf{R}_E$ being the relative position vectors from $m_E$ to $m_S$, $m_M$ to $m_S$, and $m_E$ to $m_M$, respectively.

$$ \ddot{\mathbf{r}}_{ES} = \ddot{\mathbf{r}}_S - \ddot{\mathbf{r}}_E \hspace{1cm} (2.7) $$

$$ \ddot{\mathbf{r}}_{ES} = \left[ -\frac{G m_E}{r_{ES}^3} \mathbf{r}_{ES} - \frac{G m_M}{r_{MS}^3} \mathbf{r}_{MS} \right] - \left[ \frac{G m_M}{r_{EM}^3} \mathbf{r}_{EM} + \frac{G m_S}{r_{ES}^3} \mathbf{r}_{ES} \right] m_S \hspace{1cm} (2.8) $$

$$ \ddot{\mathbf{r}}_{ES} = \left[ -\frac{G m_E}{r_{ES}^3} \mathbf{r}_{ES} - \frac{G m_S}{r_{ES}^3} \mathbf{r}_{ES} \right] \mathbf{r}_{ES} - \frac{G m_M}{r_{MS}^3} \mathbf{r}_{MS} - \frac{G m_M}{r_{EM}^3} \mathbf{r}_{EM} \hspace{1cm} (2.9) $$
Since $m_S$ is assumed to be very negligible compared to $m_E$ and $m_M$ in the CR3BP, the term $\frac{Gm_S}{r_{ES}^3}$ is neglected.

\[
\begin{align*}
\ddot{r}_{ES}^I &= -\frac{Gm_E}{r_{ES}^3}r_{ES} - \frac{Gm_M}{r_{MS}^3}r_{MS} - \frac{Gm_M}{r_{EM}^3}r_{EM} \quad (2.10) \\
\ddot{r}_{ES}^I &= -\mu_E \frac{r_{ES}}{r_{ES}^3} - \mu_M \left( \frac{r_{MS}}{r_{MS}^3} + \frac{r_{EM}}{r_{EM}^3} \right) \quad (2.11)
\end{align*}
\]

Same thing follows for $r_{MS}$:

\[
\ddot{r}_{MS}^I = -\mu_E \left[ \frac{r_{ES}}{r_{ES}^3} - \frac{r_{EM}}{r_{EM}^3} \right] - \mu_M \frac{r_{MS}}{r_{MS}^3} \quad (2.12)
\]

where $\mu_E = Gm_E$ and $\mu_M = Gm_M$ are the standard gravitation parameters of Earth and the Moon, respectively.

The position vector of the spacecraft w.r.t. the barycentric frame is defined as:

\[
R_s = x\hat{i}_B + y\hat{j}_B + z\hat{k}_B \quad (2.13)
\]

Since the barycentric frame $(B)$ rotates with respect to the inertial frame $(I)$ with an angular velocity $\omega_{B/I} = \omega_B/\hat{k}_B$

\[
\ddot{R}_s^I = \dot{R}_s^I + \omega_{B/I} \times R_s^I \quad (2.14)
\]

By taking a second derivative and while taking into consideration that the $B$ frame rotates:

\[
\ddot{R}_s^I = \ddot{R}_s^B + 2\omega_{B/I} \times \dot{R}_s^I + [\omega_{B/I}] \times R_s + \omega_{B/I} \times \omega_{B/I} \times R_s \quad (2.15)
\]
From Equation (2.4) and (2.15), it follows that:

\[
\begin{align*}
\ddot{\mathbf{R}}_s + 2\omega_B/I \times \dot{\mathbf{R}}_s + \dot{\omega}_B/I \times \mathbf{R}_s + \omega_B/I \times (\mathbf{\omega}_B/I \times \mathbf{R}_s) &= -\mu_E \frac{\mathbf{r}_{es}}{r_{es}^3} - \mu_M \frac{\mathbf{r}_{ms}}{r_{ms}^3}, \\
(2.16)
\end{align*}
\]

where \( \mathbf{r}_{es} \) and \( \mathbf{r}_{ms} \) represent the spacecraft’s position vectors with respect to \( m_E \) and \( m_M \), measured from the barycenter \( B \).

\[
\begin{align*}
\mathbf{r}_{es} &= (x + R_e) \mathbf{i}_B + y \mathbf{j}_B + z \mathbf{k}_B \quad (2.17) \\
\mathbf{r}_{ms} &= (x - R_m) \mathbf{i}_B + y \mathbf{j}_B + z \mathbf{k}_B \quad (2.18)
\end{align*}
\]

