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Analysis of the Performance, Recertification, and Sustainability of a Cessna 172N Modified with an O-360-A4M Powerplant, Variable Timing Electronic Ignition, and Tuned Exhaust

Kelsey Lee Kaht

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by

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We the undersigned committee hereby approve the attached thesis,
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Abstract

Title: Analysis of the Performance, Recertification, and Sustainability of a Cessna 172N Modified with an O-360-A4M Powerplant, Variable Timing Electronic Ignition, and Tuned Exhaust

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The purpose of this thesis is to discuss the performance, modification, and recertification of the test aircraft, a Cessna 172N that has been equipped with an O-360-A4M power plant, tuned exhaust, and variable timing electronic ignition, and to compare the aircraft to a stock aircraft and other high-performance aircraft to determine the feasibility of the modifications. 65 knot, 80 knot, and 95 knot steady state climb and 2000 ft and 5000 ft level acceleration tests were performed with the test aircraft at the Melbourne Orlando International Airport. The weather was clear with calm wind, and there was only one issue of excessive cooling when attempting idle descents during testing. The aircraft was forward loaded with 150 lbs of ballast. Time stamps, ambient temperature, and engine speed were recorded manually, and all other values, including indicated airspeed and altitude, were recorded using onboard equipment. The data was reduced, finding important values including calibrated airspeed (V_c), instrument and weight corrected power and rate of climb (P_{IW} and C_{IW}), rate of climb (ROC), and specific excess power (P_s). Standardized performance charts, such as P_{IW} vs C_{IW} and V_C vs Time, and expanded charts, such as ROC vs V_C and ROC on a Standard Day at Various Altitudes, were generated. The modified Skyhawk was compared to an unmodified stock aircraft, a Diamond DA40, a Mooney M20C, and a Grumman American AA5B Tiger using P_s vs V_c charts from a thesis by a former student, Yohan Forbes Auguste, cruise performance charts for pilot's operating handbooks (POHs), and aircraft resale marketplaces. The analysis showed that the modified Skyhawk, with a best rate of climb (ROC) of 720 ft/min at 3000 ft, has superior climb performance to that of the Stock Cessna 172N, at 400 ft/min, and comparable performance to that of the DA40, M20C, and AA5B Tiger at 700-760 ft/min. The market value for the modified Skyhawk, at $100-$175, is roughly equivalent to that of the unmodified Skyhawk. With variable timing and electronic magneto technology, the modified Skyhawk has a fuel consumption and fuel cost of 10.3 gph and $68.40/hr that is equivalent or better than that of the other high-performance aircraft at 9.2-15.1 gph and $61.10/hr-$95.60/hr. The cost of the modifications is affordable, $30k-$31k, and the average cost of an existing modified Skyhawk, $139k, is between the average costs of the other high-performance aircraft at $81k-$347k. After considering all factors, the recommendation to pilots is to upgrade a stock Skyhawk rather than purchase a newer, high-performance aircraft. If a pilot cannot afford every upgrade, it is recommended that the electronic ignition variable timing installation be done first because, in addition to providing fuel savings, it is the most cost-friendly, and potentially decreases maintenance costs. Recertification requires at least one supplemental type certificate (STC), and companies, such as Air Plains, Electroair, and Power Flow Systems provide kits for engine, variable timing electronic ignition, and tuned exhaust and approved STCs and support. It is recommended that pilots consider the inability to perform idle descents, upfront cost, higher future fuel costs over stock, and time to modify and recertify when considering the upgrades, and hire a lawyer, flight test engineer, and/or test pilot and contact the FAA early to facilitate recertification. Replacing other high-performance aircraft in the industry with 180 Hp Skyhawks with variable timing electronic ignition would not only help owners but would also increase sustainability of the aviation community by increasing flight safety, reducing fuel consumption, reducing operating costs, and providing a good, gradual solution to the energy crisis while other electronic and hybrid technologies are explored.

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Acknowledgement

I would like to Thank God and my Savior Jesus Christ for being the reason why I pursue challenges in higher learning.

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Dedication

I would like to dedicate this thesis paper to my fiancé and to my parents for always supporting me while in the military and in school. It has been a long road, and they have been encouraging the entire way. Thank you.

Nomenclature

t = time (sec)
GPS Ground Track = true course (deg)
Magnetic Heading = aircraft heading (deg)
H_{ic} = instrument corrected pressure altitude (ft)
OAT = observed ambient temperature (deg C)
V_c = calibrated airspeed (kts)
RPM = engine speed (rpm)
W_T = test weight (lbs)
dH/dt = rate of climb (ft/sec)
T_a = instrument corrected ambient temperature (deg C)
ROC_{OBC} = observed rate of climb (ft/min)
T_S = standard temperature (deg C)
ROC_{TC} = test corrected rate of climb (ft/sec)
σ = density ratio (unitless)
C_{IW} = instrument and weight corrected rate of climb (ft/min)
BHP_S = standard brake horsepower (Hp)
BHP_T = true brake horsepower (Hp)
P_{IW} = instrument and weight corrected power (Hp)
dv/dt = acceleration (ft/sec^2)
V_T = true airspeed (ft/sec)
$FHP_{inexcess}$ = thrust horsepower in excess (Hp)
W_T/W_S = weight ratio (unitless)
$(FHP_{inexcess})_{WC}$ = thrust horsepower in excess corrected for weight (Hp)
P_s = power in excess corrected for weight (Hp)
V_h = maximum achievable airspeed at maximum continuous power (kts)
V_{S1} = stalling speed or minimum steady flight speed at which the airplane is controllable in the clean configuration (kts)

Chapter 1
Introduction

1.1 Scope

The purpose of this project was to determine whether a 180 Hp O-360-A4M engine, variable timing electronic ignition, and tuned exhaust modifications on a common stock general aviation aircraft improve performance, and whether the modifications are practical. The test aircraft is a Cessna 172N Skyhawk with Tail Number N739AF. To increase aircraft performance, installing a bigger engine might seem like an obvious choice. However, there are certain other factors that could affect this decision. It is expensive and time consuming to replace an engine, install electronic ignition, tune the exhaust, and get an airplane recertified. Also, the O-360 engine weighs more than the O-320 engine, and more weight leads to more drag. One other disadvantage that was discovered during testing was that the variable timing does not allow for idle descents due to excessive cooling. On the contrary, there are also advantages to the upgrades. The engine modification increases performance characteristics such as best rate of climb and service ceiling. Variable timing electronic ignition and tuned exhaust improve fuel economy. This project entailed climb and level acceleration performance analysis.

Climb and level acceleration maneuvers were conducted in the test aircraft, data was collected and reduced, and standardized and expanded climb performance charts were generated. Then research was done to determine the average aircraft costs, modifications costs, and market values of stock and modified Skyhawks and several other common 180 Hp aircraft. Then cruise performance charts for all aircraft, specifically fuel burn rates, were gathered. Finally, climb performance, cruise performance, purchase costs, fuel costs, market values, and costs of modifications were used to compare the aircraft.

First, the modified Cessna 172N was rated against the stock Cessna 172N to show superior performance in the upgraded aircraft. Then the enhanced Skyhawk was then evaluated against the Diamond DA40, Mooney M20C, and Grumman American AA5B Tiger to show comparable performance. The goal was to make a recommendation to pilots to upgrade a stock Skyhawk rather than purchasing a newer, high-performance aircraft. The objectives of the project must be explored prior the analysis.

To create meaningful flight tests and performance charts, the span of the testing program must be defined. Several goals were set to accomplish this:

1) Gather climb and level acceleration data for the test aircraft, reduce it, and create standardized performance charts and expanded performance charts for comparison.
2) Compare the climb and level acceleration performance of the modified aircraft to that of a stock Cessna 172N quantitatively to prove that the modified aircraft has better performance
3) Determine the cost of the modifications of the aircraft
    a) Cost of parts
    b) Cost of modification
    c) Cost of recertification
4) Determine the time required for modification of the aircraft
    a) Time to purchase parts
    b) Time to modify
    c) Time to recertify
5) Compare purchase cost, fuel consumption, and fuel costs to other common general aviation aircraft with similar specifications to consider other options and fully quantify that a modification would indeed be better
6) Discuss how engine modifications could be a good gradual path to solving the energy crisis while other technologies are being pursued

1.2 Theory

To generate effective flight tests, it is important to comprehend the technology involved. Therefore, the mechanics of the modifications will be discussed. The reciprocating engine for the typical Skyhawk has many parts, but the basic operation is as follows. The camshaft opens and closes the intake and exhaust valves allowing fuel to flow in and exhaust to flow out of the cylinders [1]. Pistons move up and down in the cylinders of the engine creating translational motion [2]. This motion creates torque that turns the crankshaft. The crankshaft transfers the torque to the propeller to turn it. The turning propeller generates forward motion for the aircraft. Finally, the forward motion of the aircraft creates airflow over the wings, which generates lift. There are many other components involved in the operation of the engine, including a fuel pump, carburetor, and spark plugs [2]. The four basic stages or strokes that happen inside a cylinder are the intake, compression, power, and exhaust strokes. The power stroke is where the force is created to drive the propeller. A fuel air mixture is drawn into the cylinder, compressed, ignited, and forced out of the cylinder. The O-320 stock engine has 320 cubic inches of piston displacement and 160 Hp, while the O-360 has 360 cubic inches and 180 Hp, 40 more cubic inches of displacement and 20 more Hp than stock [3, 4, 5]. The modified engine has additional features that enhance the reciprocating engine.

The O-360-A4M engine has all the normal components of the stock O-320-H2AD engine, but also includes tuned exhaust and variable timing electronic ignition. An untuned exhaust system has the disadvantage of having a higher exhaust pressure at the outlet of the cylinder than intake pressure at the inlet of the cylinder, which degrades the flow of incoming fuel and outgoing exhaust [6]. This is because exhaust gases build up in the headers, common collector area, and the tail pipe [6]. Tuning an exhaust system optimizes the dimensions of primary exhaust pipes [6]. Gas passes through the first primary pipe forming a vacuum [6]. Then the vacuum travels to the collector, sucks exhaust gases from the next primary into the collector, and improves the overall flow of the system [6]. The benefit is higher efficiency in drawing out spent gases, improved flow of fresh fuel coming in, and improved climb rate and fuel burn [6, 7]. Electronic ignition partially replaces magnetos in an aircraft, and it involves converting battery power into voltage and transferring it to a given cylinder to spark ignition [8, 9, 10]. The benefit is more consistent charge throughout the rpm range, decreased start up time, better fuel burn, higher power at high altitude, fewer moving parts, and reduced magneto maintenance costs [8, 9]. Variable timing is a feature of electronic ignition and involves controlling the timing of an engine [11, 12]. A manifold pressure sensor measures air volume in the engine to determine power setting, a trigger mechanism measures crankshaft position to determine when top dead center occurs (TDC), and a controller uses the sensor data to send the spark at the right time [11, 12]. The benefit is improved fuel economy and higher horsepower [9]. There are pros and cons to doing these upgrades.

Chapter 2
Facility and Test Apparatus

2.1 Test Item Description

The aircraft that was used for test day was a Cessna 172N Skyhawk. Normally, a 1978 stock Skyhawk has a Lycoming O-320-H2AD engine, a fixed pitch propeller, two front seats, and a split rear seat [13]. The test aircraft has been modified by replacing the engine with a Lycoming O-360-A4M, adding tuned exhaust and variable timing electronic ignition capabilities, replacing the propeller with another matching fixed pitch propeller, and removing the rear seat. Some basic data from the stock Cessna 172N POH is different for the test aircraft. Table 1 shows the basic data, and Figure 1 shows the modified Skyhawk [5, 14].

Table 1. Basic Data for the Modified Cessna 172N Skyhawk.

Tail #	N739AF
Engine	Lycoming O-360-A4M
Propeller	Fixed Pitch
Seats	2
Empty weight	1473.2 lbs
CG	37.34 in
Moment	55008.8 in-lbs
Useful load	1077 lbs
Total Usable Fuel Capacity	40 gal

Figures 2, 3, and 4 show dimensional diagrams of the Skyhawk airframe [5].

Figure 1. Test aircraft Cessna 172N Skyhawk (N739AF), *taken from thesis, "Comparing Excess Power of General Aviation Aircraft," by Yohan Forbes Auguste [14]."*

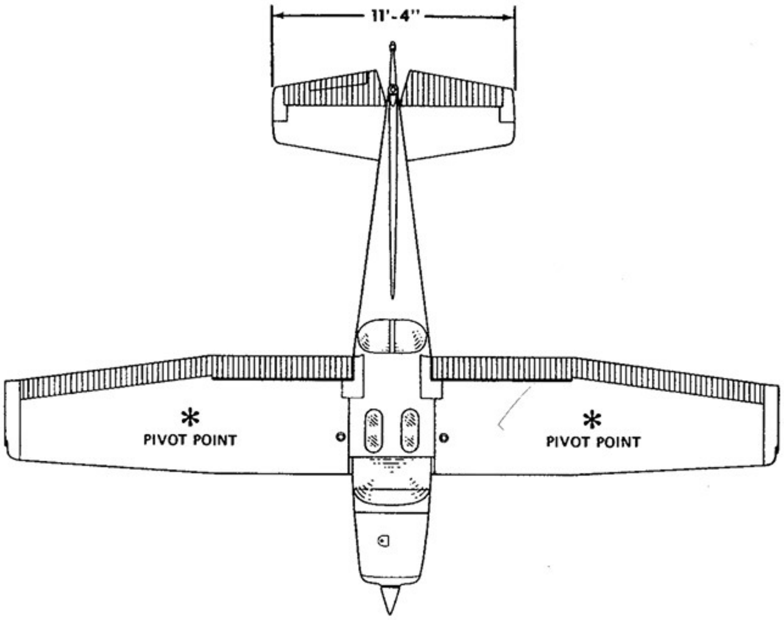


Figure 2. Top View Dimensional Diagram of the Skyhawk, *taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [5].*

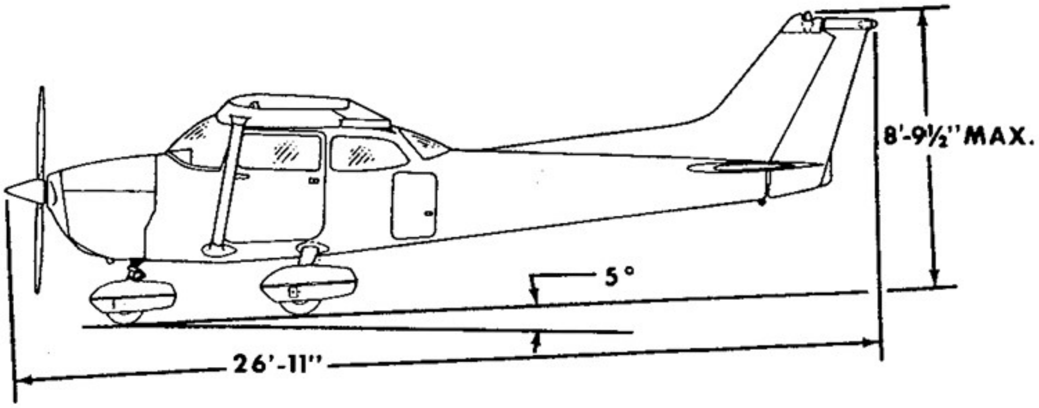


Figure 3. Side View Dimensional Diagram of the Skyhawk, *taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [5].*

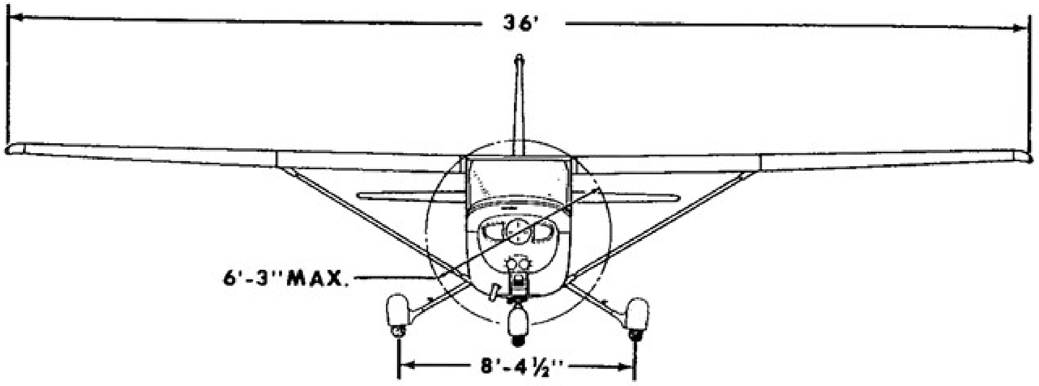
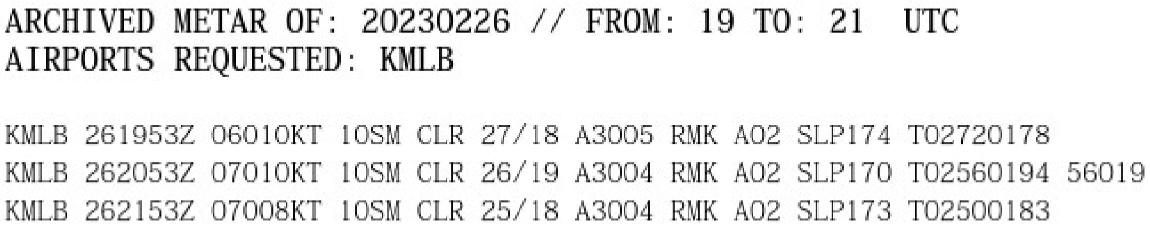


Figure 4. Front View Dimensional Diagram of the Skyhawk, *taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [5].*

2.2 Test Location and Meteorological Conditions

The testing environment was the Melbourne Orlando International Airport in Melbourne, FL. The time of day was afternoon. Preparations began at approximately 3:00 PM on Sunday, February 26th, 2023. The testing location was the surrounding area of the airport. The weather was calm and clear. Figure 5 shows the METAR report for the test day, and Figure 6 shows the location of the airport [15, 16].



```
ARCHIVED METAR OF: 20230226 // FROM: 19 TO: 21  UTC
AIRPORTS REQUESTED: KMLB

KMLB 261953Z 06010KT 10SM CLR 27/18 A3005 RMK AO2 SLP174 T02720178
KMLB 262053Z 07010KT 10SM CLR 26/19 A3004 RMK AO2 SLP170 T02560194 56019
KMLB 262153Z 07008KT 10SM CLR 25/18 A3004 RMK AO2 SLP173 T02500183
```

Figure 5. METAR Report for Test Day, February 26th, 2023, 2:53 pm to 4:53 pm EST, 1953 to 2153 ZULU, *taken from Aviation Weather Charts Archive [15].*

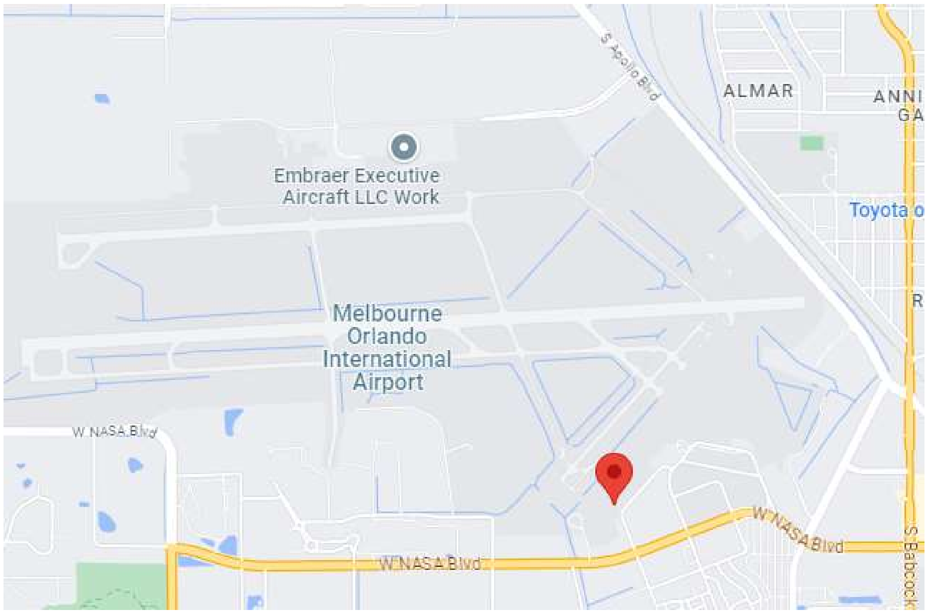


Figure 6. Picture of the Google Map Location of the Hangar at Melbourne International Airport [16].

2.3 Instrumentation

2.3.1 Equipment

Several G5 instruments were used in addition to the traditional analog gauges [17, 18]. The instrumentation used on test day was the following:

A. Test aircraft
B. Flight Test Team consisting of one pilot and one flight test engineer
C. Garmin G5 avionics: AHRS, GMU 11 magnetometer, GAD 13, GDP 59 RTD OAT probe, built-in inertial sensors, and micro SD card data logging
D. Garmin G5 digital instruments: digital clock display, airspeed indicator, altimeter, OAT gauge, heading indicator, GPS track, GPS groundspeed
E. Aircraft engine speed gauge
F. Test card

2.3.2 Calibrations

Several calibrations were needed to correct the data collected during testing. Refer to equations (1), (2), and (3) for the general theoretical formulas that were used to correct the altimeter, temperature, and airspeed for errors [19, 20].

- Standard Altimeter Calibration Equation:

$$\mathrm{H_{ic}} = \mathrm{H_i} + \Delta \mathrm{H_{ic}} + \Delta \mathrm{H_{pos}} \tag{1}$$

where H_{ic} = Calibrated Pressure Altitude
H_i = Observed Pressure Altitude
ΔH_{ic} = Altimeter Instrument Correction
ΔH_{pos} = Altimeter Position Error

- Standard Temperature Probe Calibration Equation:

  $$T_a = T_O + \Delta T_{ic} \tag{2}$$

  $$T_a(°K) = T_a(°C) + 273.15$$

  where T_a = Calibrated Ambient/Free Air Temperature

  T_O = Observed Temperature

  ΔT_{ic} = Temperature Probe Correction

- Standard Airspeed Calibration Equation:

  $$V_c = V_i + \Delta V_{ic} + \Delta V_{pos} \tag{3}$$

  where V_c = Calibrated Airspeed

  V_i = Observed Airspeed

  ΔV_{ic} = Airspeed Instrument Correction

  ΔV_{pos} = Airspeed Position Error

During testing, the barometric pressure on the altimeter was set to 29.92 in Hg to make the indicated altitude equal to pressure altitude, and thereby, make calculations simpler. The corrections for G5 instruments are not published and had to be derived. The total error for airspeed was able to be calculated from flight data using the G5 GPS ground speed and the G5 magnetic heading. The errors for altitude and temperature were unable to be calculated. However, the G5 user manual states that the altimeter is calibrated onboard the aircraft only if it fails a periodic altimeter test [21]. The G5 altimeter has low drift and hardly ever requires re-calibration according to the manual. When the calibration is done, it requires a pressure test set with an accuracy of +/- 5 ft at sea level to +/-20 ft at 30,000 ft [21]. Since the G5 relies on its own temperature sensor, and there is no data for the error of the OAT gauge and altimeter, the errors were assumed to be zero, and the ideal equations for altimeter and temperature were reduced. Refer to (4) and (5) for the reduced altitude and temperature correction equations.