Equation (2.16) can be written in component-wise form as follows:

\[
\begin{align*}
\ddot{x} - 2\omega_B/I \dot{y} - \dot{\omega}_B/I y - \omega_B/I x &= -\mu_E \frac{x + R_e}{r_{es}^3} - \mu_M \frac{x - R_m}{r_{ms}^3} \\
\ddot{y} + 2\omega_B/I \dot{x} + \dot{\omega}_B/I x - \omega_B/I y &= -\mu_E \frac{y}{r_{es}^3} - \mu_M \frac{y}{r_{ms}^3} \\
\ddot{z} &= -\mu_E \frac{z}{r_{es}^3} - \mu_M \frac{z}{r_{ms}^3} \\
(2.19, 2.20, 2.21)
\end{align*}
\]

where the norm of \( \mathbf{r}_{es} \) and \( \mathbf{r}_{ms} \) are expressed as:

\[
\begin{align*}
\mathbf{r}_{es} &= \sqrt{(x + R_e)^2 + y^2 + z^2} \\
\mathbf{r}_{ms} &= \sqrt{(x - R_m)^2 + y^2 + z^2} \\
(2.22, 2.23)
\end{align*}
\]

To simplify the equations further, nondimensional quantities are used in order to write the equations in function of one parameter.

- Time \( t \) is nondimensionalized by the system mean motion \( n \):
\[ n = \sqrt{\frac{G (m_E + m_M)}{a^3}} \]

where, \( a \) is the semimajor axis of the moon. The nondimensional time quantity \( \tau \) is also introduced, such that: \( \tau = n(t - t_0) \)

- Mass quantities are nondimensionalized such that: \( \mu_E + \mu_M = 1 \)
  The system mass parameter \( \mu \) is also defined as: \( \mu = m_M / (m_E + m_M) \), which makes:
  \[ \mu_M = \mu \quad \mu_E = 1 - \mu \]

- Distances are nondimensionalized by the semimajor axis \( a \) of the moon:
  \[ r = a \tilde{r} \quad \dot{r} = a \frac{d \tilde{r}}{d \tau} \frac{d \tau}{dt} = an \tilde{r}' \quad \ddot{r} = an^2 \frac{d \tilde{r}'}{d \tau} \frac{d \tau}{dt} = an^2 \tilde{r}'' \]
  \( \tilde{r} \) denotes a nondimensional quantity and \( \tilde{r}' \) its derivative with respect to \( \tau \)

- Angular velocities are nondimensionalized as follow:
  \[ \omega = n \tilde{\omega} \quad \dot{\omega} = n^2 \tilde{\omega}' \frac{d \tau}{dt} \frac{d \tau}{dt} = n^2 \tilde{\omega}'' \]

In the case of a CR3BP the following is also used:

- The nondimensional angular velocity for the barycentric frame \( \omega_{B/I} = 1 \)
- The distance between the moon and earth \( (r_{em}) \) is constant and therefore:
  \[ \mathbf{R}_e = -\mu \mathbf{i}_B \quad \mathbf{R}_m = (1 - \mu) \mathbf{i}_B \quad r_{em} = \mathbf{i}_B \]
Hence, the equations that govern the motion of a spacecraft in cis-lunar space can be written as follow:

\[
\ddot{x}'' - 2\dot{y}' - \ddot{x} = -(1 - \mu) \frac{\ddot{x} + \mu}{\tilde{r}_{es}^3} - \frac{\ddot{x} - 1 + \mu}{\tilde{r}_{ms}^3} 
\]

\[
\ddot{y}'' + 2\ddot{x}' - \dot{y} = -(1 - \mu) \frac{\dot{y}}{\tilde{r}_{es}^3} - \frac{\dot{y}}{\tilde{r}_{ms}^3} 
\]

\[
\ddot{z}'' = -(1 - \mu) \frac{\ddot{z}}{\tilde{r}_{es}^3} - \frac{\ddot{z}}{\tilde{r}_{ms}^3} 
\]

where:

\[
\tilde{r}_{es} = \sqrt{(\tilde{x} + \mu)^2 + \tilde{y}^2 + \tilde{z}^2} 
\]

\[
\tilde{r}_{ms} = \sqrt{(\tilde{x} - 1 + \mu)^2 + \tilde{y}^2 + \tilde{z}^2} 
\]

### 2.1.2 Relative Motion

Since what is important in this work is the motion of a deputy spacecraft relative to a chief spacecraft (see Figure 2.2). The derivation of the relative equations of motion in the barycentric coordinate frame is presented next.

The relative state vector \( \rho \) is defined as:

\[
\rho = \mathbf{R}_d - \mathbf{R}_c 
\]

and the chief’s state is given by \( \mathbf{X} = \begin{bmatrix} x_c & y_c & z_c & \dot{x}_c & \dot{y}_c & \dot{z}_c \end{bmatrix}^T \)

By taking the derivative of Equation (2.29):

\[
\frac{d}{dt} [\rho] = \frac{d}{dt} [\mathbf{R}_d] - \frac{d}{dt} [\mathbf{R}_c] 
\]
Figure 2.2: Relative three-body model in the barycentric frame.

Due to the rotation of the barycentric frame ($B$) w.r.t. the inertial frame ($I$), it follows that:

$$\ddot{\rho}_I = \dot{\rho} + \omega_{B/I} \times \rho$$  \hspace{1cm} (2.31)

and

$$\ddot{\rho}_I = \frac{d}{dt} \left[ \dot{\rho} + \omega_{B/I} \times \rho \right] + \omega_{B/I} \times \left[ \dot{\rho} + \omega_{B/I} \times \rho \right]$$  \hspace{1cm} (2.32)

$$\ddot{\rho}_I = \dot{\rho} + \omega_{B/I} \times \rho + \omega_{B/I} \times \dot{\rho} + \omega_{B/I} \times \dot{\rho} + \omega_{B/I} \times (\omega_{B/I} \times \rho)$$  \hspace{1cm} (2.33)

By taking the derivative of Equation (2.30):

$$\ddot{\rho}_I = \dddot{R}_d - \dddot{R}_c$$  \hspace{1cm} (2.34)
From Equations 2.33 and 2.34

\[
\ddot{R}_d - \ddot{R}_c = \dot{\rho} + \dot{\omega}_{B/I} \times \rho + 2\omega_{B/I} \times \dot{\rho} + \omega_{B/I} \times (\omega_{B/I} \times \rho)
\]  

(2.35)

After rearranging the above:

\[
\ddot{\rho} = -\omega_{B/I} \times \rho - 2\omega_{B/I} \times \dot{\rho} - \omega_{B/I} \times (\omega_{B/I} \times \rho) + \ddot{R}_d - \ddot{R}_c
\]  

(2.36)

From Equation (2.4) and assuming that the chaser spacecraft can be controlled, it follows that:

\[
\ddot{R}_d = -\frac{\mu_e r_{ed}}{r_{ed}^3} - \frac{\mu_m r_{md}}{r_{md}^3} + \frac{F_{\text{Thrust}}}{m_d}
\]  

(2.37)

\[
\ddot{R}_c = -\frac{\mu_e r_{ec}}{r_{ec}^3} - \frac{\mu_m r_{mc}}{r_{mc}^3}
\]  

(2.38)

By substituting in Equation (2.36):

\[
\ddot{\rho}_B = -2\omega_{B/I} \times \dot{\rho}_B - \omega_{B/I} \times \rho - \omega_{B/I} \times (\omega_{B/I} \times \rho)
\]  

(2.39)

\[
\ddot{\rho}_B = -2\omega_{B/I} \times \dot{\rho}_B - \omega_{B/I} \times \rho - \omega_{B/I} \times (\omega_{B/I} \times \rho)
\]  

(2.40)

\[
\ddot{\rho}_B = -2\omega_{B/I} \times \dot{\rho}_B - \omega_{B/I} \times \rho - \omega_{B/I} \times (\omega_{B/I} \times \rho)
\]  

(2.41)

A skew-symmetric matrix \( \Omega_{B/I} \) is defined to replace the cross-product in the equa-
\[
\Omega_{B/I} = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}
\] (2.42)

After substituting in Equation (2.41):

\[
\ddot{\rho}_B = -2\Omega_{B/I} \dot{\rho}_B - \dot{\Omega}_{B/I} \rho_B - \Omega_{B/I} \ddot{\rho}_B + \mu_e \left[ \frac{r_{ec}}{r_{ec}^3} - \frac{r_{ed}}{r_{ed}^3} \right] + \mu_m \left[ \frac{r_{mc}}{r_{mc}^3} - \frac{r_{md}}{r_{md}^3} \right] + \frac{F_{Thrust}}{m_d}
\]