- Simplified Altimeter Calibration:

  $$H_{ic} = H_i \tag{4}$$

- Simplified OAT Gauge Calibration:

  $$T_a = T_O \tag{5}$$

To calculate the G5 airspeed indicator error, several equations were used. The G5 records true airspeed directly, with its own air data, but this data was not initially trusted [22]. Since the GPS provides accurate position and airspeed data, indicated airspeed was used to calculate more accurate true and calibrated airspeed [19, 23, 24]. Refer to (6)-(10) for the equations used to calculate true airspeed, calibrated airspeed, and total error.

- Density Ratio Equation:

  $$\sigma = \frac{\left(1-\left(6.87535*10^{-6}\right)*H_{ic}\right)^{5.2561}}{\frac{273.15+T_a}{288.15}} \tag{6}$$

  where σ = density ratio

  H_{ic} = instrument corrected altitude

  T_a = ambient temperature

- GPS Groundspeed Component Along Heading Equation:

  $$GS_c = \left|\left(GS_T * \cos\left(\frac{\pi}{180} * (\text{Heading} - \text{Track})\right)\right)\right| \tag{7}$$

  where GS_C = G5 GPS groundspeed component along heading

  GS_T = G5 GPS groundspeed component along track

  Heading = G5 magnetic heading

  Track = G5 GPS ground track

- True Airspeed Equation:

  $$V_T = GS_C \tag{8}$$

  where V_T = True Airspeed

  GS_C = G5 GPS groundspeed component along heading

- Calibrated Airspeed Equation:

$$V_C = V_T * \sqrt{\sigma} \tag{9}$$

where V_C = calibrated airspeed
V_T = true airspeed
σ = density ratio

- Total Instantaneous G5 Airspeed Indicator Error Equation:

$$\Delta V_{ic} + \Delta V_{pos} = |V_C - V_I| \tag{10}$$

where ΔV_{ic} = instantaneous airspeed indicator instrument correction
ΔV_{pos} = instantaneous airspeed indicator position correction
V_C = calibrated airspeed
V_I = indicated airspeed

The recorded G5 true airspeed, which is derived from air data instruments that are subject to the boundary layer, was compared to the calculated true airspeed, which is derived from GPS position data, for a sanity check. Refer to (11)-(12) for a sample comparison.

- G5 True Airspeed Sample Value:

$$V_T = 68 \text{ kts} \tag{11}$$

- True Airspeed Sample Calculated Value:

$$V_T = 69.7 \text{ kts} \tag{12}$$

Comparison shows that the G5 true airspeed has some error. All test data was calibrated. Refer to the Appendix, section A.2, Table 23 for a partial table of calibration calculations.

2.3.3 Units

The following is a list of units for the test aircraft instruments for test day.

Airspeed indicator - indicated airspeed (kts)
Altimeter - altitude (ft)
OAT gauge - temperature (°F)
Engine speed gauge - engine speed (RPM)

2.4 Weight and Seating Considerations

2.4.1 Weight and Balance

A weight and balance check is done before every test flight to ensure that the weight is below the maximum structural limit, and the weight and center of gravity (cg) lie within the flight envelope [25, 26]. It is important that the weight and balance is within the envelope for safety reasons. It is pertinent to note that the weight and balance changed for the modified test aircraft because of installing a heavier engine and different propeller, removing the rear passenger seat, and installing variable timing ignition and tuned exhaust. The airframe was also changed to accommodate a greater maximum gross weight. The exact method of modifying the airframe was not disclosed, but it was noted that the tuned exhaust changes the outside dynamics of the aircraft slightly. A possible method that can be used to increase gross weight was found during research and is discussed in a later chapter. The flight envelope, because it is dependent on the airframe and gross weight, is changed for this aircraft. Without ballast, it was calculated that the 2,275 lbs of weight would have been dangerously close to the edge of the flight envelope at 38.6 in cg. 150 lbs of ballast was placed at the rear door post bulkhead (65.3 in arm), Figure 9-10, to put the cg safely in the envelope. Refer to Figures 7-12 for diagrams of the station arm locations and flight envelopes for the modified Skyhawk [26, 28]. Refer to Table 2 for the weight and balance calculations.

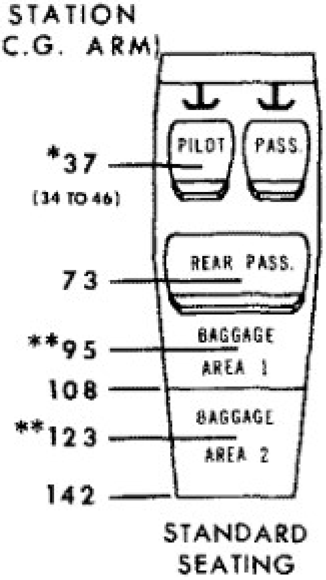


Figure 7. Side View of Station Arm Locations of the Modified Skyhawk, *taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [26].*

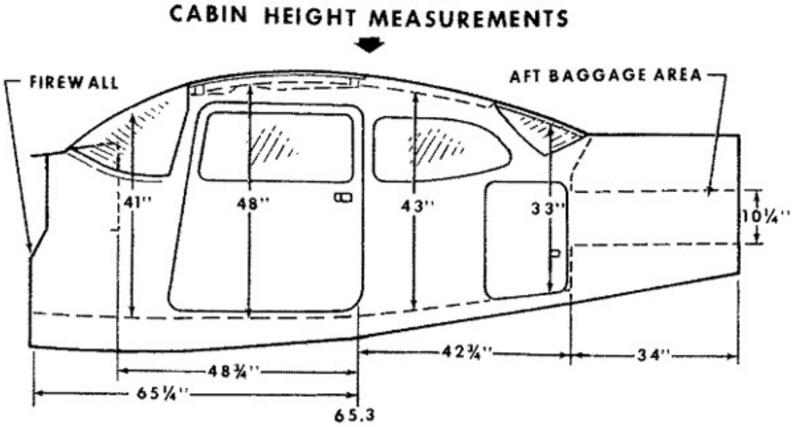


Figure 8. Side View of Station Arm Locations of the Modified Skyhawk, *taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [26].*

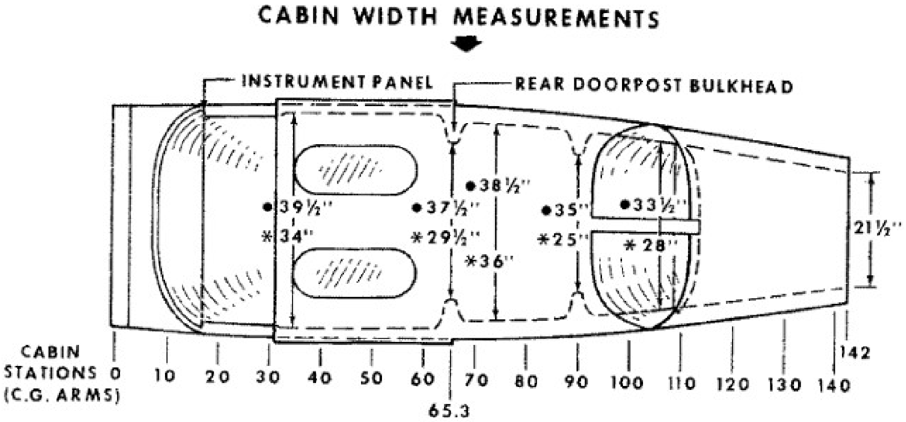


Figure 9. Top View of Station Arm Locations of the Modified Skyhawk, *taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [26].*

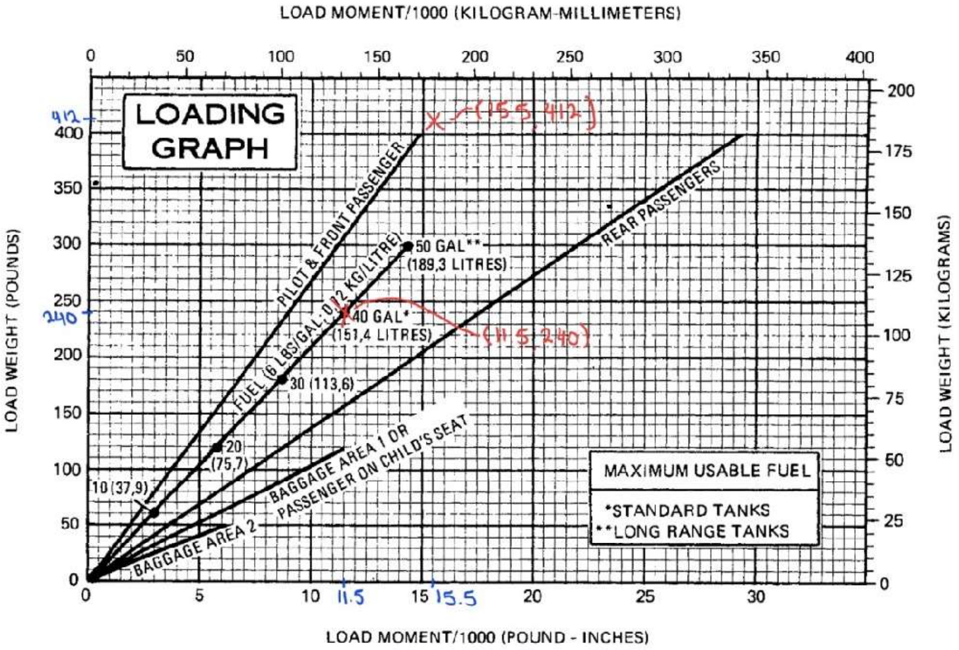


Figure 10. Component Weights and Corresponding Moments with Test Day Markings for the Modified Skyhawk, *taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [26].*

The red X’s in Figure 11 show the weight and load moments for the front seat passengers and the fuel. 240 lbs of fuel with a moment of 11,500 in-lbs, and 412 lbs of weight for front seat passengers with a moment of 15,500 in-lbs were the numbers for the flight.

Table 2. Weight and Balance for the Modified Skyhawk, *derived from Dr. Ralph Kimberlin’s FTE 5701 Class [27].*

	Weight	**Arm**	**Moment**
Item	lbs	in	in*lbs
EW	1473.2	37.34	55008.8
FSP	412	37.62	15500
Fuel	240	47.92	11500
Ballast (FWD cg limit)	150	65.3	9795
Total	2275		91804
cg	40.3		

Table 2 shows the total weight and the center of gravity (cg) location [27, 28].

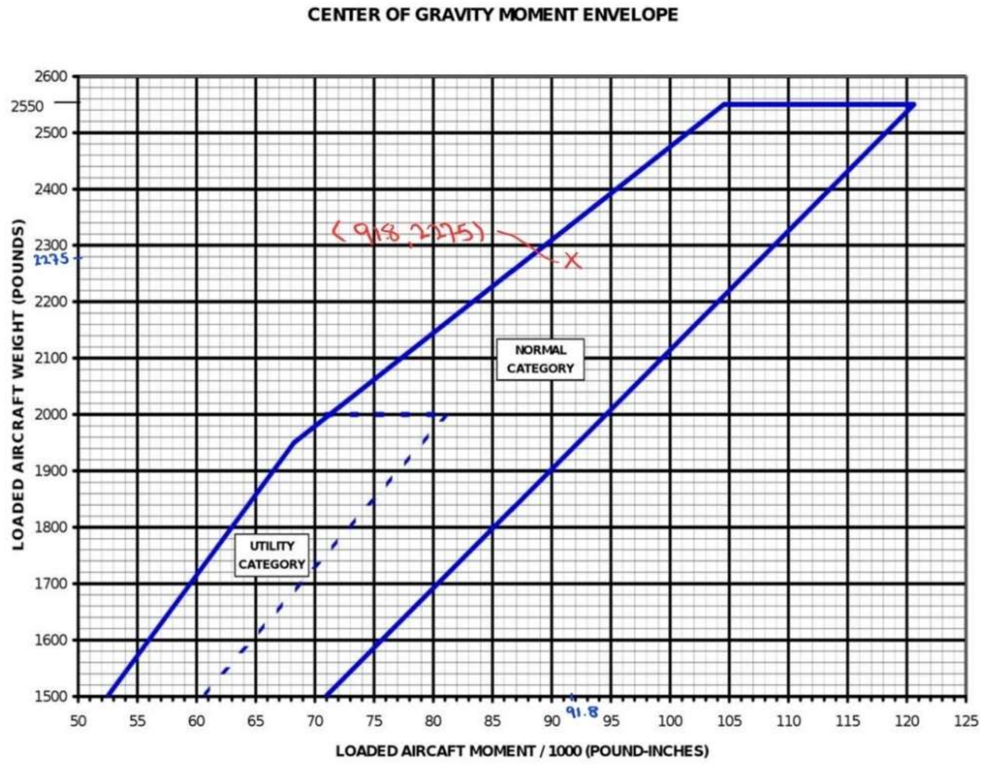


Figure 11. Flight Envelope Showing the Total Weight and Total Moment Limitations for the modified Skyhawk, *taken from Air Plains AFM Supplement [28]*.

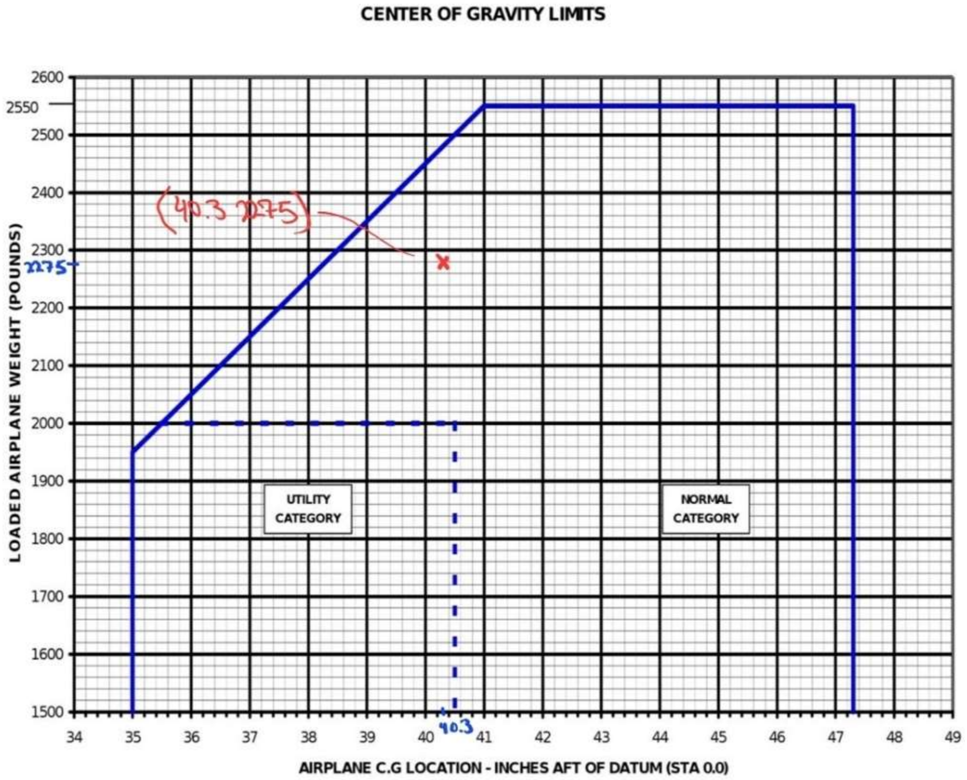


Figure 12. Flight Envelope Showing the Total Weight and CG Location Limitations for the modified Skyhawk, *taken from Air Plains AFM Supplement [28]*.

The red X's in Figures 12 and 13 show the total weight of 2275 lbs, the cg of 40.3 in, and total moment over 1000 or 91.8 in-lbs, and show that the weight and balance is within limitations of the flight envelope.

2.4.2 Test Weight Calculations

Refer to (13)-(15) for the equations used to calculate the beginning test weight, the instantaneous test weight, and the weight ratio during the climb and level acceleration performance [20].

- Standard Weight Configuration:

  $$W_T = W_E + W_P + W_S + W_F + W_B \tag{13}$$

  where W_T = Test Weight
  W_E = Empty Weight of Aircraft
  W_P = Weight of pilot and miscellaneous equipment
  W_S = Weight summation of passengers
  W_F = Weight of fuel
  W_B = Weight of ballast

The equation used for instantaneous test weight for each aircraft was as follows [17].

- Instantaneous Test Weight:

  $$W_{T2} = W_{T1} - W_{UF} \tag{14}$$

  where W_{T2} = Test weight in next instant
  W_{T1} = Test weight in first instant
  W_{UF} = Weight of used fuel

The equation used for instantaneous weight ratio for each aircraft was as follows.

- Instantaneous Weight Ratio:

  $$\text{Ratio} = W_T/W_S \tag{15}$$

  where Ratio = Instantaneous weight ratio
  W_T = Instantaneous Test weight
  W_S = Standard weight

The new empty weight of 1,473.2 lbs and standard gross weight of 2,550 lbs were provided by the pilot, and the starting test weight was 2,275 lbs. The total fuel burn of 10 gallons during testing was divided by the total test time of 58 min 22 sec, and then the fuel was assumed to be linearly burned over the course of the test at a rate of (10 gal/58.37 min)*(1 min/60 sec)*(6 lbs/1 gal) or 0.01713 lbs/sec, and 1 second per time stamp. Refer to the Appendix, section A.1, Table 22 for a partial table of the test weight calculations.

2.4.3 Seating Configuration

The following is the seating configuration for test day. Ballast was included to counteract the nose heavy condition due to having the rear seat removed. Refer to equation (16).

- Seating Configuration for Test Day:
  LH seat: Pilot Dr. Silver (16)
  RH seat: Kelsey Kaht
  Rear Doorpost Bulkhead (station 65.3): Ballast

Chapter 3
Test Procedures

3.1 Preliminaries

3.1.1 Tasks

The task assignments and timeline were as follows.

The task assignments for test day:

a) The pilot performed the test flight and communications with ATC from takeoff to landing.

b) The flight test engineer performed data collection during testing and performed collision avoidance.

3.1.2 Timeline

The approximate timeline for test day:

1) Preflight Team briefing on test plan and delegation of duties
2) Preflight inspection of the aircraft
3) Pilot briefing on test plan, flight safety, and flight operations
4) Ground run-up to test systems, verify radio communication, and gain ATC takeoff clearance
5) Takeoff to testing area from airport
6) Conduction of flight testing
7) Gaining of ATC clearance to land
8) Return to airport from testing area
9) Landing and recovery of aircraft including post-flight briefing and refuel
10) Data consolidation, post-flight team briefing, and test conclusion

3.2 Maneuvers

The most inclusive way to determine climb performance is with sawtooth climbs. However, one way to reduce the work required is to supplement steady climbs with level acceleration maneuvers. Normally, steady state climb maneuvers are required to be done at reciprocal headings to ensure accuracy. However, "Resultant data indicates that (if certain onboard test stability criteria are observed along with the use of the INS for kinetic energy corrections) the single heading method can be used [29]." The Inertial Navigation System (INS) has inertial sensors that account for wind when displaying airspeed. The G5 avionics suite also includes similar inertial sensors [22, 31]. Also, there is a GPS that includes ground track to aid in maintaining flight conditions. Therefore, the single heading method was used [29]. The steady climb and level acceleration maneuvers were done together, and the entire sequence went as follows [20, 32, 33].

Ascend to a chosen altitude of 2000 ft.

Establish steady level flight at a specific heading, 2000 ft, and 65 knots airspeed.

Perform a steady climb maneuver at the same heading and airspeed to 5000 ft:

> While maintaining heading and airspeed
>
> > Record time, altitude, and engine speed every 500 ft, starting at 2000 ft (other important values such as airspeed and heading are recorded on the micro SD card).

Establish steady level flight at a specific heading, 5000 ft, and $1.1V_{S1}$ airspeed.

Perform a steady level accelerated maneuver at the same heading, altitude, and airspeed to maximum achievable airspeed:

> While maintaining heading and maximum continuous power

Record indicated airspeed at the end of the maneuver when the maximum possible airspeed (V_h) is reached (other important values such as airspeed and heading are recorded on the micro SD card at approximately 1 second increments).

Perform a maximum power descent to 2000 ft (due to excessive cooling restrictions).

Establish steady level flight at a specific heading, 2000 ft, and $1.1V_{S1}$ airspeed.

Repeat the previous steps for a level acceleration maneuver.

Establish steady level flight at a specific heading, 2000 ft, and 80 knots.

Repeat the previous steps for a steady state climb maneuver at 80 knots to 5000 ft.

Perform a maximum power descent to 2000 ft (due to excessive cooling restrictions).

Establish steady level flight at a specific heading, 2000 ft, and 95 knots.

Repeat the previous steps for a steady state climb maneuver at 95 knots to 5000 ft.

End of testing.

3.3 Flight Conditions

3.3.1 Restrictions

The general conditions required for level acceleration performance and climb performance testing were as follows [20]. Refer to (17) and (18).

- Level Acceleration Maneuver Requirements:
  Balanced turn coordinator (ball centered) (17)
  Wings level
  Level flight
  Stabilized engine power

- Climb Maneuver Requirements:
  Stabilized engine power (18)
  Stabilized rate of climb
  Trimmed flight
  Unaccelerated flight

3.3.2 Tolerances

There are published criteria, from the FAA and the U.S. Navy, for altitude, airspeed, and heading. There was no heading tolerance available for level acceleration or climb, so a reasonable heading was defined. Normally, there would be an engine speed tolerance as well, but since the propeller is fixed pitch, the engine speed changed during the maneuver. The specific conditions required for level acceleration performance and climb performance testing were defined as follows [34, 35]. Refer to (19) and (20).

- Level Acceleration Tolerances:
  Stabilize for 60 sec at altitude and airspeed before recording (19)
  Record data from stabilization to reaching V_h
  Altitude +/- 20 feet
  Heading +/- 5 degrees

- Climb Tolerances:

  Stabilize for 60 sec at beginning altitude and airspeed before recording (20)
  Record data from beginning altitude to ending altitude
  Airspeed +/- 3 knots
  Heading Tolerance +/- 5 degrees
  Altitude Start and Stop Within Test Band +/- 20 feet

3.3.3 Recording Logs

The data required for level acceleration performance and climb performance testing were as follows [20]. Refer to (21) and (22).