Finally, the nonlinear equations of relative motion can be written in component-wise form as follow:

\[
\ddot{\rho}_x = 2\omega \dot{\rho}_y - \omega^2 \rho_x + \frac{\mu_e}{r_{mc}^3} (x_c + R_e) - \frac{\mu_e}{r_{ed}^3} (x_c + R_e + \rho_x) + \frac{\mu_m}{r_{mc}^3} (x_c - R_m) + \frac{F_{xThrust}}{m_d}
\] (2.43)

\[
\ddot{\rho}_y = -2\omega \dot{\rho}_x - \omega^2 \rho_y + \frac{\mu_e}{r_{ec}^3} y_c - \frac{\mu_e}{r_{ed}^3} (y_c + \rho_y) + \frac{\mu_m}{r_{mc}^3} y_c - \frac{\mu_m}{r_{md}^3} (y_c + \rho_y) + \frac{F_{yThrust}}{m_d}
\] (2.44)

\[
\ddot{\rho}_z = \frac{\mu_e}{r_{ec}^3} (z_c + \rho_z) - \frac{\mu_e}{r_{ed}^3} z_c - \frac{\mu_m}{r_{mc}^3} (z_c + \rho_z) + \frac{F_{zThrust}}{m_d}
\] (2.45)

The above equations govern the motion of a deputy spacecraft relative to a chief and will be referred to as nonlinear relative equations of motion in this paper.

Note that the distance from Earth to chief, Earth to deputy, Moon to chief, and Moon
to deputy are defined as follow:

\[ r_{ec} = \|R_c - R_e\| = \sqrt{(x_c + R_e)^2 + y_c^2 + z_c^2} \]  \hspace{1cm} (2.46)

\[ r_{ed} = \|R_c - R_e + \rho\| = \sqrt{(x_c + R_e + x)^2 + (y_c + y)^2 + (z_c + z)^2} \]  \hspace{1cm} (2.47)

\[ r_{mc} = \|R_c - R_m\| = \sqrt{(x_c - R_m)^2 + y_c^2 + z_c^2} \]  \hspace{1cm} (2.48)

\[ r_{md} = \|R_c - R_m + \rho\| = \sqrt{(x_c - R_m + x)^2 + (y_c + y)^2 + (z_c + z)^2} \]  \hspace{1cm} (2.49)

### 2.1.3 Linearization

Here the nonlinear relative equations of motion are linearized about an initial point \( X_0 \) using the Jacobian method.

Let \( X \) denote the relative state vector:

\[ X = \begin{bmatrix} \rho_x & \rho_y & \rho_z & \dot{\rho}_x & \dot{\rho}_y & \dot{\rho}_z \end{bmatrix}^T \]

\[ \dot{X} = \begin{bmatrix} \dot{\rho}_x & \dot{\rho}_y & \dot{\rho}_z & \ddot{\rho}_x & \ddot{\rho}_y & \ddot{\rho}_z \end{bmatrix}^T \]

In state space format, the relative equations of motion can be written in the following form:

\[ \dot{X} = A(t)X_0 + BU \]  \hspace{1cm} (2.50)

where \( A(t) \) and \( B \) are the Jacobian and input matrix, \( X_0 \) and \( U \) are the initial state and input vector, respectively.

\[ B = \begin{bmatrix} 0_{3 \times 3} \\ I_{3 \times 3} \end{bmatrix} \]

\[ U = \begin{bmatrix} F_x \\ m_d \frac{F_x}{m_d} \\ F_y \\ m_d \frac{F_y}{m_d} \\ F_z \\ m_d \frac{F_z}{m_d} \end{bmatrix} \]
where $0_{3\times3}$ and $I_{3\times3}$ are the zero and identity matrix, respectively.

And,

$$A(t) = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ A_\rho & A_\dot{\rho} \end{bmatrix}$$  \hspace{1cm} (2.51)

Where:

$$A_\rho = \begin{bmatrix} \frac{\partial \bar{a}_x}{\partial p_x} & \frac{\partial \bar{a}_y}{\partial p_y} & \frac{\partial \bar{a}_z}{\partial p_z} \\ \frac{\partial \bar{\rho}_x}{\partial p_x} & \frac{\partial \bar{\rho}_y}{\partial p_y} & \frac{\partial \bar{\rho}_z}{\partial p_z} \\ \frac{\partial \bar{\rho}_x}{\partial p_x} & \frac{\partial \bar{\rho}_y}{\partial p_y} & \frac{\partial \bar{\rho}_z}{\partial p_z} \end{bmatrix}$$  \hspace{1cm} (2.52)

$$A_\rho(1, 1) = -\omega^2 + \frac{\mu_e}{r_{ed}^3} \left[ \frac{3(x_c + R_e + \rho_x)^2}{r_{ed}^2} - 1 \right] + \frac{\mu_m}{r_{md}^3} \left[ \frac{3(x_c - R_m + \rho_x)^2}{r_{md}^2} - 1 \right]$$  \hspace{1cm} (2.53)

$$A_\rho(1, 2) = \frac{3\mu_e(y_e + \rho_y)(x_c + R_e + \rho_x)}{r_{ed}^5} + \frac{3\mu_m(y_e + \rho_y)(x_c - R_m + \rho_x)}{r_{md}^5}$$  \hspace{1cm} (2.54)

$$A_\rho(1, 3) = \frac{3\mu_e(z_c + \rho_z)(x_c + R_e + \rho_x)}{r_{ed}^5} + \frac{3\mu_m(z_c + \rho_z)(x_c - R_m + \rho_x)}{r_{md}^5}$$  \hspace{1cm} (2.55)

$$A_\rho(2, 1) = \frac{3\mu_e(y_e + \rho_y)(x_c + R_e + \rho_x)}{r_{ed}^5} + \frac{3\mu_m(y_e + \rho_y)(x_c - R_m + \rho_x)}{r_{md}^5}$$  \hspace{1cm} (2.56)

$$A_\rho(2, 2) = -\omega^2 + \frac{\mu_e}{r_{ed}^3} \left[ \frac{3(y_c + \rho_y)^2}{r_{ed}^2} - 1 \right] + \frac{\mu_m}{r_{md}^3} \left[ \frac{3(y_e + \rho_y)^2}{r_{md}^2} - 1 \right]$$  \hspace{1cm} (2.57)

$$A_\rho(2, 3) = \frac{3\mu_e(y_e + \rho_y)(z_c + \rho_z)}{r_{ed}^5} + \frac{3\mu_m(y_e + \rho_y)(z_c + \rho_z)}{r_{md}^5}$$  \hspace{1cm} (2.58)

$$A_\rho(3, 1) = \frac{3\mu_e(z_c + \rho_z)(x_c + R_e + \rho_x)}{r_{ed}^5} + \frac{3\mu_m(z_c + \rho_z)(x_c - R_m + \rho_x)}{r_{md}^5}$$  \hspace{1cm} (2.59)

$$A_\rho(3, 2) = \frac{3\mu_e(y_e + \rho_y)(z_c + \rho_z)}{r_{ed}^5} + \frac{3\mu_m(y_e + \rho_y)(z_c + \rho_z)}{r_{md}^5}$$  \hspace{1cm} (2.60)

$$A_\rho(3, 3) = \frac{\mu_e}{r_{ed}^3} \left[ \frac{3(z_c + \rho_z)^2}{r_{ed}^2} - 1 \right] + \frac{\mu_m}{r_{md}^3} \left[ \frac{3(z_c + \rho_z)^2}{r_{md}^2} - 1 \right]$$  \hspace{1cm} (2.61)
And

\[
A_\rho = \begin{bmatrix}
\frac{\partial \hat{p}_x}{\partial \hat{p}_x} & \frac{\partial \hat{p}_x}{\partial \hat{p}_y} & \frac{\partial \hat{p}_x}{\partial \hat{p}_z} \\
\frac{\partial \hat{p}_y}{\partial \hat{p}_x} & \frac{\partial \hat{p}_y}{\partial \hat{p}_y} & \frac{\partial \hat{p}_y}{\partial \hat{p}_z} \\
\frac{\partial \hat{p}_z}{\partial \hat{p}_x} & \frac{\partial \hat{p}_z}{\partial \hat{p}_y} & \frac{\partial \hat{p}_z}{\partial \hat{p}_z} \\
\end{bmatrix}
\]  

(2.63)

\[
A_\rho = \begin{bmatrix}
0 & 2\omega & 0 \\
-2\omega & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]  

(2.64)

2.2 State Estimation

In real-world applications measurement noise and disturbance can never be neglected that’s why the use of a state estimator is important. First, the Kalman filter (KMF) that is used with the linear dynamics model is discussed, and then the Unscented Kalman filter (UKF) that is used with the nonlinear dynamics model is presented.

2.2.1 Linear Model (Kalman Filter)

Kalman filter [12] has proved to be a great tool for state estimation for linear systems when the measurements are noisy and the system dynamics are not perfect due to disturbance or approximations made. KMF consists of two parts: The first part is where it predicts the next state based on the system dynamics and the control inputs applied. The second part is the update step, where it combines the prior predicted state and the measurement data to provide a better state estimate and its covariance. The equations to predict and correct the states of a discrete-time system in the following
form are presented below:

\[ X_{k+1} = AX_k + BU_k + B_w W_k \]  \hspace{1cm} (2.65)

\[ Y_k = CX_k + V_k \]  \hspace{1cm} (2.66)

where, \( A, B, \) and \( C \) are the state transition matrix, input matrix, and measurement model matrix, respectively. \( U_k \) is the input vector, \( W_k \) and \( V_k \) are the system noise and measurement noise with covariance \( Q \) and \( R \), respectively.