- Level Acceleration Data Collection:

  Airspeed (21)
  Heading
  Altitude
  Temperature
  Engine speed
  Time
  Fuel remaining

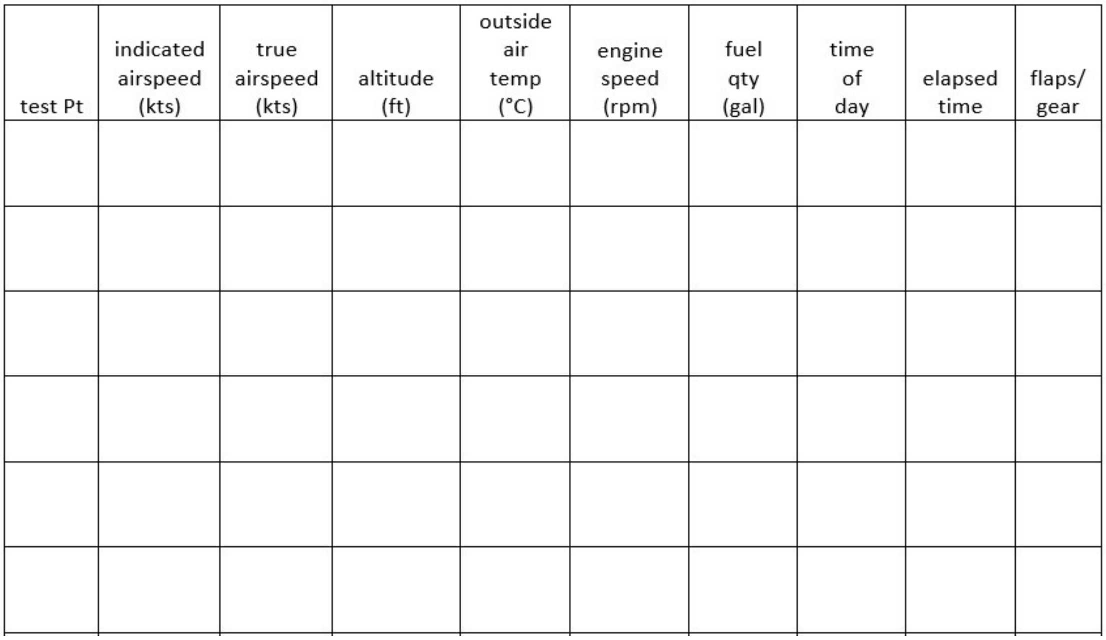
- Climb Data Collection:

  Airspeed
  Heading (22)
  Altitude
  Temperature
  Engine speed
  Time
  Fuel remaining

3.4 Test Cards

The data was collected both on the micro SD card and manually on a test card. Refer to Table 3 for a partial picture of the test card that was used for all climb and level acceleration maneuvers [32].

Table 3. Test Card for Climb and Acceleration Performance, *derived from Dr. Isaac Silver's FTE 5706 Class [32].*



test Pt	indicated airspeed (kts)	true airspeed (kts)	altitude (ft)	outside air temp (°C)	engine speed (rpm)	fuel qty (gal)	time of day	elapsed time	flaps/ gear

Chapter 4

Data Reduction

4.1 Atmospheric Calculations

The first step to data reduction involved atmospheric equations. Refer to (23) and (24) for the equations used [20, 30]. The density ratio equation was mentioned previously in the subsection, 2.3.2 Calibrations, equation (6), but is shown here again for convenience.

- Atmospheric density ratio square root:

$$\sigma = \frac{1-(6.87535*10^{-6})*H_p)^{5.2561}}{\frac{273.15+T_a}{288.15}} \tag{23}$$

where σ = atmospheric density ratio square root

H_p = pressure

T_a = instrument corrected ambient temperature

- Standard Temperature:

$$T_S = -2 * \left(\frac{H_p}{1000}\right) + 15 \tag{24}$$

where T_S = standard temperature

H_p = pressure altitude

These calculations were repeated for select points at 2600 ft and 4600 ft for the 65, 80, and 95 knot steady climbs and all points for the 2000 ft and 5000 ft level accelerations. Please refer to the Appendix, sections A.2-A.6, Tables 23, 26, 28, 31, 33, 36, 38, 40, and 42.

4.2 Engine Performance Calculations

The next step was to calculate the engine specific equations. There are several ways to determine standard brake horsepower for an engine including using an engine chart, a prop load curve, and a power required curve. Since the test aircraft does not have a manifold pressure gauge, an engine chart cannot be used, and a power required curve is hard to generate. Since the modified Skyhawk is a fixed pitch propeller, a propeller load curve can be used. In a thesis entitled, "The Functional Application of the Propeller Load Curve for Fixed Pitch Propellers," by Rebecca Speas, the P_{IW} vs V_{IW} curve and derived engine equation, and the Lycoming O-360-A engine chart, propeller load curve, and manufacturer's engine equation were all compared, and the power required equation proved to agree most with the engine chart and be the most accurate for the test aircraft. It was also found that the manufacturer's equation is more accurate at higher engine speed and is a fairly accurate substitute for the power required equation when a manifold pressure gauge is unavailable. The equation relates engine speed and horsepower [36]. A third order curve fit of the "Propeller Load Horsepower Curve," in Figure 13, for the Lycoming O-360-A engine, generates a unitless constant, $k = 9.1449*10^{-9}$, relating the standard horsepower and engine speed. Equation (25) was used to find the standard brake horsepower [37].

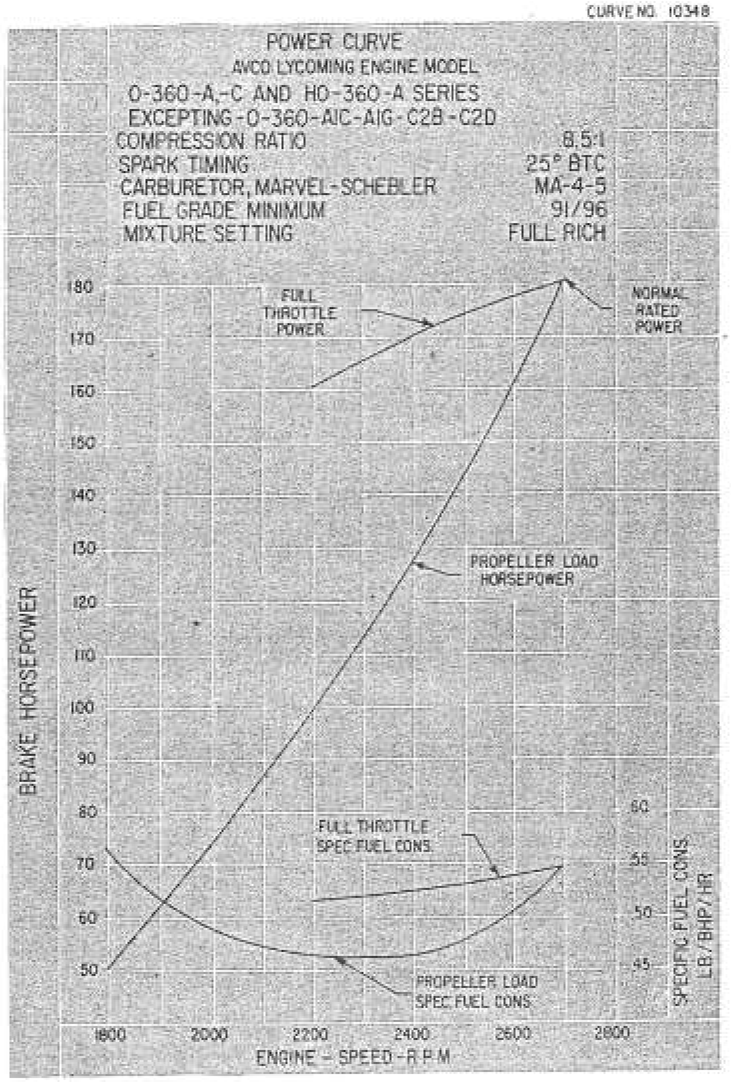


Figure 13. Manufacturer Power Curve for the O-360-A Engine, *taken from thesis, "The Functional Application of the Propeller Load Curve for Fixed Pitch Propellers," by Rebecca Speas [36].*

- Standard Brake Horsepower:

$$BHP_S = k * RPM^3 = 9.1449 * 10^{-9} * RPM^3 \quad (25)$$

where BHP_S = standard brake horsepower

k = constant

RPM = engine speed

Then the brake horsepower was corrected for atmospheric conditions. Refer to (26) for the equation used [20, 30].

- True Brake Horsepower:

  $BHP_T = BHP_S * \sqrt{(\frac{273+T_S}{273+OAT})}$ (26)

  where BHP_T = true brake horsepower

  BHP_S = standard brake horsepower

  T_S = standard temperature

  OAT = observed temperature

These calculations were repeated for chosen altitudes of 2600 ft and 4600 ft level accelerations and the 65, 80, and 95 knot climbs. Please refer to the Appendix, sections A.3-A.5, Tables 26, 28, 31, 33, 36, and 38.

4.3 Climb Performance Calculations

4.3.1 Rate of Climb

After calibrating raw data values and performing atmospheric and engine performance calculations, the next step was to plot Pressure Altitude vs Time [29, 30]. Refer to Figures 14-16 for the 65, 80, and 95 knot plots. Because the G5 has inertial sensors, the single heading method was followed [30].

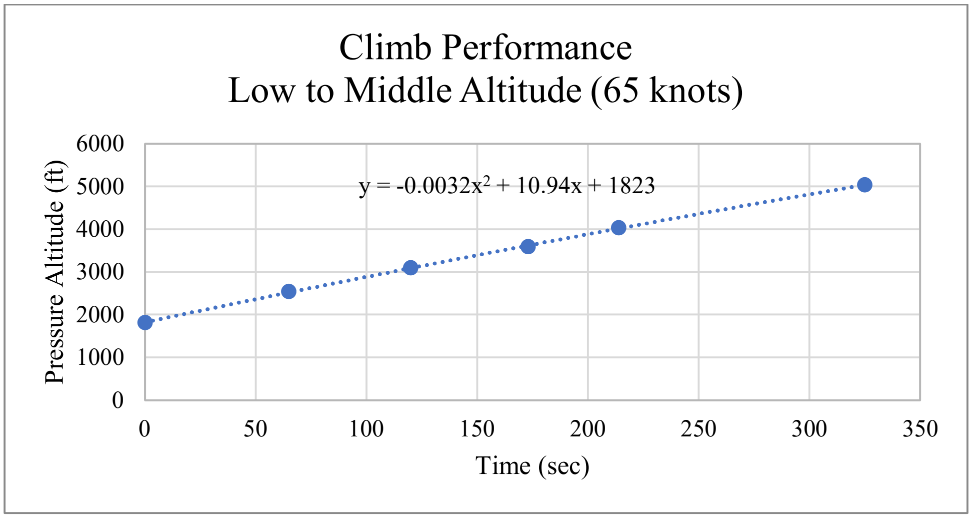


Figure 14. Pressure vs Altitude Plot for the 65 kt Steady Climb.

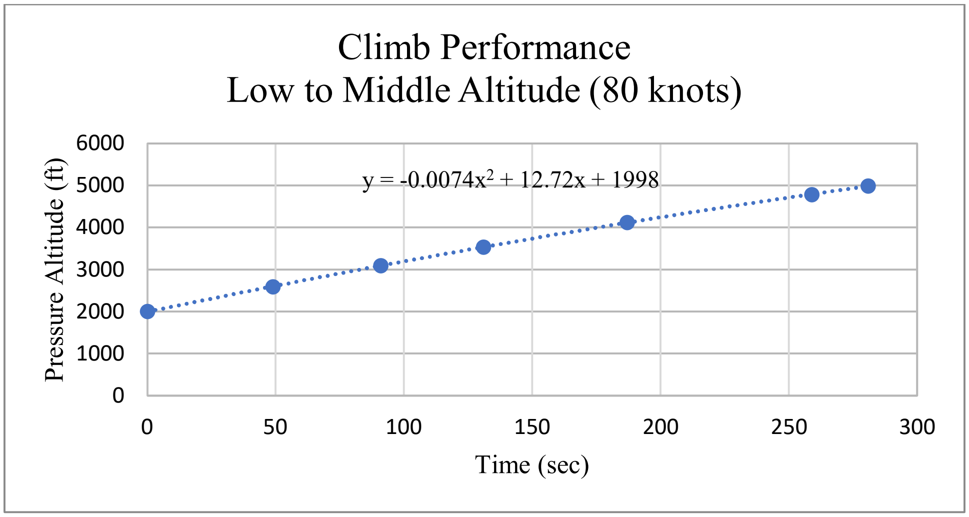


Figure 15. Pressure vs Altitude Plot for the 80 kt Steady Climb.

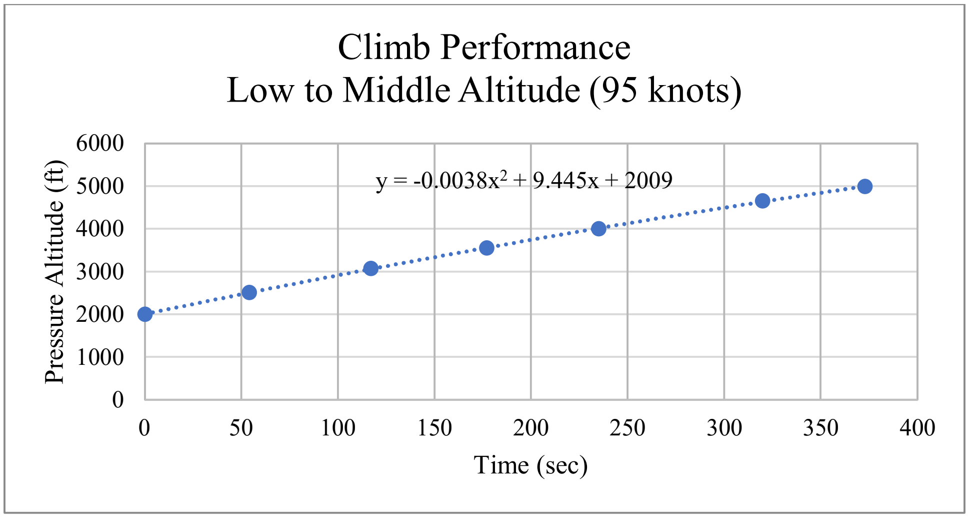


Figure 16. Pressure vs Altitude Plot for 95 kt Steady Climb.

Figures 14-16 show that pressure altitude increases over time during a steady climb. A series of equations were then calculated based on these plots [29, 30]. The curve fits from the Pressure Altitude vs Time charts, Figures 14-16, were solved for time [29, 30]. Two altitudes of 2600 ft and 4600 ft were chosen for the 65, 80, and 95 knot climbs. Refer to (27)-(29) for the time equations for the 65 knot climb.

- Instant of time at a Selected Altitude from the Pressure Altitude vs Time Curve Fit (65 knots):

$$\mathrm{t} = \frac{-10.94+\sqrt{10.94^2-4*(-0.0032)*(1823-\mathrm{H_p})}}{2*(-0.0032)} \tag{27}$$

where t = instant of time at a selected altitude from the pressure altitude vs time curve fit

H_{ic} = chosen pressure altitude

- Instant of time at a Selected Altitude from the Pressure Altitude vs Time Curve Fit (80 knots):

$$\mathrm{t} = \frac{-12.72+\sqrt{12.719^2-4*(-0.0074)*(1998-\quad_\mathrm{p})}}{2*(-0.0074)} \tag{28}$$

where t = instant of time at a selected altitude from the pressure altitude vs time curve fit

H_{ic} = chosen pressure altitude

- Instant of time at a Selected Altitude from the Pressure Altitude vs Time Curve Fit (95 knots):

$$\mathrm{t} = \frac{-9.445+\sqrt{9.445^2-4*(-0.0038)*(2009-\mathrm{H_p})}}{2*(-0.0038)} \tag{29}$$

where t = instant of time at a selected altitude from the pressure altitude vs time curve fit

H_{ic} = chosen pressure altitude

The instantaneous rate of climb or derivatives of equations (27)-(29), the Pressure Altitude vs Time curve fits, were then found [29, 30]. Refer to (30)-(32) for the derivative equations.

- Instantaneous Rate of Climb at a Selected Altitude from the Pressure Altitude vs Time Curve Fit Derivative (65 knots):

  $$\mathrm{ROC} = \frac{\mathrm{dH}}{\mathrm{dt}} = -0.0064\mathrm{t} + 10.94 \tag{30}$$

  where ROC = dH/dt = instantaneous rate of climb at a selected altitude from the pressure altitude vs time curve fit derivative

  t = instant of time

- Instantaneous Rate of Climb at a Selected Altitude from the Pressure Altitude vs Time Curve Fit Derivative (80 knots):

  $$\mathrm{ROC} = \frac{\mathrm{dH}}{\mathrm{dt}} = -0.0148\mathrm{t} + 12.72 \tag{31}$$

  where ROC = dH/dt = instantaneous rate of climb at a selected altitude from the pressure altitude vs time curve fit derivative

  t = instant of time

- Instantaneous Rate of Climb at a Selected Altitude from the Pressure Altitude vs Time Curve Fit Derivative (95 knots):

  $$\mathrm{ROC} = \frac{\mathrm{dH}}{\mathrm{dt}} = -0.0078\mathrm{t} + 9.445 \tag{32}$$

  where ROC = dH/dt = instantaneous rate of climb at a selected altitude from the pressure altitude vs time curve fit derivative

  t = instant of time

4.3.2 Corrections

After the rate of climb was calculated at the two altitudes, it was test corrected and instrument corrected [29, 30]. Refer to (33)-(35) for the equations used.

- Test Corrected Rate of Climb:

$$\mathrm{ROC_{TC}} = \mathrm{ROC_{OBS}} * \frac{(\mathrm{T_a}+273.15)}{(\mathrm{T_S}+273.15)} \tag{33}$$

where $\mathrm{ROC_{TC}}$ = test corrected rate of climb

$\mathrm{ROC_{OBS}}$ = observed rate of climb

Ta = instrument corrected ambient temperature

$\mathrm{T_S}$ = standard temperature

- Instrument and Weight Corrected Rate of Climb:

$$\mathrm{C_{IW}} = \frac{ROC_{TC}*\sqrt{\sigma}}{\sqrt{\frac{\mathrm{W_T}}{\mathrm{W_S}}}} \tag{34}$$

where $\mathrm{C_{IW}}$ = instrument and weight corrected weight of climb

$\mathrm{ROC_{TC}}$ = test corrected rate of climb

σ = density ratio

$\mathrm{W_T}$ = test weight

$\mathrm{W_S}$ = standard weight

Finally, the power was corrected [29, 30]. Refer to (57)-(58) for the equation used.

- Instrument and Weight Corrected Power:

$$\mathrm{P_{IW}} = \frac{\mathrm{BHP_T}*\sqrt{\sigma}}{\left(\frac{\mathrm{W_T}}{\mathrm{W_S}}\right)^{1.5}} \tag{35}$$

where $\mathrm{P_{IW}}$ = instrument and weight corrected power

$\mathrm{BHP_T}$ = true brake horsepower

σ = density ratio

$\mathrm{W_T}$ = test weight

$\mathrm{W_S}$ = standard weight

4.3.3 Summary of Important Values Calculated

For the 2000-5000 ft steady climbs, these calculations were repeated at chosen altitudes of 2600 ft and 4600 ft respectively. Then the normalized plot of $\mathrm{P_{IW}}$ vs $\mathrm{C_{IW}}$ and the expanded plot of Pressure Altitude vs ROC were generated [29, 30]. Refer to Tables 4-6 for summaries of the important values calculated for 65, 80, and 95 knots.

Table 4. Important Calculations for Generating Standardized and Expanded Power Required and Rate of Climb Charts for Climb Performance (65 knots).

H_p	C_{IW}	P_{IW}	ROC_{TC}
Pressure Altitude	Instrument and Weight Corrected ROC	Instrument and Weight Corrected Power	Test Corrected ROC
ft	ft/min	Hp	ft/min
2600	654	134	653
4600	555	114	570

Table 5. Important Calculations for Generating Standardized and Expanded Power Required and Rate of Climb Charts for Climb Performance (80 knots).

H_p	C_{IW}	P_{IW}	ROC_{TC}
Pressure Altitude	Instrument and Weight Corrected ROC	Instrument and Weight Corrected Power	Test Corrected ROC
ft	ft/min	Hp	ft/min
2600	752	141	748
4600	560	130	572

Table 6. Important Calculations for Generating Standardized and Expanded Power Required and Rate of Climb Charts for Climb Performance (95 knots).

H_p	C_{IW}	P_{IW}	ROC_{TC}
Pressure Altitude	Instrument and Weight Corrected ROC	Instrument and Weight Corrected Power	Test Corrected ROC
ft	ft/min	Hp	ft/min
2600	562	160	557
4600	426	156	434

4.4 Level Acceleration Performance Calculations

4.4.1 Acceleration

After calibrating raw data values and calculating atmospheric, airspeed, engine performance, and climb performance equations, a plot of calibrated airspeed over time was made for both level acceleration maneuvers [29, 37]. Refer to Figures 17 and 18 for the high and low altitude plots.

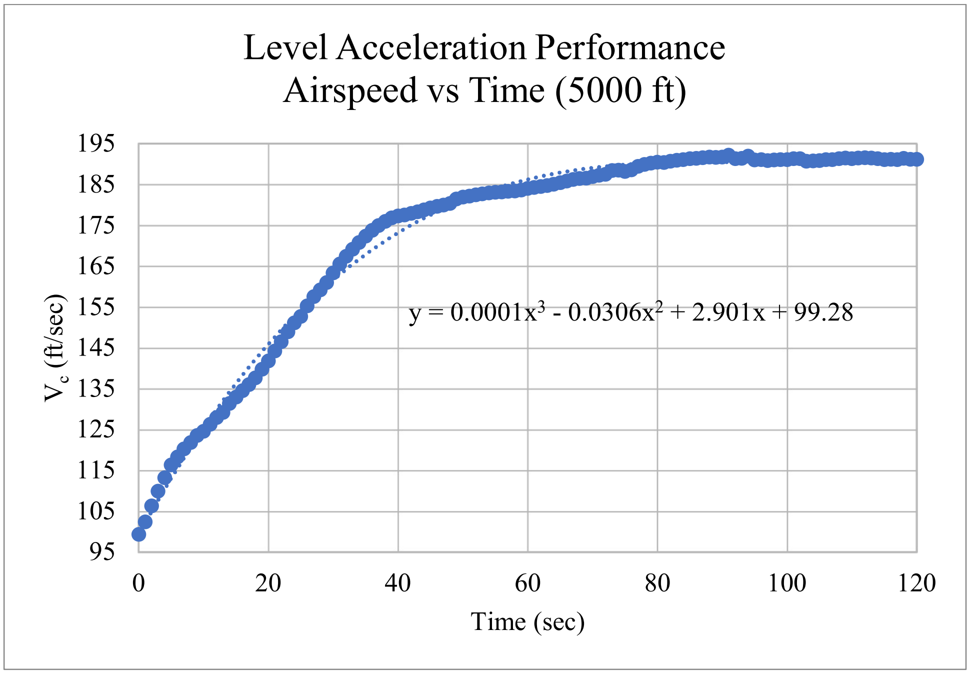


Figure 17. Plot of Pressure Altitude Over Time at 5000 ft.

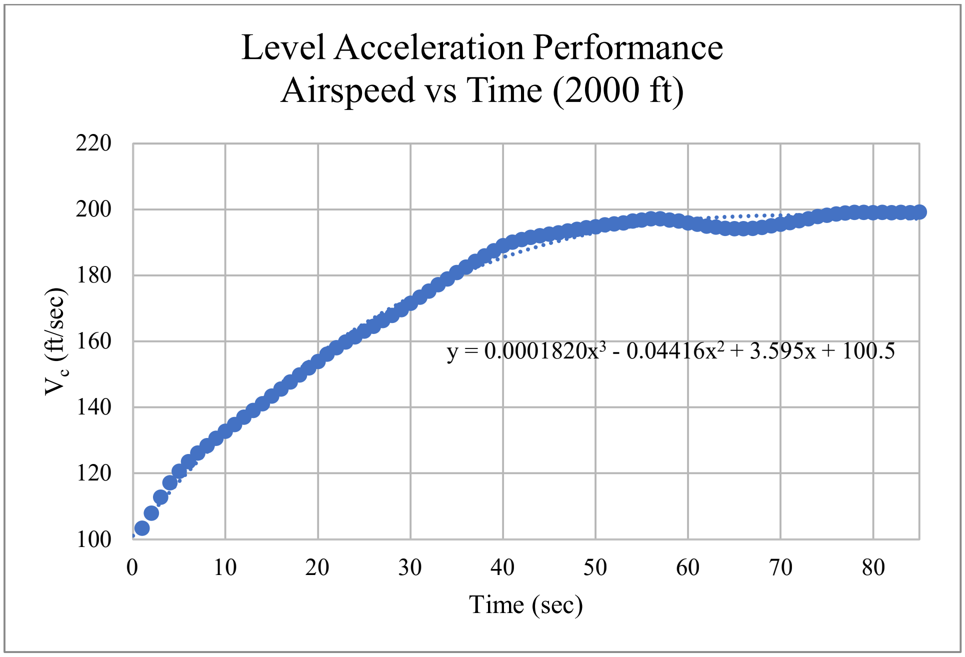


Figure 18. Plot of Pressure Altitude Over Time at 2000 ft.

Figures 18 and 19 show that the calibrated airspeed first increases quite rapidly over time, and then starts to asymptotically peak towards the end of the maneuver, when maximum achievable airspeed at maximum continuous power (V_h) is reached. Figures 17 and 18 were then used to find the derivative of velocity over time to find the instantaneous acceleration [29, 37]. Refer to (36) and (37) for the high and low altitude derivatives.

- Instantaneous Acceleration for High Altitude:

$$\frac{dv}{dt} = 0.0003t^2 - 0.0612t + 2.901 \tag{36}$$

where dv/dt = instantaneous acceleration

t = an instant of time

- Instantaneous Acceleration for Low Altitude:

$$\frac{dv}{dt} = 0.000546t^2 - 0.08832t + 3.595 \tag{37}$$

where dv/dt = instantaneous acceleration

t = an instant of time

4.4.2 Excess Power

Next, excess thrust, rate of climb, and excess power calculations were performed [29, 37, 38]. The same equations were used for high and low altitude. Refer to (38)-(41) for the equations.

- Thrust Horsepower in Excess:

$$\mathrm{FHP_{inexcess}} = \frac{W_T}{32.2} * \frac{dv}{dt} * V_T * \frac{1}{55} \tag{38}$$

where $\mathrm{FHP_{inexcess}}$ = thrust horsepower in excess

W_T = instantaneous test weight

dv/dt = instantaneous acceleration

V_T = true airspeed

- Thrust Horsepower in Excess Corrected for Weight:

$$(\mathrm{FHP_{inexcess}})_{\mathrm{WC}} = \frac{\mathrm{FHP_{inexcess}}}{\left(\frac{W_T}{W_S}\right)^{1.5}} \tag{39}$$

where $(\mathrm{FHP_{inexcess}})_{\mathrm{WC}}$ = thrust horsepower in excess corrected for weight

$\mathrm{FHP_{inexcess}}$ = thrust horsepower in excess

W_T/W_S = weight ratio

- Rate of Climb:

$$\mathrm{ROC} = \frac{(\mathrm{FHP_{inexcess}})_{\mathrm{WC}}}{W_S} * 550 * 60 \tag{40}$$

where ROC = rate of climb

$(\mathrm{FHP_{inexcess}})_{\mathrm{WC}}$ = thrust horsepower in excess corrected for weight

W_S = standard weight

- Power in Excess Corrected for Weight:

$$P_S = \mathrm{ROC} * \left(\frac{W_T}{W_S}\right)^{0.5} \tag{41}$$

where P_S = power in excess corrected for weight

ROC = rate of climb

W_T/W_S = weight ratio

4.4.3 Summary of Important Values Calculated

Calculations were repeated for all points in the 2000 and 5000 ft level acceleration maneuvers. Then standardized ROC, Specific Excess Thrust Horsepower, and Specific Excess Power vs Calibrated Airspeed were plotted from the reduced data [29, 37]. Refer to Tables 7 and 8 for partial summaries of the important values calculated for high and low altitude.

Table 7. Important Calculations for Generating Standardized Charts for Climb Performance High Altitude.

V_c	ROC	$(FHP_{inexcess})_{WC}$	P_s
Calibrated Airspeed	Rate of Climb	Thrust Horsepower in Excess Corrected for Weight	Power in Excess Corrected for Weight
53	0	0	0
67.0	652	50.4	614
68.9	655	50.6	617
70.0	652	50.3	613
71.2	647	50.0	609
72.1	641	49.5	603
73.2	635	49.1	598
73.8	625	48.3	589
74.8	619	47.8	583
75.8	612	47.3	576
76.5	603	46.6	567
77.8	598	46.2	563
78.7	590	45.6	555
79.7	582	45.0	548
80.5	573	44.3	540
81.5	565	43.7	532
112	6.1	0.47	5.8

Table 8. Important Calculations for Generating Standardized Charts for Climb Performance Low Altitude.

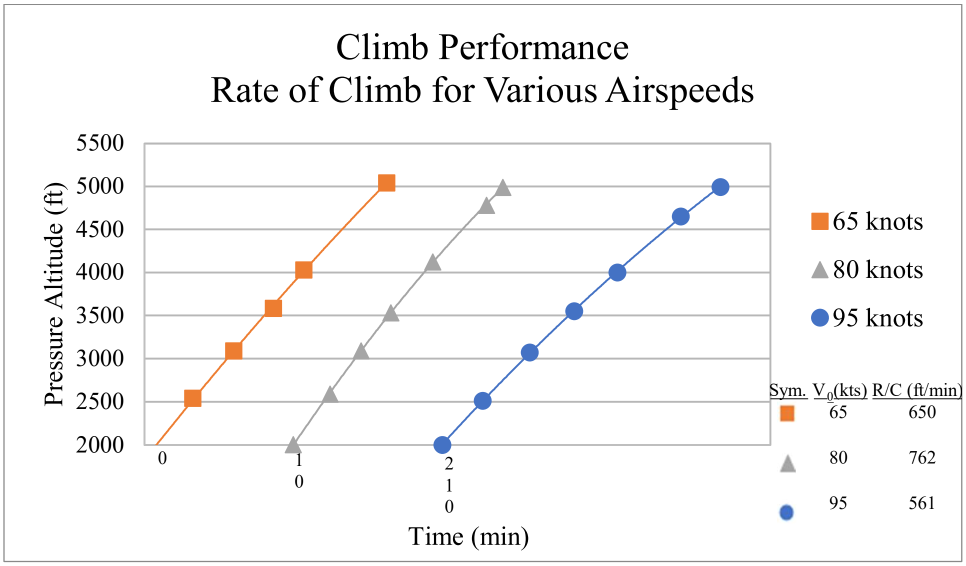
V_c	ROC	$(FHP_{inexcess})_{WC}$	P_s
Calibrated Airspeed	Rate of Climb	Thrust Horsepower in Excess Corrected for Weight	Power in Excess Corrected for Weight
53	0	0	0
69.4	792	61.2	744
71.5	795	61.4	747
73.2	793	61.3	745
74.7	788	60.9	740
76.1	781	60.3	734
77.4	773	59.7	726
78.6	764	59.0	718
79.8	754	58.3	708
81.2	745	57.6	700
82.4	734	56.8	690
83.6	724	55.9	680
118	0.748	9.67	9.09

Chapter 5
Analysis and Results

5.1 Discussion of Modified Cessna 172N Climb Performance

5.1.1 Analysis of Standardized Performance Charts

The first step in data analysis was to standardize the calculations. Because both steady climb and acceleration maneuvers were done back-to-back, the standardized curves are shown together. The first step to finding best rate of climb speed was to perform steady state climbs at several airspeeds [29]. Three steady climbs at 65, 80, and 95 knots were conducted from 2000 to 5000 ft. The single heading method was followed because the G5 is equipped with inertial sensors [29]. Then pressure altitude over time was plotted, and curve fits were created [29, 30]. Refer to Figure 19.



Sym.	V_0(kts)	R/C (ft/min)
■	65	650
▲	80	762
●	95	561

Figure 19. Rate of Climb at Several Airspeeds for the Modified Skyhawk.

Figure 19 shows that the pressure altitude over time varies linearly during a steady state climb. The plot also shows that the slope stays roughly constant as the airspeed increases from 65 to 80 knots. At 95 knots, the slope is shallower. Theoretically, the slope should stay constant [29]. The error was probably due to more difficulty in maintaining flight conditions at a higher airspeed. If steady state climbs are performed at several different altitudes and airspeeds, the rate of climb vs airspeed can be plotted to find the best rate and angle of climb [29]. This process takes time, however.

In order to shorten the time required, level acceleration maneuvers were performed instead at 2000 and 5000 ft. First, rate of climb versus calibrated airspeed was plotted, and then curve fits were created [29, 37]. The ROC reaches 0 ft/min at stall speed and maximum achievable airspeed at maximum continuous power (V_h) [14]. To make the trendline at each altitude, two parabola curve fits were combined [14, 29]. This method was demonstrated in thesis, "Comparing Specific Excess Power of General Aviation Aircraft," by Yohan Auguste. One parabola curve fit contains raw data values leading up to the peak ROC, and is anchored at the stall speed, and the other parabola curve fit contains a range of raw data values following ROC and is anchored at the V_h airspeed. The parabolas meet at the peak ROC value. The stall speed for the aircraft at 2300 lbs, flaps 0 degrees, and angle of bank 0 degrees is 53 KCAS [39]. At 2000 ft, V_h was 118 KCAS, and at 5000 ft, V_h was 112 KCAS. Refer to Figure 20 for an example and 21 for the results.

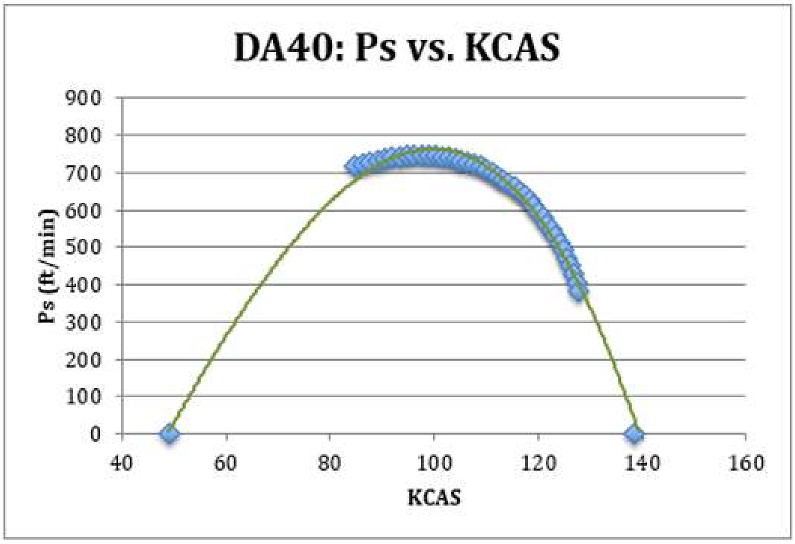


Figure 20. Example Curve Fit Anchored at Stall Speed and V_h Speed, *taken from “Comparing Specific Excess Power of General Aviation Aircraft,” by Yohan Auguste [14].*

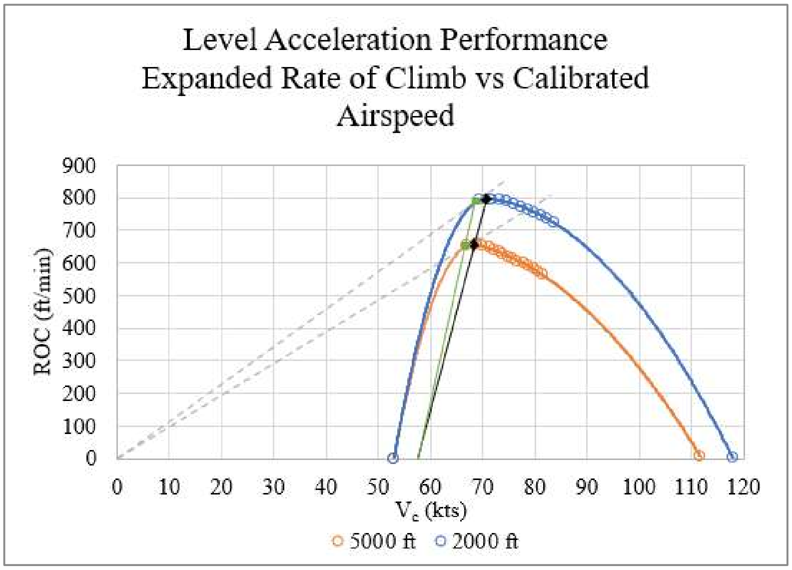


Figure 21. Best Angle and Rate of Climb at Various Altitudes for the Modified Skyhawk.

Figure 21 shows that the rate of climb first increases, then peaks, then starts to decrease as the calibrated airspeed increases during the maneuver. As the altitude increases, the behavior of the rate of climb stays the same, however, the airspeed range and peak airspeed decrease. The most important information that can be derived from the plot is best rate and angle of climb at 2000 and 5000 ft. The best or maximum rate of climb occurs at the peak of the curve [29, 37]. The best angle of climb occurs where a line drawn from zero is tangent to the curve [29, 37]. Refer to Tables 9 and 10.

Table 9. Best Rate of Climb from Level Acceleration Performance for the Modified Skyhawk.

H_p	V_y (Best Rate)
Pressure Altitude	Calibrated Airspeed
ft	kts
2000	70.7
5000	68.3

Table 10. Best Angle of Climb from Level Acceleration Performance for the Modified Skyhawk.

H_p	V_x (Best Angle)
Pressure Altitude	Calibrated Airspeed
ft	kts
2000	68.5
5000	66.5

Tables 9 and 10 show that the best rate of climb and the best angle of climb decrease as altitude increases. Normally, after V_y is determined, check climbs or steady state climbs would then be flown at this speed to generate best rate of climb performance charts, but there was not sufficient time [29, 37]. For data reduction, the P_{IW} vs C_{IW} method was used [29, 37]. First, the values in the tables were used to generate Figure 22.

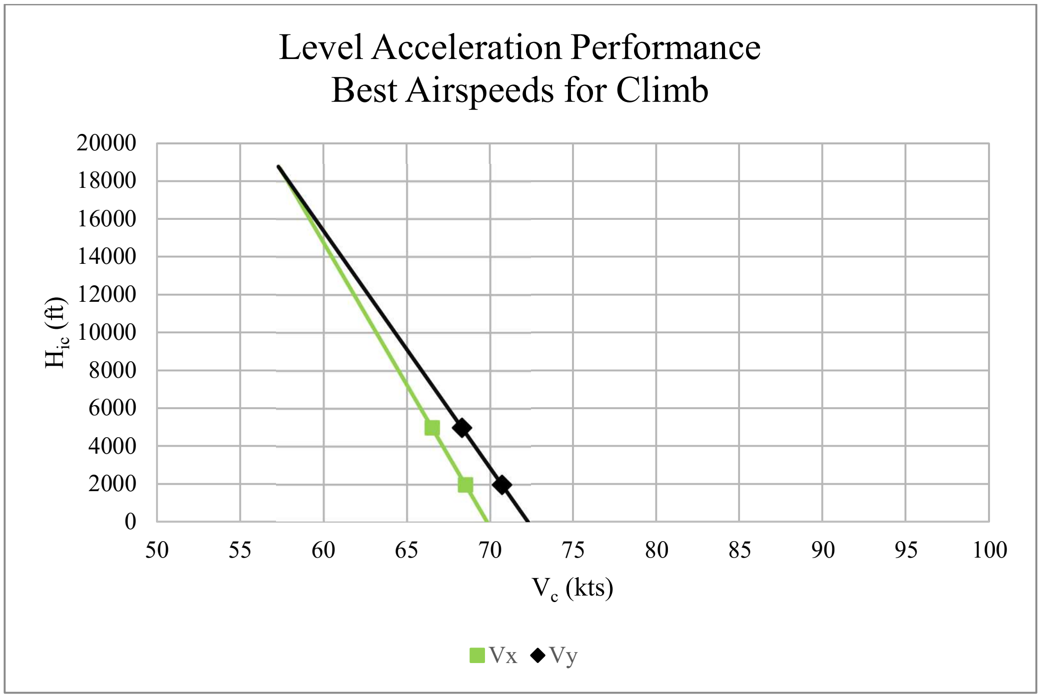


Figure 22. Best Rate and Angle of Climb Over Various Altitudes for the Modified Skyhawk.

Figure 22 shows that the best angle of climb and best rate of climb airspeed decrease as altitude increases. Theoretically, V_y should actually remain fairly constant [29, 37]. This deviation was probably due to errors in maintaining flight conditions and recording values during testing. The V_x and V_y converge as altitude increases, until the ceiling of the aircraft, 18,300 ft, is reached at the peak. Next, P_{IW} vs C_{IW} plots for the steady climbs were generated. Since no check climbs were flown, the data at best rate of climb speed is unavailable. The best rate of climb speed lies between the 65 knot and 85 knot curves. Therefore, the P_{IW} vs C_{IW} curve at 65 knots and 80 knots are shown to estimate what the best rate of climb curve values are. The P_{IW} vs C_{IW} method utilizes the theory that power varies linearly with rate of climb [29]. Using the method, curves were generated by finding P_{IW} and C_{IW} at two different altitudes during the steady climb, and then generating a linear curve fit [29, 37]. This was done at each airspeed. Refer to Figures 23 and 24 for the results at 65 and 80 knots and Tables 11 and 12 for important values derived from the plots.

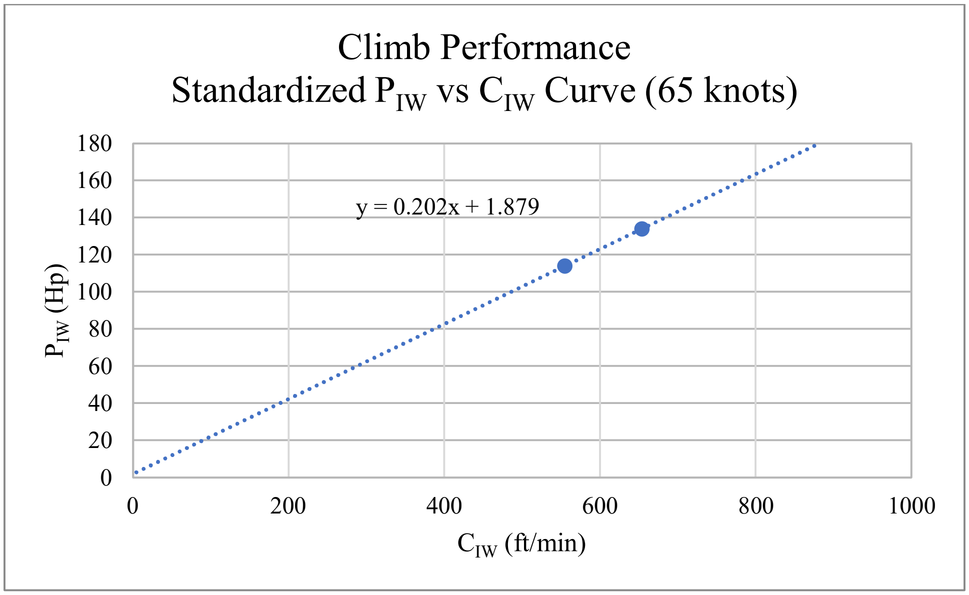


Figure 23. Standardized P_{IW} vs C_{IW} at 65 knots for the Modified Skyhawk.

Table 11. Important Values from the P_{IW} vs C_{IW} Curve at 65 knots for the Modified Skyhawk.

180 Hp	2 Hp
ROC_{max}	ROC_{min}
ft/min	ft/min
882	0

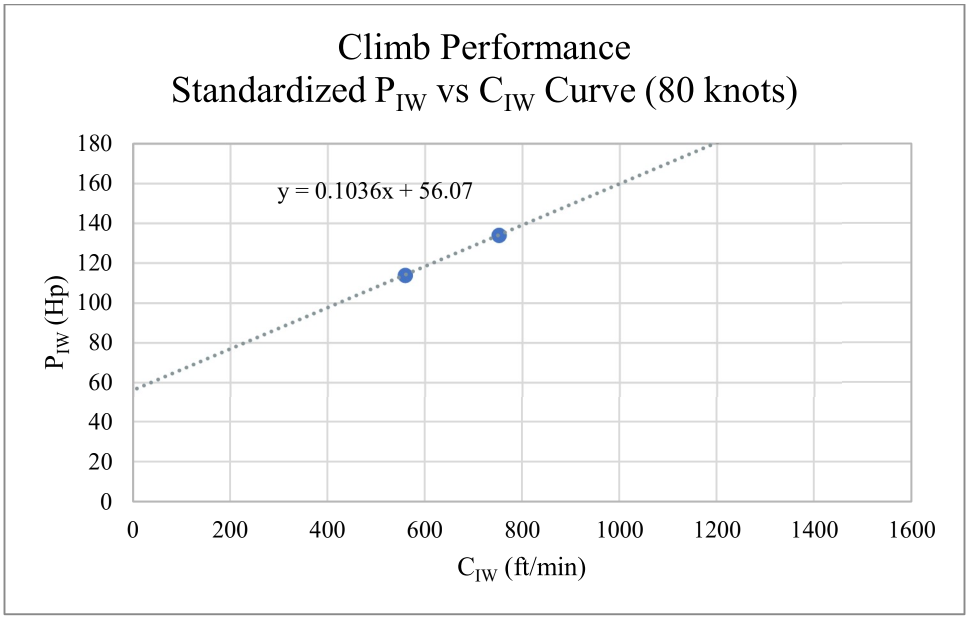


Figure 24. Standardized P_{IW} vs C_{IW} at 80 knots for the Modified Skyhawk.

Table 12. Important Values from the P_{IW} vs C_{IW} Curve at 80 knots for the Modified Skyhawk.

180 Hp	56 Hp
ROC_{max}	ROC_{min}
ft/min	ft/min
1120	0

The data points and linear curve fits that were generated per the P_{IW} vs C_{IW} method show an increase of corrected power with an increase in corrected rate of climb. Important values can be derived from the P_{IW} vs C_{IW} plot [29, 37]. The "y" value in the curve fit was set to 180 Hp and solved for x to get the maximum rate of climb at maximum power. The "x" value in the curve fit was set to 0 and solved for y to get the power at minimum rate of climb. Refer to Tables 11 and 12. According to Table 9, the best rate of climb remains fairly constant across altitude, and is approximately 67 knots, the average of the values at 2000 ft and 5000 ft. Observing the level acceleration raw data in the Appendix, sections A.6-A.7, Tables 39 and 41, shows that the corrected rate of climb increases as the airspeed increases, until it reaches a peak value at about 70 knots, and then starts to decrease, which aligns with the 67 knot approximation. The best rate of climb of 67 knots P_{IW} vs C_{IW} curve lies between the 65 and 80 knot curves. There is not enough information to generate a P_{IW} vs C_{IW} best rate of climb curve, but it can be predicted that the maximum rate of climb is at least 882 ft/min at 180 Hp, and the horsepower required for a zero rate of climb is below 2 Hp.