First, the predicted state \( X_{k+1}^* \) and covariance \( P_{k+1}^* \) for the next time step are calculated using:

\[ X_{k+1}^* = A \hat{X}_k + BU_k \]  \hspace{1cm} (2.67)

\[ P_{k+1}^* = AP_k A^T + Q \]  \hspace{1cm} (2.68)

And then, the corrected state \( \hat{X}_k \), Kalman gain \( K_k \) and state covariance matrix \( P_k \) are calculated using the following equations:

\[ \hat{X}_k = X_k^* + K_k [Y_k - CX_k^*] \]  \hspace{1cm} (2.69)

\[ K_k = P_k^* C^T [R + CP_k C^T]^{-1} \]  \hspace{1cm} (2.70)

\[ P_k = P_k^* [I - K_k C] \]  \hspace{1cm} (2.71)
$P_k$ is the state covariance matrix and $K_k$ is the Kalman gain.

The covariance matrix is initialized by:

$$P_0 = \begin{bmatrix}
\sigma_{X_1(0)} & 0 & 0 & 0 \\
0 & \sigma_{X_2(0)} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \sigma_{X_n(0)}
\end{bmatrix}$$

or $P_0 = I$ if $\sigma_{X_i(0)}$ are unknown.

And $X_0^* = 0$ if $X_0^*$ it’s unknown.

### 2.2.2 Non-linear Model (Unscented Kalman Filter)

For the nonlinear model, the unscented Kalman filter [9] is used. The UKF approximates the probability distribution of the non-linear system using a set of points (sigma points) that are chosen from a Gaussian distribution in a way that enables them to capture the mean and covariance of the distribution. These points are then propagated through the system and used to estimate the mean and covariance of the predicted state. And just like in the Kalman filter, the predicted state along with the measurement are used to make the correction.

For a nonlinear system of the form:

$$X_{k+1} = f(X_k, U_k, V_k, k)$$

$$Y_k = h(X_k, U_k, k) + W_k$$

Where, $V_k$ and $W_k$ are the system and measurement noise vectors with covariance $Q$ and $R$, respectively. Just like in the regular Kalman filter, it is assumed that the noise vectors are zero-mean (white Gaussian noise).
First, sigma points are calculated:

\[ X_0 = \bar{X} \quad \quad \quad \quad W_0 = \frac{\lambda}{n + k} \] (2.74)

\[ X_i = \bar{X} + \left( \sqrt{(n + \lambda)P_{xx}} \right)_i \quad \quad \quad \quad W_i = \frac{1}{2(n + \lambda)} \] (2.75)

\[ X_{i+n} = \bar{X} - \left( \sqrt{(n + \lambda)P_{xx}} \right)_i \quad \quad \quad \quad W_{i+n} = \frac{1}{2(n + \lambda)} \] (2.76)

Where: \( \bar{X} \) and \( P_{xx} \) are the mean and covariance of \( X \). And \( \lambda \in \mathbb{R} \), \( W_i \) is the weight associated with the \( i \)th point, and \( \left( \sqrt{(n + \lambda)P_{xx}} \right)_i \) is the \( i \)th row (or column) of the matrix square root \( (n + \lambda)P_{xx} \).

The predicted mean is calculated using:

\[ \hat{X} = \sum_{i=0}^{2n} W_i f(X_i) \] (2.77)

The predicted covariance:

\[ \hat{P} = \sum_{i=0}^{2n} W_i \left\{ f(X_i) - \hat{X} \right\} \left\{ f(X_i) - \hat{X} \right\}^T \] (2.78)

The same thing goes for the measurement:

\[ \hat{Y} = \sum_{i=1}^{2n} W_i h(X_i) \] (2.79)

\[ \hat{S} = \sum_{i=0}^{2n} W_i \left\{ h(X_i) - \hat{Y} \right\} \left\{ h(X_i) - \hat{Y} \right\}^T + R \] (2.80)

And then, the cross-covariance matrix is calculated from:

\[ \hat{P}_{xy} = \sum_{i=0}^{2n} W_i \left\{ f(X_i) - \hat{X} \right\} \left\{ h(X_i) - \hat{Y} \right\}^T \] (2.81)
Finally, the update is performed using the following equations:

\[
X = \hat{X} + WV \tag{2.82}
\]

\[
P = \hat{P} - W\hat{S}W^T \tag{2.83}
\]

such that,

\[
V = Y - \hat{Y} \tag{2.84}
\]

\[
W = \hat{P}_{xy}\hat{S}^{-1} \tag{2.85}
\]

### 2.3 Controller Design

A Linear Quadratic Regulator is chosen as the controller since it provides an optimal solution to the control problem by minimizing a cost function that takes into consideration input limitations. Another benefit of using LQR is its robustness to disturbance and uncertainties in the system.