5.1.2 Analysis of Expanded Performance Charts

The next step to data reduction was to expand the standardized data [29, 30]. Because check climbs were not done at the best rate of climb speed of 67 knots, the expanded values for best rate of climb were estimated using the 65 and 80 knots curves. Refer to Figures 25-26 and Tables 13-14.

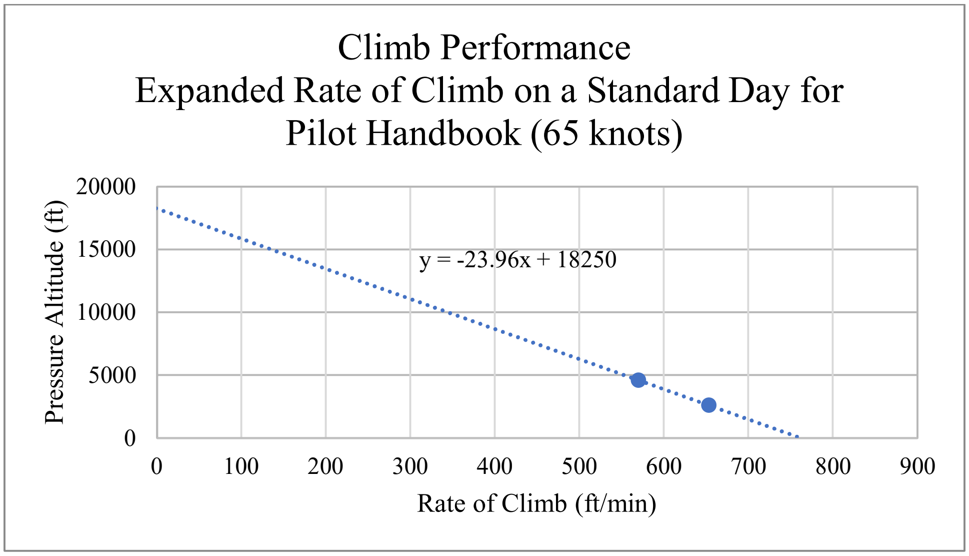


Figure 25. Expanded Rate of Climb at 65 knots for the Modified Skyhawk.

Table 13. Important Values from the Expanded Curve at 65 knots for the Modified Skyhawk.

Best ROC at SL	Ceiling
ft/min	ft
762	18,300

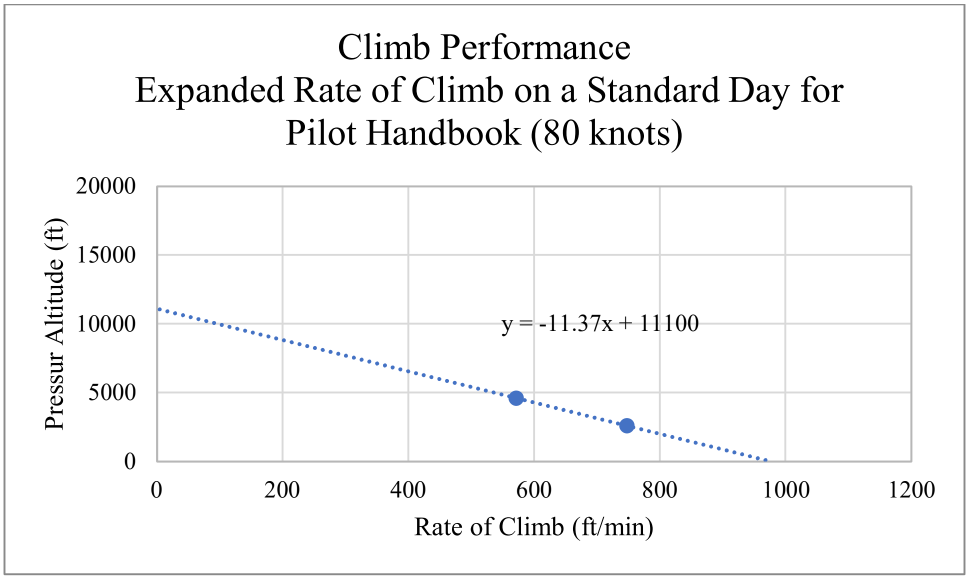


Figure 26. Expanded Rate of Climb at 80 knots for the Modified Skyhawk.

Table 14. Important Values from the Expanded Curve at 80 knots for the Modified Skyhawk.

Best ROC at SL	Ceiling
ft/min	ft
976	11,100

Observing the level acceleration raw data in the Appendix, sections A.6-A.7, Tables 39 and 41, shows that the rate of climb increases as the airspeed increases, until it reaches a peak value at about 70 knots, and then starts to decrease, which aligns with the 67 knot approximation. Theoretically, the absolute ceiling should do the same. The exact behavior is unknown because the best rate of climb chart is between 65 knots and 80 knots. It can be predicted, however, that, at 67 knots, the sea level best rate of climb is at least 760 ft/min and absolute ceiling is at least 18,300 ft. This result agrees with the level acceleration prediction of 18,300 ft in Figure 22. The absolute service ceiling of the stock Skyhawk is 14,200 ft, providing evidence that the modifications have increased the ceiling by about 4,000 ft.

5.2 Rating of the Modified Cessna 172N Against the Stock Cessna 172N, Diamond DA40, Mooney M20C, and Grumman American AA-5B Tiger

5.2.1 Comparison of the Modified Aircraft to a Stock Aircraft

Climb Performance

P_s charts provide standardized data that has been corrected for atmosphere and weight [14]. The rate of climb in Figure 21 was converted into specific excess thrust horsepower and specific excess power to compare performance to other aircraft easily [29, 37]. Basically, the instantaneous acceleration was converted to thrust horsepower in excess, then the thrust horsepower in excess was corrected for weight, then the specific thrust horsepower in excess was used to find rate of climb, and finally, the rate of climb was used to find specific excess power [29, 37]. Refer to Figures 27 and 28.

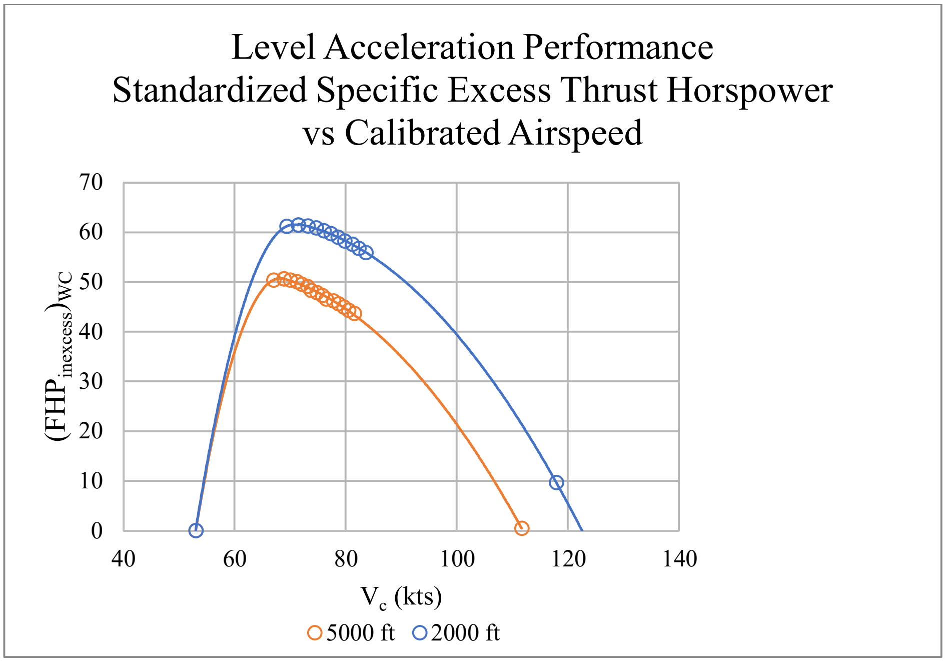


Figure 27. Specific Excess Thrust for the Modified Cessna 172N.

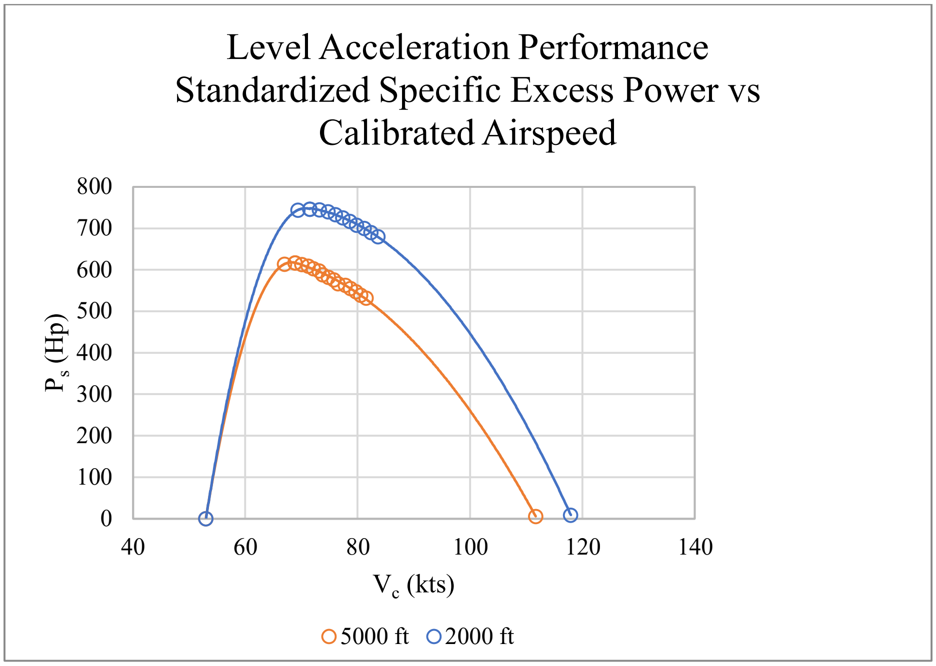


Figure 28. Specific Excess Power for the Modified Cessna 172N.

Figures 27 and 28 show that the excess thrust horsepower and excess power first increase, peak, and then decrease with increasing airspeed. Also, the excess thrust horsepower and excess power decrease with increasing altitude. The excess power can be used to show that the climb performance of the modified Skyhawk is indeed better than that of a stock Cessna 172N aircraft. To make this comparison, performance charts from a previous thesis entitled, “Comparing Specific Excess Power of General Aviation Aircraft,” by Yohan Forbes Auguste were used [14]. Refer to Figures 29 and 30.

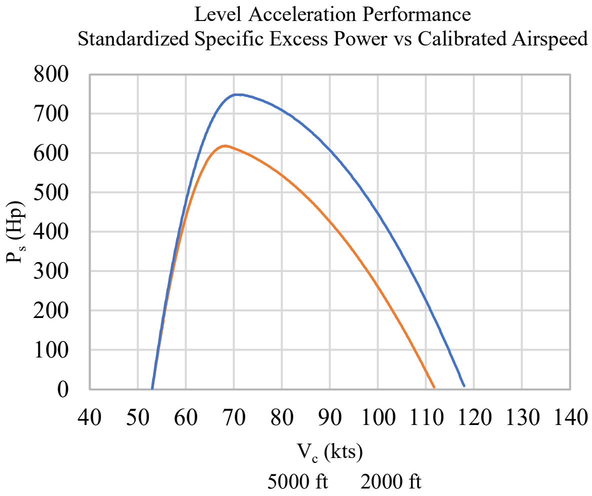


Figure 29. Specific Excess Power Trendline for the Modified Cessna 172N.

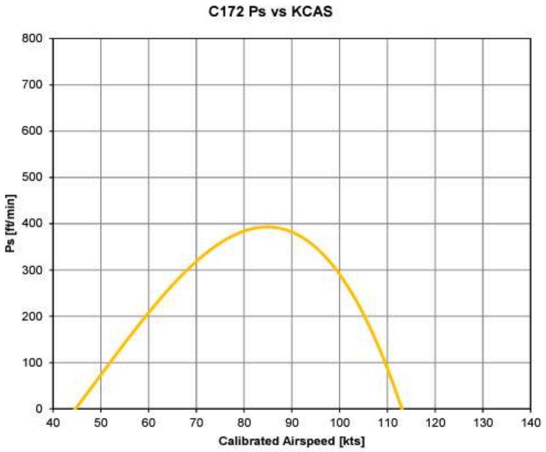


Figure 30. Specific Excess Power of the Test Cessna 172N Aircraft Before the Engine Modification, *taken from thesis, "Comparing Excess Power of General Aviation Aircraft," by Yohan Forbes Auguste [14].*

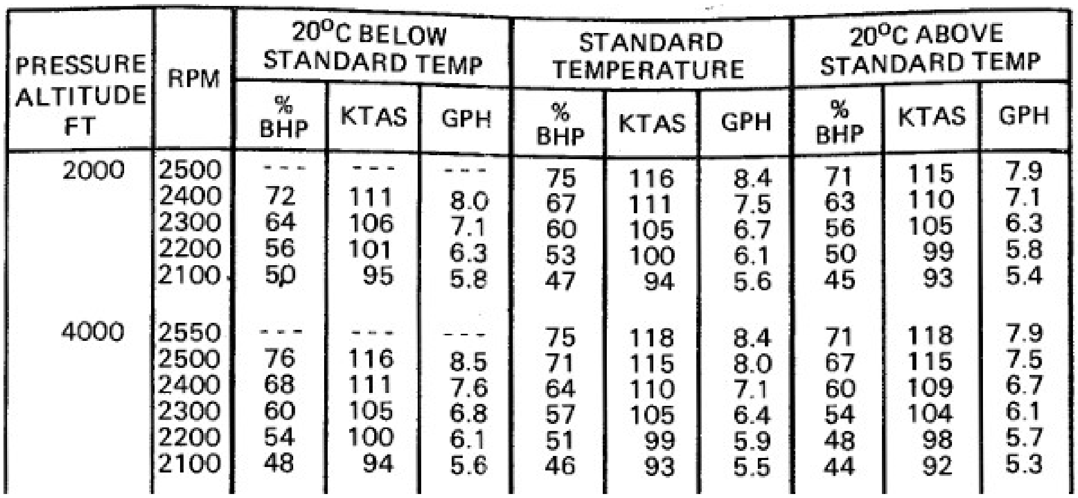
Figure 29 shows that the peak specific excess power was around 750 ft/min at 2000 ft and 660 ft/min at 5000 ft. The low altitude testing in this project was done at 2000 ft, and the testing in the thesis was done at 3000 ft. Using a linear interpolation to get an estimate, the peak at 3000 ft is approximately 720 ft/min for the modified aircraft. When Figures 29 and

30 are compared, it shows that before the engine modification, the peak specific excess power was around 400 ft/min, and after the modification, the specific excess power increased to around 1.8 times as much. For the same weight ratio, at peak specific excess power, the aircraft had a rate of climb of 400 ft/min before modification and has a rate of climb of 720 ft/min after modification for a weight ratio of 1. The POH boasts a 630 ft/min climb rate at standard temperature and maximum weight, which seems to be exaggerated when compared to actual flight performance from the thesis [40]. The climb performance is better for the modified Skyhawk, but does this modification come at a cost? A heavier engine means more weight, higher drag, and a higher fuel burn rate. It is possible that the other features of the modification, variable timing and tuned exhaust, might improve the efficiency of the reciprocating engine enough to counteract the worsened fuel economy that normally is associated with higher performance aircraft. The best way to see if fuel economy is affected is to compare cruise performance.

Cruise Performance

There is no standardized data for cruise performance, but the stock Cessna 172N fuel burn rate can be compared to the test fuel burn rate to get an idea of fuel economy and cost [40]. Refer to Table 15.

Table 15. Cruise Performance for the Stock Cessna 172N*, taken from Pilot's Operating Handbook Cessna 1978 Skyhawk Cessna Model 172N [40].*



PRESSURE ALTITUDE FT	RPM	20°C BELOW STANDARD TEMP			STANDARD TEMPERATURE			20°C ABOVE STANDARD TEMP		
		% BHP	KTAS	GPH	% BHP	KTAS	GPH	% BHP	KTAS	GPH
2000	2500	---	---	---	75	116	8.4	71	115	7.9
	2400	72	111	8.0	67	111	7.5	63	110	7.1
	2300	64	106	7.1	60	105	6.7	56	105	6.3
	2200	56	101	6.3	53	100	6.1	50	99	5.8
	2100	50	95	5.8	47	94	5.6	45	93	5.4
4000	2550	---	---	---	75	118	8.4	71	118	7.9
	2500	76	116	8.5	71	115	8.0	67	115	7.5
	2400	68	111	7.6	64	110	7.1	60	109	6.7
	2300	60	105	6.8	57	105	6.4	54	104	6.1
	2200	54	100	6.1	51	99	5.9	48	98	5.7
	2100	48	94	5.6	46	93	5.5	44	92	5.3

The fuel burn rate for the stock Skyhawk was interpolated from the chart using test conditions and an array of values to make the comparison accurate. The test aircraft has a fixed pitch propeller. The available engine speed data from testing ranged from 2000 ft to 5000 ft, 2300 rpm to 2700 rpm, and 10 deg C above standard. First, the fuel burn rates for 2000 ft and 2300-2500 rpm in Table 15 were averaged at 0 deg C above standard and 20 deg C above standard to get 7.5 gph and 7.1 gph. Then the values were linearly interpolated to get a 7.3 gph fuel burn rate at 10 deg C above standard and 2000 ft. Then similarly, the fuel burn rates for 4000 ft and 2300-2550 rpm in Table 15 were averaged at 0 deg C above standard and 20 deg C above standard to get 7.5 gph and 7.1 gph. Then the values were linearly interpolated to get a 7.3 gph fuel burn rate at 10 degrees above standard and 4000 ft. Finally, the fuel burn rates at 2000 ft and 4000 ft were averaged to get an approximate fuel burn rate of 7.3 gph for the stock Cessna. The fuel burn rate for the modified Cessna was estimated to be 10.3 gph. The burn rate of the modified aircraft is roughly 3 gallons more per hour. The cost is approximately $68.40/hr for the modified aircraft and $48.50/hr for the stock aircraft [41]. This means that the higher climb performance of the modified aircraft sacrifices some cruise performance. This modification could still be worth it, however, because of the performance gained. Fuel is not the only cost that is important to consider.

Cost Benefit Analysis

It is difficult to judge the average cost and market value of an aircraft because there is no standard or blue book value. For this study, market value was considered the range of prices that an aircraft is being resold for, from lowest to highest, based on several marketplaces in different states. The average resale cost of an aircraft was found based on the same range of prices. Several reputable sources were used to get an idea of these values for an unmodified and modified Skyhawk [42-56]. For the unmodified aircraft, the criteria were to look for standard instruments, no electronic ignition variable timing, and with or without Garmin GNS 430/530 equipment. This is because most aircraft on the market are either stock or equipped with older GPS technology. The aircraft with the newest Garmin GTN 650 or G5 technology were excluded from the data. For the modified aircraft, the criteria were to look for the prior mentioned criteria, and additionally, the 180 Hp engine upgrade, and with or without electronic ignition variable timing. There were plenty of data points for the standard aircraft, but not many data points for existing modified aircraft, as is shown in the Appendix, section A.8, Table 43. The average cost of an unmodified Cessna 172N, years 1976-1980, is about $121k, and the market value of existing modified aircraft with the same year range is $85k-$168k [42-52]. The cost of the engine modification by one company, Air Plains, is $21.995k, the cost of the electronic ignition and variable timing modifications from two other companies, Surefly and Electroair, is $1.755k and $2.595k, respectively, and the cost of tuned exhaust from a fourth company, Power Flow Systems, is $6.2k. The average cost of an existing modified aircraft, years 1976-1980, is $139k, and the market value of existing modified aircraft is $100-$175k [9, 52-59]. The modifications are affordable if an owner has an airplane already; it increases the market value by about $11.2k, and there are affordable aircraft in the market with the upgrade. Buying an unmodified aircraft and an engine upgrade is the same as buying an existing modified aircraft. If a pilot were to have a stock Cessna 172N already, the modifications are certainly feasible. If a pilot is looking for a higher performance aircraft and has not purchased an airplane yet, to see if the modifications are still worth it, the performance and costs are contrasted against other high-performance aircraft.

5.2.2 Comparison of the Modified Aircraft to Other Common General Aviation Aircraft

Climb Performance

P_s charts show standardized data that has been corrected for atmosphere and weight. The chart for the modified Cessna 172N was compared to three other higher performance aircraft in the industry, the Diamond DA40, Mooney M20C, and the Grumman American AA5B Tiger. Refer to Figures 31 and 32 and Table 16.

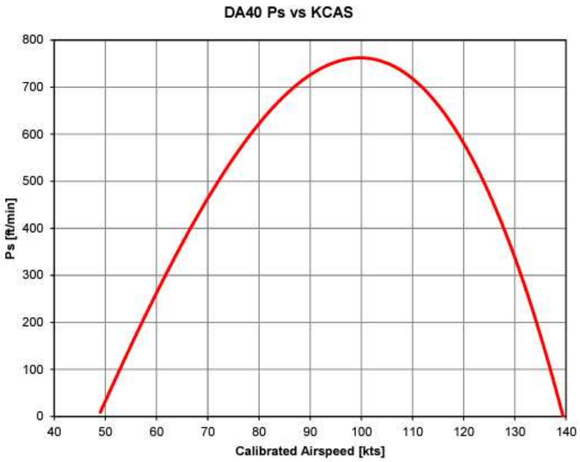


Figure 31. Climb Performance Specific Excess Power for the DA40*, taken from thesis, "Comparing Excess Power of General Aviation Aircraft," by Yohan Forbes Auguste [14].*

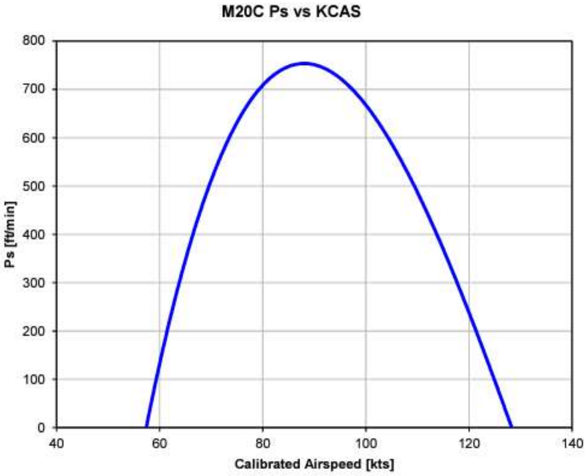
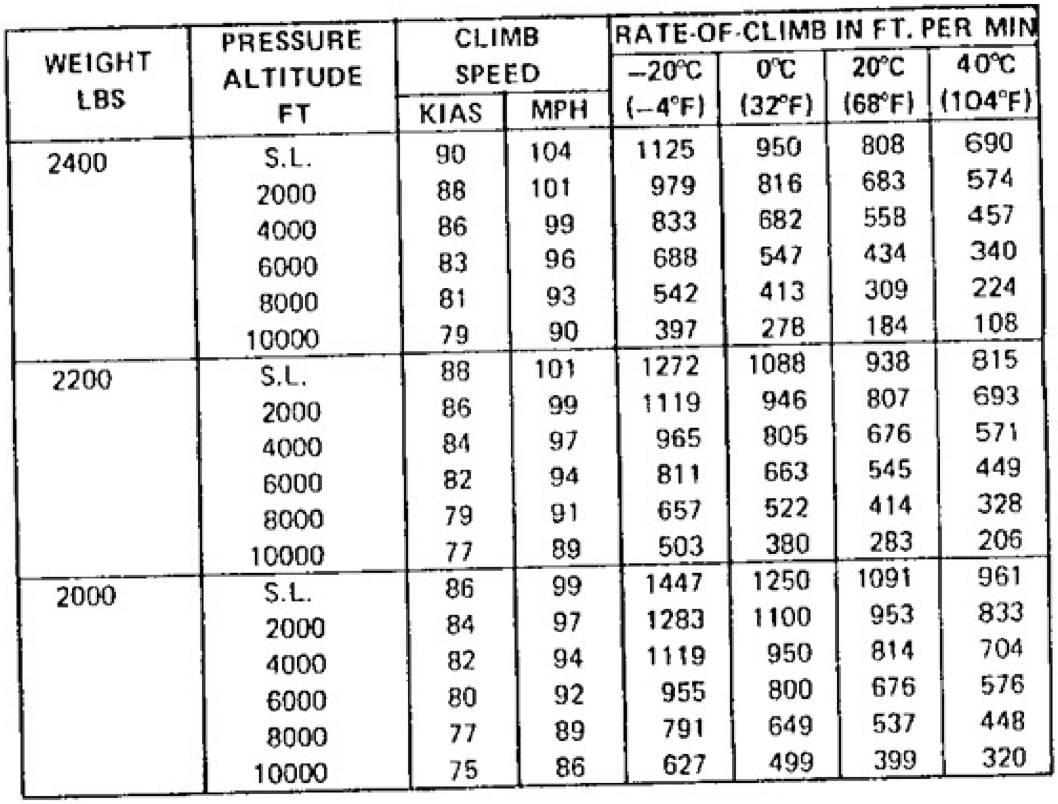


Figure 32. Climb Performance Specific Excess Power for the M20C, *taken from thesis, "Comparing Excess Power of General Aviation Aircraft," by Yohan Forbes Auguste [14].*

Figures 31 and 32 show that the peak specific excess power is 760 ft/min for the DA40 and 750 ft/min for the M20C. For the same weight ratio, at peak specific excess power, the modified aircraft has a rate of climb of 720 ft/min, and the DA40 and M20C have a rate of climb of 760 ft/min and 750 ft/min, respectively, for a weight ratio of 1. The ROC is comparable for all three aircraft. There is no standard P_s curve for the AA5B Tiger, however, the POH can be referenced [60]. Refer to Table 16.

Table 16. AA5B Climb Performance, *taken from Pilot's Operating Handbook AA-5B Tiger [60].*



WEIGHT LBS	PRESSURE ALTITUDE FT	CLIMB SPEED		RATE-OF-CLIMB IN FT. PER MIN			
		KIAS	MPH	−20°C (−4°F)	0°C (32°F)	20°C (68°F)	40°C (104°F)
2400	S.L.	90	104	1125	950	808	690
	2000	88	101	979	816	683	574
	4000	86	99	833	682	558	457
	6000	83	96	688	547	434	340
	8000	81	93	542	413	309	224
	10000	79	90	397	278	184	108
2200	S.L.	88	101	1272	1088	938	815
	2000	86	99	1119	946	807	693
	4000	84	97	965	805	676	571
	6000	82	94	811	663	545	449
	8000	79	91	657	522	414	328
	10000	77	89	503	380	283	206
2000	S.L.	86	99	1447	1250	1091	961
	2000	84	97	1283	1100	953	833
	4000	82	94	1119	950	814	704
	6000	80	92	955	800	676	576
	8000	77	89	791	649	537	448
	10000	75	86	627	499	399	320

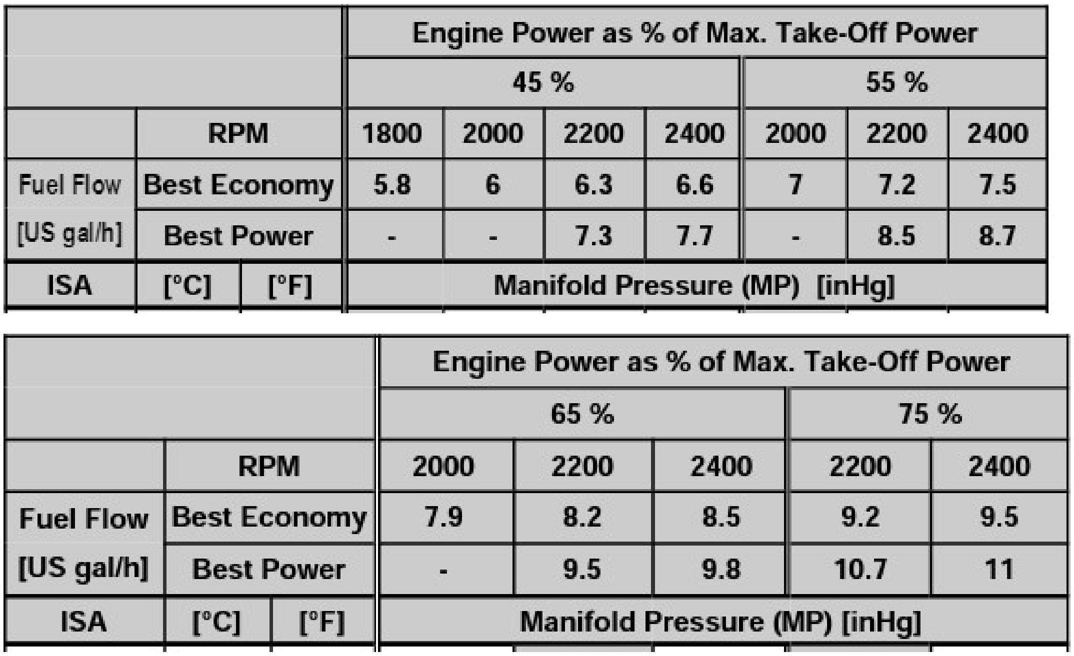
The high-performance aircraft were compared at a weight ratio of one or maximum weight. Therefore, data was taken at maximum weight for the AA5B Tiger or 2400 lbs. First, the standard temperatures at 2000 ft and 4000 ft were calculated to get 11 deg C and 7 deg C. Then, Table 16 data points at 2400 lbs and 2000 ft, (0 deg C, 816 ft/min) and (20 deg C, 683 ft/min), were linearly interpolated to get an approximate rate of climb of 743 ft/min at standard temperature. Then, Table 16 data points at 2400 lbs and 4000 ft, (0 deg C, 682 ft/min) and (20 deg C, 558 ft/min), were interpolated to get an approximate rate of climb of 639 ft/min at standard temperature. Finally, calculated points (2000 ft, 743 ft/min) and (4000 ft, 639 ft/min) were interpolated to get an approximate rate of climb of 700 ft/min at

3000 ft. At maximum weight and weight ratio of one, this gives a 700 ft/min specific excess power or power in excess corrected for weight. The modified Skyhawk, at 720 ft/min specific excess power, and 720 ft/min rate of climb at a weight ratio of one, has comparable performance to the AA5B Tiger. Climb performance is not the only important factor, however. Cruise performance will now be discussed.

Cruise Performance

There is no standardized data for cruise performance, but the DA40, M20C, and AA5B Tiger fuel burn rates can be compared to the test fuel burn rate to get an idea of fuel economy and cost [60-62]. Refer to Tables 17-19.

Table 17. DA40 Cruise Performance, *taken from Airplane Flight Manual DA 40 [61].*



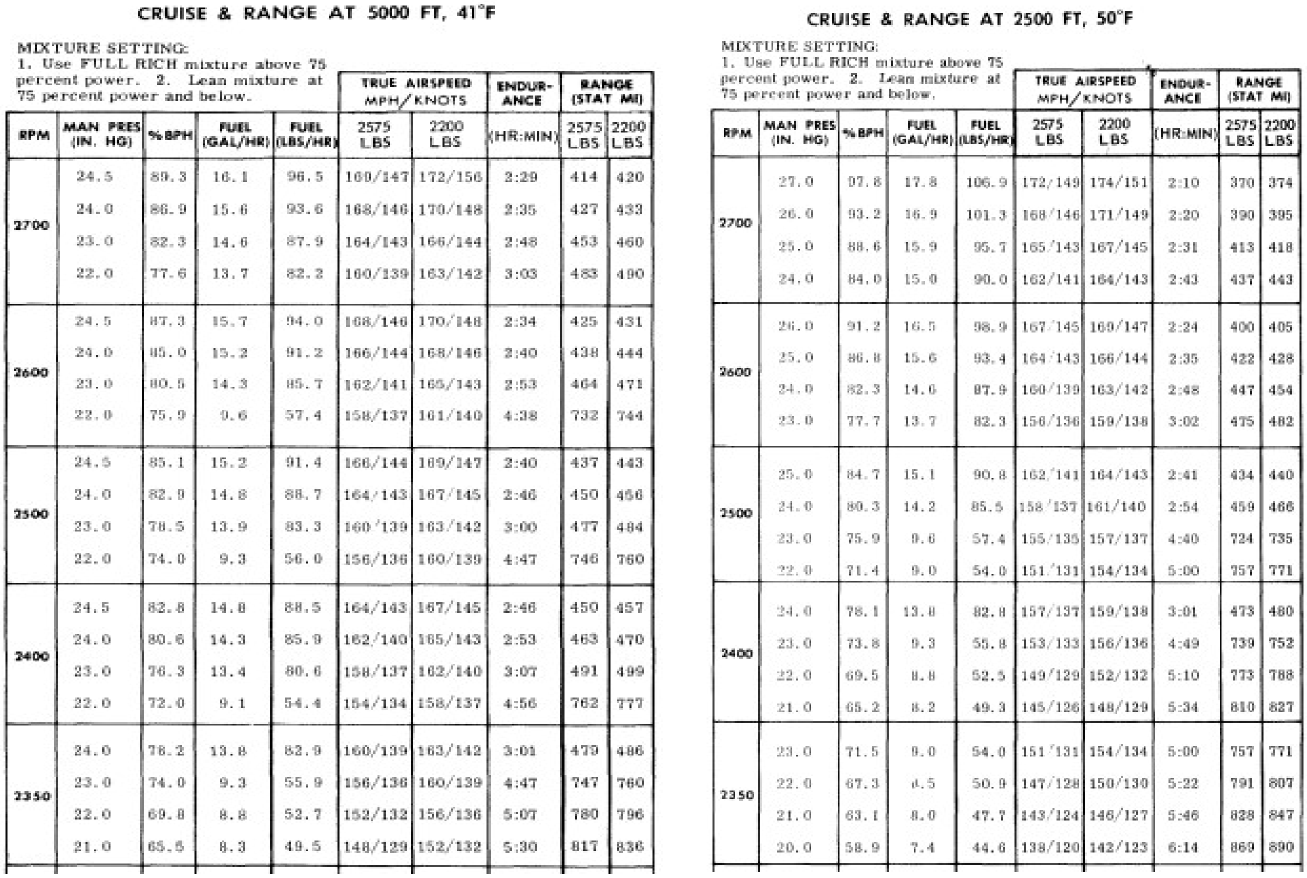
		Engine Power as % of Max. Take-Off Power						
		45 %				55 %		
	RPM	1800	2000	2200	2400	2000	2200	2400
Fuel Flow	Best Economy	5.8	6	6.3	6.6	7	7.2	7.5
[US gal/h]	Best Power	-	-	7.3	7.7	-	8.5	8.7
ISA	[°C] [°F]	Manifold Pressure (MP) [inHg]						

		Engine Power as % of Max. Take-Off Power				
		65 %			75 %	
	RPM	2000	2200	2400	2200	2400
Fuel Flow	Best Economy	7.9	8.2	8.5	9.2	9.5
[US gal/h]	Best Power	-	9.5	9.8	10.7	11
ISA	[°C] [°F]	Manifold Pressure (MP) [inHg]				

During testing, the throttle or power setting was at several different levels, and the engine speed was 2300-2700 rpm. Table 17 fuel burn rates at 2400 rpm, best power and best economy, and 55%-75% power were averaged to get an approximate fuel burn rate of 9.2 gph for the DA40. The test burn rate was approximated at 10.3 gph. The burn rate of the modified aircraft is more than the DA40, but not by much, a little over a gallon per hour.

The cost is approximately $68.40/hr for the modified aircraft and $61.10/hr for the DA40 [41]. With a comparable rate of climb and fuel burn rate, the modification is an equivalent choice to purchasing a DA40 so far. The next aircraft is the M20C. Refer to Table 18.

Table 18. M20C Cruise Performance, *taken from Mooney Ranger Operators Manual [62].*



CRUISE & RANGE AT 5000 FT, 41°F

MIXTURE SETTING:
1. Use FULL RICH mixture above 75 percent power. 2. Lean mixture at 75 percent power and below.

					TRUE AIRSPEED MPH/KNOTS		ENDUR-ANCE	RANGE (STAT MI)	
RPM	MAN PRES (IN. HG)	%BPH	FUEL (GAL/HR)	FUEL (LBS/HR)	2575 LBS	2200 LBS	(HR:MIN)	2575 LBS	2200 LBS
2700	24.5	89.3	16.1	96.5	169/147	172/156	2:29	414	420
	24.0	86.9	15.6	93.6	168/146	170/148	2:35	427	433
	23.0	82.3	14.6	87.9	164/143	166/144	2:48	453	460
	22.0	77.6	13.7	82.2	160/139	163/142	3:03	483	490
2600	24.5	87.3	15.7	94.0	168/146	170/148	2:34	425	431
	24.0	85.0	15.2	91.2	166/144	168/146	2:40	438	444
	23.0	80.5	14.3	85.7	162/141	165/143	2:53	464	471
	22.0	75.9	9.6	57.4	158/137	161/140	4:38	732	744
2500	24.5	85.1	15.2	91.4	166/144	169/147	2:40	437	443
	24.0	82.9	14.8	88.7	164/143	167/145	2:46	450	456
	23.0	78.5	13.9	83.3	160/139	163/142	3:00	477	484
	22.0	74.0	9.3	56.0	156/136	160/139	4:47	746	760
2400	24.5	82.8	14.8	88.5	164/143	167/145	2:46	450	457
	24.0	80.6	14.3	85.9	162/140	165/143	2:53	463	470
	23.0	76.3	13.4	80.6	158/137	162/140	3:07	491	499
	22.0	72.0	9.1	54.4	154/134	158/137	4:56	762	777
2350	24.0	76.2	13.8	82.9	160/139	163/142	3:01	479	486
	23.0	74.0	9.3	55.9	156/136	160/139	4:47	747	760
	22.0	69.8	8.8	52.7	152/132	156/136	5:07	780	796
	21.0	65.5	8.3	49.5	148/129	152/132	5:30	817	836

CRUISE & RANGE AT 2500 FT, 50°F

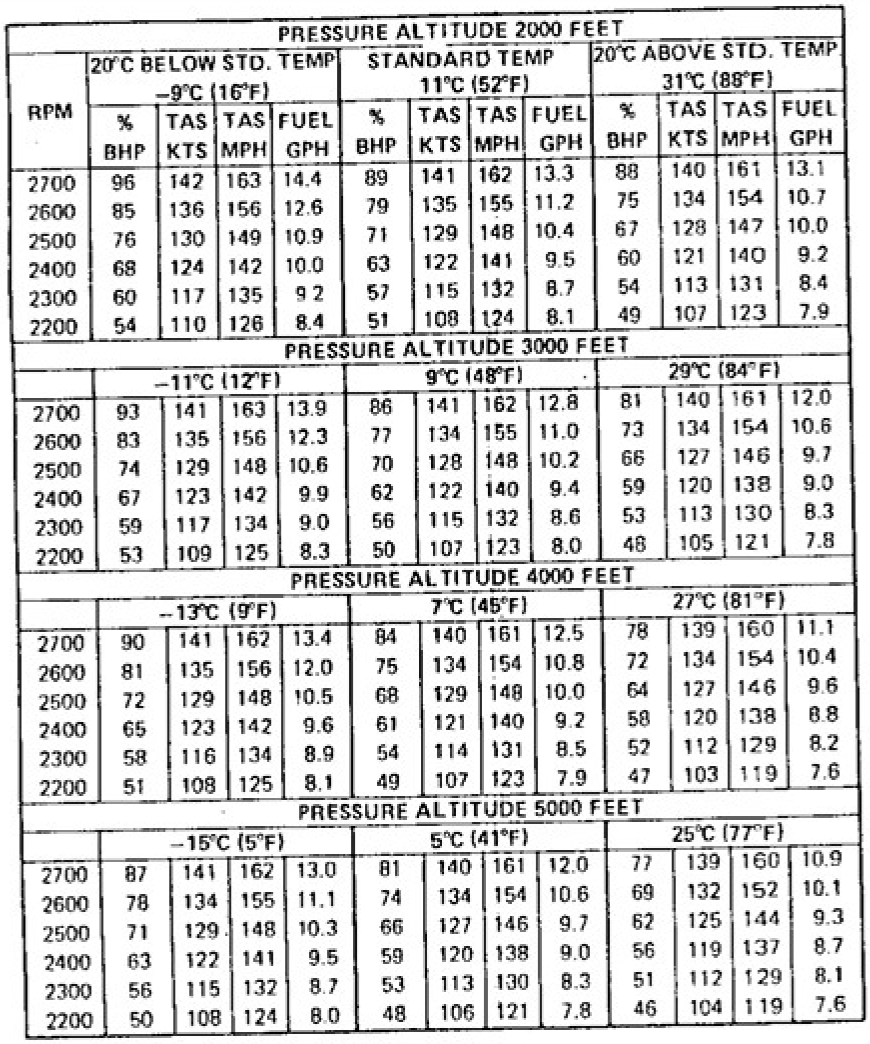
MIXTURE SETTING:
1. Use FULL RICH mixture above 75 percent power. 2. Lean mixture at 75 percent power and below.

					TRUE AIRSPEED MPH/KNOTS		ENDUR-ANCE	RANGE (STAT MI)	
RPM	MAN PRES (IN. HG)	%BPH	FUEL (GAL/HR)	FUEL (LBS/HR)	2575 LBS	2200 LBS	(HR:MIN)	2575 LBS	2200 LBS
2700	27.0	97.8	17.8	106.9	172/149	174/151	2:10	370	374
	26.0	93.2	16.9	101.3	168/146	171/149	2:20	390	395
	25.0	88.6	15.9	95.7	165/143	167/145	2:31	413	418
	24.0	84.0	15.0	90.0	162/141	164/143	2:43	437	443
2600	26.0	91.2	16.5	98.9	167/145	169/147	2:24	400	405
	25.0	86.8	15.6	93.4	164/143	166/144	2:35	422	428
	24.0	82.3	14.6	87.9	160/139	163/142	2:48	447	454
	23.0	77.7	13.7	82.3	156/136	159/138	3:02	475	482
2500	25.0	84.7	15.1	90.8	162/141	164/143	2:41	434	440
	24.0	80.3	14.2	85.5	158/137	161/140	2:54	459	466
	23.0	75.9	9.6	57.4	155/135	157/137	4:40	724	735
	22.0	71.4	9.0	54.0	151/131	154/134	5:00	757	771
2400	24.0	78.1	13.8	82.8	157/137	159/138	3:01	473	480
	23.0	73.8	9.3	55.8	153/133	156/136	4:49	739	752
	22.0	69.5	8.8	52.5	149/129	152/132	5:10	773	788
	21.0	65.2	8.2	49.3	145/126	148/129	5:34	810	827
2350	23.0	71.5	9.0	54.0	151/131	154/134	5:00	757	771
	22.0	67.3	8.5	50.9	147/128	150/130	5:22	791	807
	21.0	63.1	8.0	47.7	143/124	146/127	5:46	828	847
	20.0	58.9	7.4	44.6	138/120	142/123	6:14	869	890

First, Table 18 fuel burn rates at 2500 ft, 2350-2700 rpm, and maximum manifold pressure setting were averaged to get an approximate fuel burn rate of 14.4 gph at low altitude. Then, Table 18 fuel burn rates at 5000 ft, 2350-2700 rpm, and maximum manifold pressure setting were averaged to get an approximate fuel burn rate of 15.1 gph at high altitude. Finally, the calculated fuel burn rates at 2500 ft and 5000 ft were averaged to get an approximate 14.7 gph fuel burn rate for the M20C. The test burn rate was estimated at 10.3 gph. The burn rate of the modified aircraft is roughly 5 gallons less per hour. The fuel cost is approximately $68.40/hr for the modified aircraft and $95.60/hr for the MA20C [41].

With a comparable rate of climb and lower fuel burn rate, the modification is a better choice over the M20C so far. The next aircraft is the AA5B Tiger.

Table 19. AA5B Tiger Cruise Performance, *taken from Pilot's Operating Handbook AA-5B Tiger [60].*



PRESSURE ALTITUDE 2000 FEET												
RPM	20°C BELOW STD. TEMP −9°C (16°F)				STANDARD TEMP 11°C (52°F)				20°C ABOVE STD. TEMP 31°C (88°F)			
	% BHP	TAS KTS	TAS MPH	FUEL GPH	% BHP	TAS KTS	TAS MPH	FUEL GPH	% BHP	TAS KTS	TAS MPH	FUEL GPH
2700	96	142	163	14.4	89	141	162	13.3	88	140	161	13.1
2600	85	136	156	12.6	79	135	155	11.2	75	134	154	10.7
2500	76	130	149	10.9	71	129	148	10.4	67	128	147	10.0
2400	68	124	142	10.0	63	122	141	9.5	60	121	140	9.2
2300	60	117	135	9.2	57	115	132	8.7	54	113	131	8.4
2200	54	110	126	8.4	51	108	124	8.1	49	107	123	7.9
PRESSURE ALTITUDE 3000 FEET												
	−11°C (12°F)				9°C (48°F)				29°C (84°F)			
2700	93	141	163	13.9	86	141	162	12.8	81	140	161	12.0
2600	83	135	156	12.3	77	134	155	11.0	73	134	154	10.6
2500	74	129	148	10.6	70	128	148	10.2	66	127	146	9.7
2400	67	123	142	9.9	62	122	140	9.4	59	120	138	9.0
2300	59	117	134	9.0	56	115	132	8.6	53	113	130	8.3
2200	53	109	125	8.3	50	107	123	8.0	48	105	121	7.8
PRESSURE ALTITUDE 4000 FEET												
	−13°C (9°F)				7°C (45°F)				27°C (81°F)			
2700	90	141	162	13.4	84	140	161	12.5	78	139	160	11.1
2600	81	135	156	12.0	75	134	154	10.8	72	134	154	10.4
2500	72	129	148	10.5	68	129	148	10.0	64	127	146	9.6
2400	65	123	142	9.6	61	121	140	9.2	58	120	138	8.8
2300	58	116	134	8.9	54	114	131	8.5	52	112	129	8.2
2200	51	108	125	8.1	49	107	123	7.9	47	103	119	7.6
PRESSURE ALTITUDE 5000 FEET												
	−15°C (5°F)				5°C (41°F)				25°C (77°F)			
2700	87	141	162	13.0	81	140	161	12.0	77	139	160	10.9
2600	78	134	155	11.1	74	134	154	10.6	69	132	152	10.1
2500	71	129	148	10.3	66	127	146	9.7	62	125	144	9.3
2400	63	122	141	9.5	59	120	138	9.0	56	119	137	8.7
2300	56	115	132	8.7	53	113	130	8.3	51	112	129	8.1
2200	50	108	124	8.0	48	106	121	7.8	46	104	119	7.6

First, the fuel burn rates, in Table 19, for 2000 ft and 2300-2700 rpm were averaged at 0 deg C above standard and 20 deg C above standard to get 10.6 gph and 10.3 gph. Then the values were linearly interpolated to get a fuel burn rate of 10.5 gph at 10 deg C above standard. Then this process was repeated at 3000 ft, 4000 ft, and 5000 ft, to get fuel burn rates of 10.2 gph, 9.9 gph, and 9.7 gph at 10 degrees above standard. Then the fuel burn rates were averaged for 2000 ft through 5000 ft to get an approximate fuel burn rate of 10.1 gph for the AA5B Tiger. The fuel burn rate for the modified Cessna was estimated at 10.3 gph. The burn rate of the modified aircraft is roughly equivalent to that of the AA5B Tiger. The cost is approximately $68.40/hr for the modified aircraft and $67.10/hr for the AA5B Tiger [41]. With a better climb rate and equivalent fuel economy, the modified Skyhawk is the better choice so far. Fuel is not the only cost that is important to consider.