For a system written in the form:

\[
X_{k+1} = AX_k + BU_k \tag{2.86}
\]

\[
Y_k = CX_k + D \tag{2.87}
\]

The controller is designed such that the optimal gain matrix \( K \) in:

\[
u_k = -Kx_k \tag{2.88}
\]
minimizes the following quadratic cost function:

\[ J(u) = \sum_{n=1}^{\infty} (x_k^T Q x_k + u_k^T R u_k) \]  \hspace{1cm} (2.89)

Where, \( Q \) and \( R \) are the state-cost and input-cost weighted matrices.

To do so, the discrete-time Riccati equation is solved:

\[ A^T S A - S - (A^T S B) (B^T S B + R)^{-1} (B^T S A) + Q = 0 \]  \hspace{1cm} (2.90)

to finally get the infinite horizon solution gain:

\[ K = (B^T S B + R)^{-1} B^T S A \]  \hspace{1cm} (2.91)
Chapter 3

Results

This Chapter presents the results of state estimation using both linear and nonlinear system dynamics as discussed earlier. It also includes a simulation of a docking scenario where a KMF and LQR are used to estimate the states and optimally control the thrust.

3.1 State Estimation

The KMF and UKF algorithms are first compared by starting the deputy and chief spacecraft on an L2 Southern Halo orbit. Here, the true states are obtained by propagating the chief’s and deputy’s states using the nonlinear CR3BP equations. Measurements are then created by adding white noise with a standard deviation $\sigma$ to the true states in order to simulate a real-world scenario.

Note: The ”ode113” solver is used because it is usually more efficient than ”ode45” at strict tolerances or if the ordinary differential equation function is expensive to evaluate.
The chief’s initial state w.r.t. barycentric frame is set to:

\[ X_c = \begin{bmatrix} 428673.3 & 0 & -78330.3 & 0 & -0.22 & 0 \end{bmatrix}^T \text{ (km, km/s)} \]

And the deputy’s initial state w.r.t. barycentric frame is set to:

\[ X_d = \begin{bmatrix} 428666.81 & 11.81 & -78337.89 & 0 & -0.22 & 0 \end{bmatrix}^T \text{ (km, km/s)} \]

The measurement and control update frequency is considered to be equal to 1Hz.

The standard deviation of the measurements is set to:

\[ \sigma = \begin{bmatrix} 0.005 & 0.005 & 0.005 & 0.001 & 0.001 & 0.001 \end{bmatrix} \text{ (km, km/s)} \]

### 3.1.1 KMF

Since there is no input, the equations can also be written in the following format:

\[ \dot{X} = A(t)X_0 \]  \hspace{1cm} (3.1)

The analytical solution is expressed as:

\[ X(t_{k+1}) = e^{A(t_k)\Delta t}X(t_k) \]  \hspace{1cm} (3.2)

such that,

\[ e^{A\Delta t} = \left[ I + A\Delta t + \frac{A^2\Delta t^2}{2!} + \frac{A^3\Delta t^3}{2!} + \ldots \right] \]  \hspace{1cm} (3.3)
Typical standard deviation values of relative position/velocity measurement instruments deployed on LEO satellites are used. And the measurement covariance matrix is set to:

\[
R = \begin{bmatrix}
0.005^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.005^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.005^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.001^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.001^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.001^2
\end{bmatrix}
\]

Here, the control input noise is set to zero since the deputy is not controlled. Thus, the system covariance matrix is set to: \( Q = 10^{-20}I_{6 \times 6} \).

Only the first 100 seconds of the simulation are shown on the plots for presentation purposes.

Figure 3.1: Comparison between measured, estimated (KMF), and true position (x-axis)
Figure 3.2: Comparison between measured, estimated (KMF), and true position (y-axis)

Figure 3.3: Comparison between measured, estimated (KMF), and true position (z-axis)
Figure 3.4: Comparison between measured, estimated (KMF), and true velocity ($x$-axis)

Figure 3.5: Comparison between measured, estimated (KMF), and true velocity ($y$-axis)
From the relative position plots, it is obvious that the estimated states were very close to the true values, especially in the case of the $y$ and $z$ axes.