Cost and Benefit Analysis

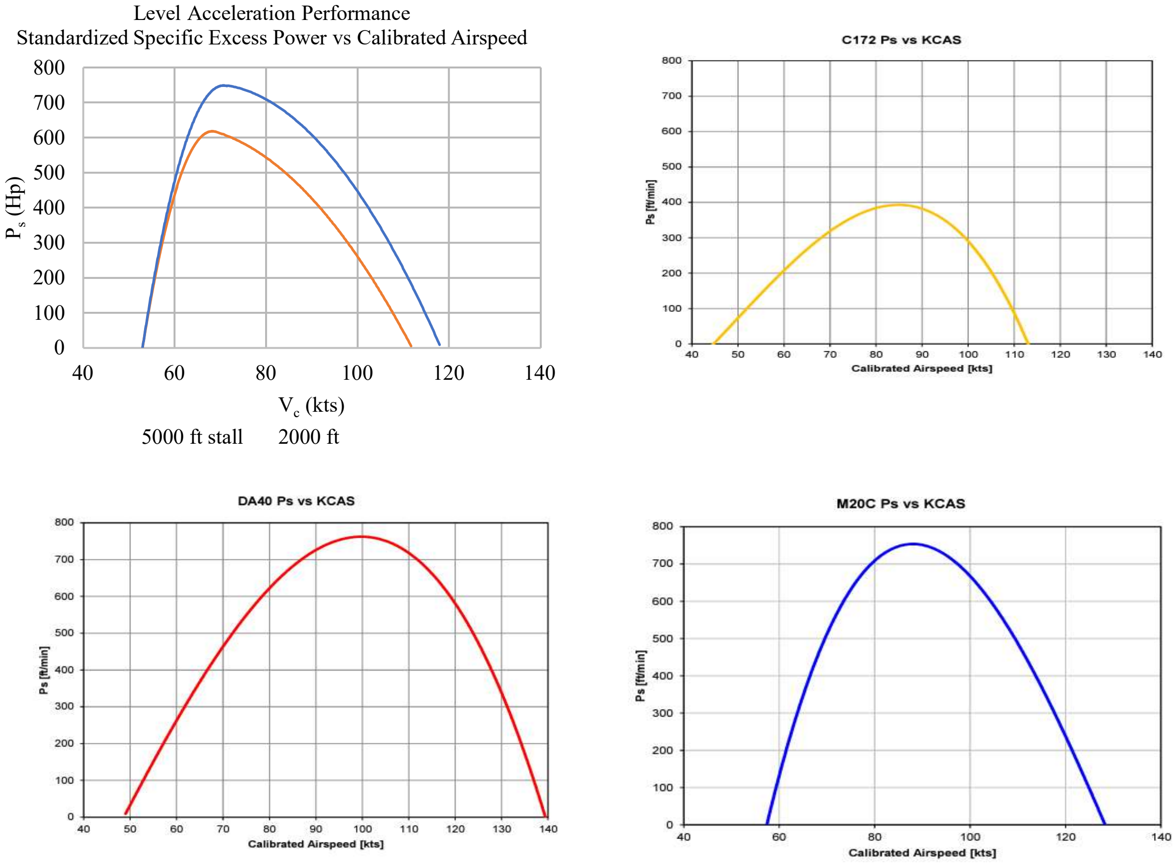
The average cost of a stock Cessna 172N, years 1976-1980, is about $121k, and the market value is $85k-$167.5k. The cost of the engine modification by one company, Aero Plains is $21.995k, the cost of the electronic ignition and variable timing by two other companies, Surefly and Electroair, is $1.755k and $2.795k, respectively, the cost of tuned exhaust by one company, Power Flow Systems, Inc., is $6.2k, and the market value of existing modified aircraft is $100-$175k. The average cost of an existing modified aircraft, years 1976-1980, is $139k. The average cost of a DA40, years 2004-2008, is about $347k, and the market value is $295k-$400k [63, 64]. The average cost of an M20C, years 1964-1969, is $82k, and the market value is $60k-$129k [65-71]. The average cost of a AA5B Tiger, years 1975-1979, is $139k, and the market value is $78k-180k [72-81]. In the Appendix, Section A.8, Table 43, shows the range of prices that were found for each aircraft. The modifications are much less than the purchase of any higher performance aircraft. Also, the market value is greater than the M20C and equivalent to the AA5B Tiger. However, the total cost of purchasing a stock airplane along with the modifications would be greater than just purchasing an M20C and could potentially be greater than purchasing an AA5B Tiger. If a pilot were to have a stock Cessna 172N already, the modification is appealing. If a pilot is looking for a higher performance aircraft and has not purchased a plane yet, additional factors must be considered to see if it is still beneficial. There are additional disadvantages to the modifications that were discovered during testing that will now be discussed.

5.2.3 Additional Considerations

It was observed during testing that the modified aircraft is incapable of doing idle descents. The engine health monitor would show an excessive cooling error if the power was too low during descent because too much cooling in the engine can cause ice. This can be a problem for cruise performance because the aircraft must be flown at a higher power to descend which burns more fuel. This aircraft has also had the rear seat removed to increase performance, but there can only one be one additional passenger. Another disadvantage is that it takes more time to modify and recertify an aircraft than it does to purchase a high-performance aircraft. These factors must be considered to make a final recommendation.

5.2.4 Recommendations

It is helpful to look at the results side by side [82-87]. Refer to Figure 33 and Table 20. The AA5B Tiger is not included in the figure because there is no P_s curve available.



Level Acceleration Performance
Standardized Specific Excess Power vs Calibrated Airspeed

C172 Ps vs KCAS

DA40 Ps vs KCAS

M20C Ps vs KCAS

Figure 33. Side by Side Comparison of Ps Curves for the High Performance Aircraft [14].

Table 20. Summary of All Information for All Aircraft.

	Stock Cessna 172N	Modified Cessna 172N	DA40	M20C	AA5B Tiger
Horsepower rating (Hp)	160	180	180	180	180
Engine speed (rpm)	2700	2700	2700	2700	2700
Engine type	O-320-H2AD	O-360-A4M	IO-360-M1A	O-360-A1D	O-360-A4K
Maximum Certified Weight (standard weight, lbs)	2300	2550	2535	2575	2400
Standard Empty Weight (lbs)	1379	1473	1620	1525	1398
Maximum Useful Load (lbs)	921	1077	915	1050	1002
Fuel tank capacity (gallons)	40	40	36	52	51
Fuel Burn (gph)	7.3	10.3	9.2	15.1	10.1
Best rate of climb at low altitude (ft/min)	400	720	760	750	700
Service Ceiling (ft)	14,200	18,300	16,400	17,200	13,800
Cost of fuel per hour ($/hr)	$48.50	$68.40	$61.10	$95.60	$67.10
Average cost ($)	$121k	$139k ($30k-$31k for mods)	$347k	$82k	$139k
Market value ($)	$85k-$168k	$100k-$175k	$295k-$400k	$60k-$129k	$78k-$180k

The rates of climb for the high-performance aircraft are comparable in Figure 33. The modified aircraft fuel burn rate and fuel cost is much less than the M20C, and equivalent to the DA40 and AA5B Tiger, according to Table 20. The existing modified aircraft average cost is much less than that of the DA40, significantly more than that of the M20C, and roughly equivalent to that of the AA5B Tiger. The maximum gross weight of the modified aircraft is equivalent to the DA40 and M20C, and significantly greater than the AA5B Tiger. The service ceiling of the modified Skyhawk is also greater than that of the DA40, M20C, and AA5B Tiger. With maximum gross weight, climb performance, service ceiling, cruise performance, and cost in mind, the upgraded Skyhawk is the best choice overall. Therefore, the recommendation to pilots is to upgrade a stock Skyhawk rather than purchase a newer, high-performance aircraft. If a pilot cannot afford every upgrade, it is recommended that the electronic ignition variable timing installation be done first because, in addition to providing fuel cost savings, it is the most cost-friendly, and potentially decreases maintenance costs. The pilot will need to consider the inability to perform idle descents, upfront cost, higher future fuel costs compared to stock, limited seating, and more time to modify and recertify.

Chapter 6
Recertification

6.1 Supplemental Type Certificate

The modifications that were done on the test aircraft required a few Supplemental Type Certificates (STCs) to recertify the aircraft. According to the FAA, an STC is, "… a type certificate (TC) issued when an applicant has received FAA approval to modify an aeronautical product from its original design. The STC, which incorporates by reference the related TC, approves not only the modification but also how that modification affects the original design [88]." Engine/propeller modifications, magneto modifications, and tuned exhaust modifications fall under this umbrella. Normally, a pilot would have to apply for his or her own STC to do this modification, but some research showed that there are existing STCs, SA2196CE, SA4428SW, SA02987CH, and SA01801AT, that lay the groundwork and could be used to get this modification approved. The first two STCs were created by a company called Air Plains, the second by a company called Electroair, and the fourth by Power Flow Systems, Inc., and these companies also offer kits to facilitate the modifications, support to get recertified, supplemental manuals, continued airworthiness instructions, and maintenance procedures [9, 57, 58, 59, 87, 88]. A pilot does not have to use these pathways to apply for recertification, but they could help make the process easier.

6.2 Applicable Regulations

6.2.1 14 CFR Part 23 Regulations

The following regulations explain the specific installation requirements, flight tests, manual requirements, and continued airworthiness requirements that must be done for recertification after the engine modification [91]. The magneto and exhaust modifications might involve additional regulations. Certified parts are easier to get approved.

14 CFR Part 23-Airworthiness Standards: Normal Category Airplanes

- §23.1529 Instructions for continued airworthiness

Subpart A-General

- §23.2000 Applicability and definitions
- §23.2005 Certification of normal category airplanes
- §23.2010 Accepted means of compliance

Subpart B-Flight

- §23.2100 Weight and center of gravity
- §23.2105 Performance data
- §23.2110 Stalling speed
- §23.2115 Takeoff performance
- §23.2130 Landing
- §23.2135 Controllability
- §23.2140 Trim
- §23.2145 Stability
- §23.2150 Stall characteristics, stall warning, and spins
- §23.2155 Ground and water handling characteristics

Subpart E-Powerplant

- §23.2400 Powerplant installation
- §23.2410 Powerplant installation hazard assessment
- §23.2415 Powerplant ice protection
- §23.2425 Powerplant operational characteristics
- §23.2435 Powerplant induction and exhaust systems
- §23.2440 Powerplant fire protection

Subpart F-Equipment

- §23.2500 Airplane level systems requirements
- §23.2505 Function and installation
- §23.2510 Equipment, systems, and installation
- §23.2505 Function and installations
- §23.2515 Electrical and electronic system lightning protection
- §23.2520 High-Intensity Radiated Fields (HIRF) protection
- §23.2525 System power generation, storage, and distribution

Subpart G-Flightcrew Interface and Other Information

- §23.2605 Installation and operation
- §23.2610 Instrument markings, control markings, and placards
- §23.2620 Airplane flight manual

Appendix A to Part 23-Instructions for Continued Airworthiness

- A23.1 General
- A23.2 Format
- A23.3 Content
- A23.4 Airworthiness limitations

6.2.2 14 CFR Part 21 Regulations

The following regulations explain the specific requirements for STC holders after they receive their certificates [92].

14 CFR Part 21-Certification Procedures for Products and Articles

- §21.3 Reporting of failures, malfunctions, and defects
- §21.5 Airplane or rotorcraft flight manual
- §21.49 Availability
- §21.50 Instructions for continued airworthiness and manufacturer's maintenance manuals having airworthiness limitations sections.
- §21.99 Required design changes

6.2.3 Advisory Circulars

The following ACs explain the process of getting an STC, doing the modification, and flight testing the aircraft afterword [93-96].

- AC 21-40A Application Guide for Obtaining a Supplemental Type Certificate
- AC 23-8C Flight Test Guide for Certification of Part 23 Airplanes
- AC 23-17B Systems and Equipment Guide for Certification of Part 23 Airplanes and Airships
- AC 23.1309-1E System Safety Analysis and Assessment for Part 23 Airplanes

6.2.4 Orders

The following explains what an STC is and how to develop a foundation for approval [97, 98].

- 8110.4 Type Certificate
- 8110.48 How to Establish the Certification Basis for Changed Aeronautical Products

6.3 Required Documentation

6.3.1 STC SA4428SW

This STC lays the foundation for gaining approval for several airframes for a 180 Hp engine upgrade and propeller combination [99, 100]. The Cessna 172N is listed as one of the models, and the O-360-A4M is listed as one of the engines in the STC. McCauley 1A170/CFA7660 and Sensenich76EM8S are listed as propeller choices for the test aircraft engine model [99, 100]. The STC is owned by Air Plains Service Corporation.

6.3.2 STC SA2196CE

This STC lays the foundation for gaining approval for several airframes for a gross weight increase [101]. The Cessna 172N is listed as one of the models. The STC is owned by Air Plains Service Corporation, and the company provides details on how the gross weight is increased [102].

6.3.3 STC SA2987CH

This STC lays the foundation for gaining approval for several airframes for the installation of an electronic ignition system [103]. The Cessna 172N is listed as one of the models. The STC is owned by Electroair Acquisition Corporation.

6.3.4 STC SA01801AT

This STC lays the foundation for gaining approval for Cessna 172 models with O-360 engines for making stock exhaust changes per STC SA4418SW [104]. The STC is owned by Power Flow Systems, Inc.

6.3.5 Form FAA 8110-12 Application for Type Certificate, Production Certificate, or Supplemental Type Certificate

This form will have to be filled out to apply for the STC [105]. The form asks if any existing STCs are being used to gain approval. If the aircraft owner only wants to do an engine and propeller swap with the original gross weight, STC SA4428SW would be listed. If the owner wants to add additional gross weight, STC SA2196CE would be listed on the form too. If the owner wishes to incorporate electronic ignition and variable timing, STC SA02987CH would be listed in addition. If the owner wishes to do the tuned exhaust STC SA01801AT would be listed as well.

6.3.6 Form 337 Major Repair and Alteration

This form will have to be filled out when performing the modification [106]. The FAA can explain the exact process, but the initial guess is that one form can encompass the engine replacement, propeller replacement, variable timing electronic ignition kit installation, and tuned exhaust installation, and seat removal.

6.4 Application Process

There is a series of steps that are laid out on the FAA website for obtaining an STC for an aircraft. The steps listed on the website have been consolidated here for convenience [107].

Contacting the FAA

The FAA encourages an STC applicant to connect with his or her local FAA Aircraft Certification Office (ACO) [108]. Hiring a lawyer or flight test engineer to facilitate discussion with the FAA and completion of paperwork is recommended as well.

STC Approval Process

This section basically has two parts which are taken directly from the FAA website.

- Establishment of Certification Basis [108]:
    1) Applicant applies for STC (Form FAA 8110-12)
    2) Familiarization and preliminary type certification board (TCB) meetings
    3) FAA develops certification program plan
    4) Establishment of certification basis by FAA
    5) Applicant submits data for approval
    6) FAA design evaluation
    7) FAA and applicant hold specialists and interim type certification meetings, as required
    8) FAA performs conformity inspections
    9) Engineering compliance examinations
    10) Pre-flight TCB Meeting
    11) Applicant performs ground inspections, ground tests, and flight tests
    12) FAA reviews manufacturer's flight test results and issues TIA
    13) FAA performs conformity inspection, witnesses tests, performs official certification flight tests and flight standards evaluations
    14) Functional and reliability testing
    15) FAA approves flight manual supplement or supplemental flight manual and holds final TCB meeting
    16) AEG completes continuing airworthiness determination
    17) FAA issues STC

- STC is issued [108]:

  An STC will only be issued if:

  1) The pertinent technical data have been examined and found satisfactory
  2) All necessary tests and compliance inspections have been completed
  3) The alteration has been found to conform with the technical data

Permission for an Existing STC

Since the STCs listed above belong to Air Plains, Electroair, and Power Flow Systems, Inc., the pilot must ask permission to use the STC and all drawings, data, specifications, supplemental manuals, and maintenance procedures for continued airworthiness [109].

Responsibilities After STC Issuance

The following, taken directly from the FAA website, are the responsibilities of the STC holder once the STC is issued [110]:

- Report failures, malfunctions, and defects (14 CFR §21.3)
- Make the type certificate available to FAA and National Transportation Safety Board, upon request (14 CFR §21.49)
- Make Instructions for Continued Airworthiness available to owner/operator (14 CFR §21.50)
- Make required design changes to address Airworthiness Directives and make them available (14 CFR §21.99)
- Make flight manuals supplements and supplemental flight manuals available with each alteration (14 CFR §21.5 and §23.1581)

6.5 Required Flight Testing

An all-inclusive flight test program may not be required for a modification like this, but certain tests are still required. Tony Morris, a test pilot for the C.A.A. gives some suggestions from a previous test program [111]. The FAA will ultimately determine exactly what tests are needed, but analysis of past programs can give an idea of what to expect. An aircraft owner might benefit from hiring a flight test engineer and/or test pilot with extensive background in aircraft modifications to aid in writing the test plan for FAA approval and to perform flight tests [111].

6.5.1 Weight and Balance

A new weight and balance analysis must be done for FAA approval. Refer to the STCs below [111].

- STC SA4428SW (no gross weight change)
    1) The aircraft will have to be weighed without pilot, passengers, usable fuel, and usable oil to determine the empty weight.
    2) A weight and balance at maximum limitations will have to be done to show compliance to flight envelope.

- STC SA2196CE (gross weight change)
    1) The same steps as STC SA 4428SW will have to be performed.
    2) A new empty weight and useful load will have to be defined according to the stock gross weight.
    3) A new moment arm will have to be established for the empty weight.
    4) In addition, a new flight envelope will have to be defined that reflects the change in gross weight.

6.5.2 Stall

Some new stall calculations, demonstrations, and processes must be done for FAA approval [111].

Performance

Determine new calculations for the following tests.

- Level, turning, and accelerated stalling period, speed, and warning speed

Successfully demonstration the following maneuvers.

- Level, turning, and accelerated stall

6.5.3 Takeoff and Landing

Some new takeoff and landing calculations, demonstrations, and processes must be done for FAA approval [111].

Handling

Determine the differences in how the aircraft behaves during takeoff and landing.

Performance

Determine new calculations for the following tests.

- Normal, short-field, and takeoff distances, speeds, ground and flight paths, and runs
- Normal, short-field, and balked landing distance and speeds
- Accelerate-stop distance

Define new emergency procedures for abnormal conditions.

- Takeoff climb with engine inoperative

Successful demonstration the following maneuvers.

- Normal and short field takeoff
- Normal, short field, and balked landing

6.5.4 Longitudinal, Lateral, and Directional Stability

Some new stability calculations, processes, and demonstrations must be done for FAA approval [111].

Handling

Determine the differences in how stable the aircraft is longitudinally, laterally, and directionally [111].

- Ground and in-flight Lateral/Directional
- Ground and in-flight Longitudinal (trim, control, damping)
- Control during landings and takeoffs
- Elevator control forces in maneuvers
- Operation on unpaved surfaces
- Vibration and buffeting
- High speed characteristics

Performance

Determine new calculations for the following tests.

- Minimum control speed

Successfully demonstrate the following controlled maneuvers.

- Level flight
- Controlled landing
- Controlled takeoff
- Minimum speed flight
- High speed flight

6.5.5 Engine Performance

Handling

Determine the differences in how the engine behaves.

- Minimum control speed

Performance

Define new emergency procedures for engine abnormalities.

- In-flight restart
- Engine-inoperative/abnormal

6.5.6 Additional Resources

The following were listed as additional resources by Tony Morris, Test Pilot for the CAA, for a pilot doing this type of modification [111].

- CAA
- FAA Regulatory and Guidance Library (RGL)

6.6 Final Thoughts

The exact process that was used to recertify and test the modified Cessna 172N was not disclosed, however, SA4428SW, SA2196CE, SA2987CH, and SA01801AT STCs might have been used. When an aircraft owner is ready to move forward with an aircraft upgrade, it is important to familiarize with the regulations, requirements, process, cost, and time of recertification. Several companies give the impression that, if the modification is done through them with one of their kits, they help with the entire process and provide supplemental manuals and continued airworthiness documentation. If an owner wishes to buy his or her own parts and take another route to gain an STC, it might be helpful to hire a flight test engineer or lawyer to facilitate the process with the FAA and take care of the paperwork. Finally, regardless of the path that the pilot chooses, it is best to contact the FAA early on to make the process as easy as possible.

Chapter 7
Level of Error and Uncertainty

To provide the level of uncertainty in the data calculations for level accelerated and climb flight testing, the mean and standard deviation were calculated for all sets of raw data points that were supposed to remain constant. Many of the measurements contained a range of values, and this is a way of quantifying the variation in that range. For the steady climbs, heading and airspeed were supposed to be constant, and for level accelerations, heading and altitude were supposed to be constant. These errors are important because the airspeed, altitude, and heading are used for many calculations, and deviations cause error. Section A.9, Tables 44 and 45, in the Appendix, contain all of the raw data values and statistical calculations. Refer to Table 21 for a summary of the standard deviations and means for the 65, 80, and 95 knot climbs and 2000 ft and 5000 ft level accelerations [112, 113].

Table 21. Summary of Means and Standard Deviations for All Flight Maneuvers.

		SSC 65 knots	**SSC 80 knots**	**SSC 95 knots**	**LA 2000 ft**	**LA 5000 ft**
Magnetic Heading	*Mean*	146	334	169	339	173
	Standard Deviation	3.49	3.55	3.33	2	2.21
Indicated Airspeed	*Mean*	66.5	79.9	94.6		
	Standard Deviation	1.32	0.625	0.654		
Indicated Altitude	*Mean*				2037	5010
	Standard Deviation				20.0	0.364

Assuming the data for all maneuvers is normalized, Gaussian distributions or bell curves can be used to approximate the spread of raw data values from the various means [114]. For the 65 knot steady climb, the error from the mean value of 146 degrees is very small. Roughly 68% of values fall within one standard deviation of the mean or 142-149, and 95% lie within two standard deviations or 139-153. In actuality, 83% lie within one standard deviation, and 100% lie within two. The discrepancy is because the sample size is small. The larger the sample size, the more accurate the bell curve model becomes [115]. Looking at how the raw data is spread with reference to mean and standard deviation can help the reader feel more confidence in the data collecting technology and methods used in the flight tests.