### 3.1.2 UKF

Here, the Euler discretization method is used to transform the continuous-time system to a discrete-time system ($\tau = 0.1s$):

$$X_{k+1} = A_dX_k + B_dU$$

(3.4)

where, $A_d = I + A\tau$ and $B_d = B\tau$. $\tau$ being a very small time step ($\tau = 0.1s$).

Also, the tuning parameters are set to: $\alpha = 1 \times 10^{-3}$, $\beta = 2$, and $\kappa = 0$. 

26
Same as for the KMF, the process covariance matrix is set to $Q = 10^{-20}I_{6 \times 6}$.

\[
R = \begin{bmatrix}
0.005^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.005^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.005^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.001^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.001^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.001^2 \\
\end{bmatrix}
\]

Figure 3.7: Comparison between measured, estimated (UKF), and true position ($x$-axis)
Figure 3.8: Comparison between measured, estimated (UKF), and true position (y-axis)

Figure 3.9: Comparison between measured, estimated (UKF), and true position (z-axis)
Figure 3.10: Comparison between measured, estimated (UKF), and true velocity ($x$-axis)

Figure 3.11: Comparison between measured, estimated (UKF), and true velocity ($y$-axis)
From the above plots, it is obvious that the KMF and UKF performed almost similarly and the position and velocity estimates were very close to the true values. To see which one of the two filters yielded more accurate results the error percentage for each state is calculated and plotted below.

\[
error\% = \frac{|estimatedstate - truestate|}{|state|} \times 100\% \tag{3.5}
\]
Figure 3.13: Comparison between position percentage errors from KMF and UKF ($x$-axis)

Figure 3.14: Comparison between position percentage errors from KMF and UKF ($y$-axis)
Figure 3.15: Comparison between position percentage errors from KMF and UKF (z-axis)

Figure 3.16: Comparison between velocity percentage errors from KMF and UKF (x-axis)
From the error percentage plots, it is clear that the UKF has performed slightly
better than the KMF overall since its percentage error was mostly smaller than that of the KMF. For the next step, it was decided to use the linear dynamics equations due to the great advantages of a linear system when designing a stable and optimal controller.

### 3.2 Docking Simulation

Here a KMF combined with an LQR is used to achieve the goal of docking with the chief spacecraft. The same initial conditions as before are used and the continuous system is transformed into a discrete system.

\[
\dot{X} = A(t)X_0 + BU \quad (3.6)
\]

Euler discretization method is used to transform the continuous-time system to a discrete-time system \((\tau = 0.1s)\):

\[
X_{k+1} = A_dX_k + B_dU \quad (3.7)
\]
where, $A_d = I + A\tau$ and $B_d = B\tau$. $\tau$ being a very small time step ($\tau = 0.1s$).

The covariance matrices for the KMF are set to:

$$Q_{KMF} = 10^{-20}I_{6\times6}$$

$$R_{KMF} = \begin{bmatrix}
0.005^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.005^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.005^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.001^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.001^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.001^2 \\
\end{bmatrix}$$

And the LQR covariance matrices are chosen as:

$$Q_{lqr} = 10^{-2}I_{6\times6}$$

$$R_{lqr} = 10^7I_{3\times3}$$

The input-cost weighted matrix $R_{lqr}$ had to be chosen carefully so that the control effort is constrained properly.
Figure 3.19: Plot of relative position (x-axis).

Figure 3.20: Plot of relative position (y-axis).
Figure 3.21: Plot of relative position ($z$-axis).

Figure 3.22: Plot of relative velocity ($x$-axis).
From the relative position and velocity plots, it is obvious that every state converged to zero (i.e., the deputy and chief docked successfully) after around 25 minutes.
In the below plot, the control effort or thrust acceleration applied in each axis is shown.

Figure 3.25: Plot of the control input in the three axes.
Chapter 4

Conclusion

In this paper, the relative equations of motion in the circular restricted three-body problem were derived, a comparison between the use of nonlinear equations of motion and linear equations of motion was made and finally, a docking scenario of a deputy and chief spacecraft was simulated. The simulation was conducted in the presence of noisy measurements, that required the use of a KMF in the case where linear equations were used and UKF in the case where nonlinear equations were used.

The comparison between the KMF and UKF algorithms showed that there wasn’t a significant improvement when the nonlinear equations were used. Therefore it was decided that using the linear equations is sufficient to simplify the design of a stable optimal controller.

An LQR controller was later added to the system along with the KMF. So that the deputy spacecraft can be optimally controlled to dock with the chief spacecraft.

The simulation results demonstrated the effectiveness of the proposed approach. The linearized version of the nonlinear CR3BP relative equations of motion proved to give results that were accurate enough to enable a deputy spacecraft to successfully dock with its target in the presence of measurement noise.
Bibliography