The bell curve or Gaussian Distribution is only an estimate of the data. Another observation that can provide assurance is that the majority of the data set values fall within the tolerances that were defined in sub section 3.3.2 of Chapter 3, +/- 5 degrees heading, +/- 3 kts airspeed, and +/- 20 ft altitude for the level accelerations. For the magnetic heading, 83% for the 65 knot steady climb, 86% for the 80 and 95 knot steady climbs, 95% for the 2,000 ft level acceleration, and 94% for the 5,000 ft level acceleration values were within tolerance. For the indicated airspeed, all values were within tolerance. There was the most variation with altitude. 100% of values were within tolerance for the 5,000 ft level acceleration, but the 2000 ft level acceleration only had 65% of values within the +/- 20 ft tolerance. The outliers were not drastically far outside of this tolerance, with 80% within +/- 25 ft and 92% within +/- 30 ft. If this performance testing was for airworthiness, the data would have to lie within the tolerance values per the FAA, and the run would have to be redone. For this aircraft comparison performance testing, there is room for higher tolerances or +/-30 ft, and the data, although suspicious, can still be used. The errors were probably due to taking the first data points too early, marking down incorrect time stamps, or human error in maintaining flight conditions. "Using the guidance of the rules for uncertainty from the AIAA, in order to provide readers with the ability to understand and replicate the results of the lab in this paper, the methods of data collection, data reduction, and performance chart generation were fully described, and statistical methods for calculating uncertainty in the data were provided [116]."

Chapter 8
Sustainability

When deciding whether the modifications are merited, sustainability in the aviation community must be considered. A 180 Hp engine, electronic ignition with variable timing, and tuned exhaust can benefit an aircraft owner, but how about the industry?

It was shown that the modified aircraft has superior climb performance to the stock aircraft and comparable performance to similar 180 Hp aircraft. The upgraded Skyhawk climbs at 720 ft/min at low altitude, while the stock Cessna 172N climbs at 400 ft/min. This means that the modified aircraft can gain 300 ft more per minute to avoid sudden clouds or weather, mountains, and other aircraft. At sea level, a 300 ft/min boost in ROC gives greater ability to clear obstacles surrounding the airport more quickly. Upgraded climb performance at altitude and during takeoff boosts the pilot's ability to maneuver, practice collision avoidance, and clear obstacles. The tuned exhaust contains no welds in the portion of piping that provides heat to the cabin which means less discontinuities and potential cracking [6]. This improved design greatly reduces the potential of carbon monoxide leakage in the cabin [6]. Both the enhanced collision avoidance capability and decreased likelihood of carbon monoxide poisoning increase flight safety. Another important factor is cruise performance.

The fuel economy of 10.3 gph of the modified Skyhawk, due to the variable timing electronic ignition and tuned exhaust, is not as good as the stock Cessna 172N at 8.2 gph, but it is comparable or better than that of several other aircraft with similar engines at 9.2-15.1 gph. A two-hour flight would equal 30.2 gallons at 15.1 gph and would equal 20.6 gallons at 10.3 gph, saving about 10 gallons over just a short flight. As the flight lengthens, the better fuel economy adds up, with 30 gallons saved over a long six-hour flight. There is already an abundance of 180 Hp aircraft in the industry. Having more modified Skyhawks to do the work of these higher performance aircraft, such as the M20C, would conserve fuel, and make the better fuel consumption of the modified aircraft over the other higher performance aircraft outweigh the worse full consumption than stock. Also, the FAA has already approved unleaded fuel, and the Cessna 172N and O-360-A4M engine upgrade, as long as it is a gravity fed fuel system, is unleaded fuel capable with STC approval [117]. This means that the engine modification, in the very least, is not only beneficial, but might also be necessary, due to the eventual removal of 100LL fuel. The combination of upgraded fuel economy and unleaded fuel capability of the modified Skyhawk allows for higher fuel conservation, better fuel costs, less emissions, cleaner operations, and most importantly, continued operation when unleaded fuel is the only type available. Marketability is also crucial.

It was shown that the average cost of a modified aircraft, \$139k, is much less than that of the DA40, \$347k. Choosing a modified Skyhawk over a DA40 saves a pilot \$208k on average. This means more possibility for higher performance for a greater amount of pilots. For each M20C that is replaced with a modified Skyhawk, \$27.20/hr in fuel cost is saved. At two hours a day, 730 hours a year, this would be \$20k/yr of fuel cost savings per aircraft. Lower cost or more accessibility to higher performance aircraft and savings in fuel costs have the potential to increase interest in aircraft ownership, pursuing training, and flying more in general, which helps sustain the aviation economy. Introducing more modified Skyhawks can increase flight safety, conserve fuel, and reduce resale, fuel, and maintenance costs, and thereby, help support the aviation community. Modifying older airframes with better engines and technology might be a good, intermediate option, while other clean energy and electric aircraft technologies are being researched.

Chapter 9
Conclusion

The purpose of this research was to prove that modifying a Cessna 172N with a 180Hp engine, matching propeller, electronic ignition variable timing, and tuned exhaust is the best option for higher performance, and that introducing more upgraded Skyhawks into the market helps resolve some of the issues in the aviation industry. The modifications, the recertification process, and the feasibility of doing the upgrades were also discussed. The test aircraft was a Cessna 172N, upgraded with a O-360-A4M engine, matching propeller, and the other attributes. Test methods, data reduction, and performance analysis were reported. Results were compared to performance of the aircraft before modification and other similar aircraft. The modified Skyhawk has superior climb performance, but worse fuel economy to the stock Cessna 172N. However, the upgraded aircraft has comparable climb performance and comparable or superior fuel economy to the Diamond DA40, Mooney M20C, and Grumman American AA5B Tiger. Overall, the modification is a better choice than obtaining another aircraft for higher climb performance, higher service ceiling, better fuel consumption, lower future fuel costs, higher maximum gross weight, and better technology. Therefore, the recommendation to pilots is to upgrade a stock Skyhawk rather than purchase a newer, high-performance aircraft. If a pilot cannot afford every upgrade, it is recommended that the electronic ignition variable timing installation be done first because, in addition to providing fuel cost savings, it is the most cost-friendly, and potentially decreases maintenance costs. It is also advised that the owner consider the associated significant upfront cost, recertification requirements, and excessive cooling issues before upgrading. At least one Supplemental Type Certificate (STC) is needed for the engine modification, and other STCs are needed for variable timing electronic ignition and tuned exhaust. Kits, services, recertification support, airworthiness instructions, manuals, and STC paperwork are becoming available to pilots. It is advised that pilots contact the FAA early for help, and consider hiring a lawyer, flight test engineer, and/or test pilot to assist with flight planning and paperwork. This thesis provides a wealth of knowledge to flight test engineers on how to make a test plan, execute a test plan, and apply for recertification for a modified GA aircraft, and to aircraft owners on frugal choices with aircraft modifications. Introducing more modified aircraft like this into the market also enhances flight safety and sustainability and provides a temporary option as clean energy technologies are explored. Electronic ignition variable timing is advertised to replace magneto technology at a feasible cost and reduce maintenance costs [8, 9, 12]. "Most parts … are not life limited (the MTH [Magneto Timing Housing] is recommended to be changed at overhaul of the engine; spark plug wires on a regular interval) - this combined with reduced spark plug fouling means lower maintenance costs [9]." "Not only are points and condensers a point of failure in magneto ignitions, so are rotors and caps as they carbon track and provide alternate paths to ground as resistance builds in the system. [The technology] … utilizes none of these antiquated technologies. [The] … components are all solid state and industrial-grade electronics. The result is zero maintenance, zero overhauls, no rebuilds, and no maintenance costs whatsoever [12]." Over a thousand operating hours and decreased wear and tear is offered. Research of failure trends over time of electronic ignition could solidify these claims. Future work could include long term research on fuel consumption, maintenance intervals, and fuel and maintenance costs of high-performance aircraft to better identify electronic magneto failure trends and the aircraft with best cruise performance and maintenance requirements. This study could lead to better understanding of what technology improves energy conservation and quality, and if the identified aircraft are used to replace aircraft in the market, it can help sustain the aviation community.

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Appendix

A.1 Test Weight and Weight Ratio for Raw Data

Table 22. Partial Table of Sample Test Weight and Weight Ratio Calculations for Raw Data.

	t	W_T	W_T/W_S
	Time	Test Weight	Weight Ratio
	sec	lbs	unitless
20:23:16	0	2275	0.8922
20:23:16	1	2275	0.8922
20:23:17	2	2275	0.8922
20:23:18	3	2275	0.8922
20:23:19	4	2275	0.8922
20:23:20	5	2275	0.8922
20:23:21	6	2275	0.8922
20:23:22	7	2275	0.8922
20:23:23	8	2275	0.8922
20:23:24	9	2275	0.8922
20:23:25	10	2275	0.8922
20:23:26	11	2275	0.8922
20:23:27	12	2275	0.8922
20:23:28	13	2275	0.8921
20:23:29	14	2275	0.8921
20:23:30	15	2275	0.8921
20:23:31	16	2275	0.8921
20:23:32	17	2275	0.8921
20:23:33	18	2275	0.8921
20:23:34	19	2275	0.8921
20:23:35	20	2275	0.8921
20:23:36	21	2275	0.8921
20:23:37	22	2275	0.8921
20:23:38	23	2275	0.8921
20:23:39	24	2275	0.8921

A.2 Calibrations for Raw Data

Table 23. Partial Table of Sample Calibration Calculations for the G5 Airspeed Indicator.

	H_{ic}	T_c	IAS	GS_T			σ	GS_c, V_t	V_c	$\Delta V_{ic} + \Delta V_{pc}$
	Calibrated Altitude	Calibrated Temperature	Indicated Airspeed	GPS Ground Speed Component Along Track	GPS Ground Track	Magnetic Heading	Density Ratio	GPS Ground Speed Component Along Heading / True Airspeed	Calibrated Airspeed	Total Instrument Error
	ft	deg C	kts	kts	deg	deg	unitless	kts	kts	kts
20:35:10	4623	15.5	66.2	80.7	133	146.6	0.8424	78.44	71.99	5.791
20:35:11	4631	15.4	66.2	80.7	133	147.1	0.8424	78.27	71.84	5.638
20:35:12	4639	15.4	66.2	80.8	133	147.2	0.8422	78.33	71.88	5.685
20:35:13	4646	15.4	66.3	80.8	133	146.8	0.8420	78.47	72.00	5.701
20:35:14	4655	15.4	66.1	80.7	133	146.4	0.8417	78.50	72.02	5.921
20:35:15	4665	15.4	66	80.6	133	146.3	0.8414	78.44	71.95	5.948
20:35:16	4675	15.3	66	80.6	133	146.5	0.8413	78.37	71.89	5.887
20:35:17	4684	15.3	65.9	80.5	133	146.3	0.8411	78.34	71.85	5.946
20:35:18	4692	15.3	65.9	80.6	133	146.4	0.8408	78.41	71.89	5.995
20:35:19	4701	15.3	66.1	80.6	133	146.8	0.8405	78.27	71.76	5.661

A.3 65 knot Steady State Climb Data Reduction

Table 24. Raw Data for the 65 knot Steady Climb.

	Time			H_{ic}	OAT	V_c	RPM	W_T
		GPS Ground Track	Magnetic Heading	Pressure Altitude	Temperature	Calibrated airspeed	Engine Speed	Test Weight
	sec	deg	deg	ft	Celsius	kts	rpm	lbs
20:30:34	0	134	138.7	1812	22.9	67.26	2380	2268
20:31:39	65	135	143.6	2543	21.1	67.44	2370	2267
20:32:34	120	135	146.7	3094	19.5	69.42	2350	2266
20:33:27	173	134	148.2	3588	18.3	72.13	2330	2265
20:34:08	214	134	148.4	4033	17.3	70.16	2310	2264
20:35:59	325	133	148	5042	14.3	71.66	2225	2262

Table 25. Data Reduction Calculations for a Specific Point for the 65 knot Steady State Climb.

2600 ft	Time	ROC, dH/dt	T_a	W_T	RPM
		Rate of Climb	Temperature	Test Weight	Engine Speed
	sec	ft/sec	Celsius	lbs	rpm
145 deg	72.57	10.48	20.9	2267	2368

Table 26. Data Reduction Calculations for a Specific Point for the 65 knot Steady State Climb.

ROC_{OBS}	ROC_{OBS}	T_a	T_S	ROC_{TC}	σ	W_T	C_{IW}	RPM	BHP_S	BHP_T	P_{IW}
Observed Rate of Climb	Observed Rate of Climb	Observed Temperature	Standard Temperature	Test Corrected ROC	Density ratio	Test Weight	Instrument and Weight Corrected ROC	Engine Speed	Standard Brake Horsepower	True Brake Horsepower	Instrument and Weight Corrected Power
10.48	628.7	20.9	9.8	653	0.8913	2267	654	2368	121.4	119.1	134

Table 27. Data Reduction Calculations for a Specific Point for the 65 knot Steady State Climb.

4600 ft	Time	ROC, dH/dt	T_a	W_T	RPM
		Rate of Climb	Temperature	Test Weight	Engine Speed
	sec	ft/sec	Celsius	lbs	rpm
145 deg	276.1	9.173	15.6	2263	2262

Table 28. Data Reduction Calculations for a Specific Point for the 65 knot Steady State Climb.

ROC_{OBS}	ROC_{OBS}	T_a	T_S	ROC_{TC}	σ	W_T	C_{IW}	RPM	BHP_S	BHP_T	P_{IW}
Observed Rate of Climb	Observed Rate of Climb	Observed Temperature	Standard Temperature	Test Corrected ROC	Density ratio	Test Weight	Instrument and Weight Corrected ROC	Engine Speed	Standard Brake Horsepower	True Brake Horsepower	Instrument and Weight Corrected Power
ft/sec	ft/min	Celsius	Celsius	ft/min	unitless	lbs	ft/min	rpm	Hp	Hp	Hp
9.175	550.5	15.6	5.8	570	0.8428	2263	555	2262	105.8	104.0	114

A.4 80 knot Steady State Climb Data Reduction

Table 29. Raw Data for the 80 knot Steady Climb.

	Time			H_{ic}	OAT	V_c	RPM	W_T
		GPS Ground Track	Magnetic Heading	Pressure Altitude	Temperature	Calibrated airspeed	Engine Speed	Test Weight
	sec	deg	deg	ft	Celsius	kts	rpm	lbs
20:48:49	0	335	339.2	2004	21.9	78.78	2420	2249
20:49:38	49	336	337.3	2591	20.7	78.10	2400	2248
20:50:20	91	336	335.1	3095	19.6	75.84	2390	2247
20:51:00	131	336	333	3537	18.5	75.55	2380	2247
20:51:56	187	335	331.5	4126	17	74.24	2360	2246
20:53:08	259	335	329.2	4784	15.1	73.34	2350	2244
20:53:30	281	335	329.5	4991	14.7	71.14	2320	2244

Table 30. Data Reduction Calculations for a Specific Point for the 80 knot Steady State Climb.

2600 ft	Time	ROC, dH/dt	T_a	W_T	RPM
		Rate of Climb	Temperature	Test Weight	Engine Speed
	sec	ft/sec	Celsius	lbs	rpm
335 deg	48.71	12.00	20.7	2248	2399

Table 31. Data Reduction Calculations for a Specific Point for the 80 knot Steady State Climb.

ROC_{OBS}	ROC_{OBS}	T_a	T_S	ROC_{TC}	σ	W_T	C_{IW}	RPM	BHP_S	BHP_T	P_{IW}
Observed Rate of Climb	Observed Rate of Climb	Observed Temperature	Standard Temperature	Test Corrected ROC	Density ratio	Test Weight	Instrument and Weight Corrected ROC	Engine Speed	Standard Brake Horsepower	True Brake Horsepower	Instrument and Weight Corrected Power
ft/sec	ft/min	Celsius	Celsius	ft/min	unitless	lbs	ft/min	rpm	Hp	Hp	Hp
12.00	719.9	20.7	9.8	748	0.8919	2248	752	2399	126.3	123.9	141

Table 32. Data Reduction Calculations for a Specific Point for the 80 knot Steady State Climb.

4600 ft	Time	ROC, dH/dt	T_a	W_T	RPM
		Rate of Climb	Temperature	Test Weight	Engine Speed
	sec	ft/sec	Celsius	lbs	rpm
335 deg	237.2	9.208	15.6	2245	2353

Table 33. Data Reduction Calculations for a Specific Point for the 80 knot Steady State Climb.

ROC_{OBS}	ROC_{OBS}	T_a	T_S	ROC_{TC}	σ	W_T	C_{IW}	RPM	BHP_S	BHP_T	P_{IW}
Observed Rate of Climb	Observed Rate of Climb	Observed Temperature	Standard Temperature	Test Corrected ROC	Density ratio	Test Weight	Instrument and Weight Corrected ROC	Engine Speed	Standard Brake Horsepower	True Brake Horsepower	Instrument and Weight Corrected Power
ft/sec	ft/min	Celsius	Celsius	ft/min	unitless	lbs	ft/min	rpm	Hp	Hp	Hp
9.206	552.3	15.6	5.8	572	0.8428	2245	559	2353	119.1	117.1	130

A.5 95 knot Steady State Climb Data Reduction

Table 34. Raw Data for the 95 knot Steady Climb.

		GPS Ground Track	Magnetic Heading	Pressure Altitude	Temperature	Calibrated airspeed	Engine Speed	Test Weight
	sec	deg	deg	ft	Celsius	kts	rpm	lbs
21:00:08	0	155	161.3	2002	21.6	93.72	2500	2237
21:01:02	54	155	166.9	2512	20.5	92.78	2490	2236
21:02:05	117	155	168.3	3076	19.1	91.17	2490	2235
21:03:05	177	156	169.5	3553	17.9	91.57	2480	2234
21:04:03	235	155	171	4002	16.8	92.53	2480	2233
21:05:28	320	156	171.1	4653	15.2	94.81	2490	2232
21:06:21	373	156	171.5	4996	14.6	95.97	2490	2231

Table 35. Data Reduction Calculations for a Specific Point for the 95 knot Steady State Climb.

2600 ft	Time	ROC, dH/dt	T_a	W_T	RPM
		Rate of Climb	Temperature	Test Weight	Engine Speed
	sec	ft/sec	Celsius	lbs	rpm
170 deg	64.23	8.944	20.3	2236	2490

Table 36. Data Reduction Calculations for a Specific Point for the 95 knot Steady State Climb.

ROC_{OBS}	ROC_{OBS}	T_a	T_S	ROC_{TC}	σ	W_T	C_{IW}	RPM	BHP_S	BHP_T	P_{IW}
Observed Rate of Climb	Observed Rate of Climb	Observed Temperature	Standard Temperature	Test Corrected ROC	Density ratio	Test Weight	Instrument and Weight Corrected ROC	Engine Speed	Standard Brake Horsepower	True Brake Horsepower	Instrument and Weight Corrected Power
ft/sec	ft/min	Celsius	Celsius	ft/min	unitless	lbs	ft/min	rpm	Hp	Hp	Hp
8.944	536.6	20.3	9.8	557	0.8931	2236	562	2490	141.2	138.6	160

Table 37. Data Reduction Calculations for a Specific Point for the 95 knot Steady State Climb.

4600 ft	Time	ROC, dH/dt	T_a	W_T	RPM
		Rate of Climb	Temperature	Test Weight	Engine Speed
	sec	ft/sec	Celsius	lbs	rpm
170 deg	314.0	6.996	15.3	2232	2490

Table 38. Data Reduction Calculations for a Specific Point for the 95 knot Steady State Climb.

ROC_{OBS}	ROC_{OBS}	T_a	T_S	ROC_{TC}	σ	W_T	C_{IW}	RPM	BHP_S	BHP_T	P_{IW}
Observed Rate of Climb	Observed Rate of Climb	Observed Temperature	Standard Temperature	Test Corrected ROC	Density ratio	Test Weight	Instrument and Weight Corrected ROC	Engine Speed	Standard Brake Horsepower	True Brake Horsepower	Instrument and Weight Corrected Power
ft/sec	ft/min	Celsius	Celsius	ft/min	unitless	lbs	ft/min	rpm	Hp	Hp	Hp
6.996	419.8	15.3	5.8	434	0.8437	2232	426	2490	141.2	138.8	156

A.6 5000 ft Level Acceleration Data Reduction

Table 39. Raw Data for the 5000 ft Level Acceleration.

	t			H_{ic}	T_a	V_c	V_c
	Time	GPS Ground Track	Magnetic Heading	Calibrated Pressure Altitude	Calibrated Temperature	Calibrated Airspeed	Calibrated Airspeed
	sec	deg	deg	ft	deg C	kts	ft/sec
20:38:30	0	155	175.1	5017	14.2	58.83	99.29
20:38:31	1	155	175.3	5015	14.1	60.65	102.4
20:38:32	2	156	175.5	5011	14.1	62.94	106.2
20:38:33	3	156	175.3	5005	14	65.11	109.9
20:38:34	4	156	175.5	5000	14	67.01	113.1
20:38:35	5	157	176.2	4997	13.9	68.88	116.3
20:38:36	6	157	176.7	4995	13.9	70.04	118.2
20:38:37	7	157	176.4	4995	13.9	71.21	120.2
20:38:38	8	157	176.5	4997	13.8	72.12	121.7
20:38:39	9	158	176.9	5001	13.8	73.16	123.5
20:38:40	10	158	177.1	5004	13.8	73.76	124.5
20:38:41	11	159	177.5	5008	13.8	74.80	126.2
20:38:42	12	159	177.3	5011	13.7	75.76	127.9
20:38:43	13	159	177.8	5014	13.7	76.49	129.1
20:38:44	14	160	178	5016	13.7	77.80	131.3
20:38:45	15	160	178.3	5017	13.6	78.72	132.9
20:38:46	16	160	178.3	5018	13.6	79.67	134.5
20:38:47	17	160	178.3	5018	13.6	80.54	135.9
20:38:48	18	160	178.4	5019	13.6	81.53	137.6
20:38:49	19	160	178.1	5018	13.6	82.80	139.8

20:38:50	20	159	177	5018	13.5	83.99	141.8
20:38:51	21	159	176.4	5019	13.5	85.41	144.2
20:38:52	22	159	175.9	5020	13.5	86.77	146.5
20:38:53	23	159	175.3	5020	13.4	88.20	148.9
20:38:54	24	159	174.9	5020	13.4	89.52	151.1
20:38:55	25	158	174.4	5020	13.4	90.44	152.6
20:38:56	26	158	173.5	5020	13.4	91.99	155.3
20:38:57	27	158	172.9	5021	13.4	93.31	157.5
20:38:58	28	157	172.1	5021	13.3	94.30	159.2
20:38:59	29	157	171.9	5021	13.3	95.36	160.9
20:39:00	30	157	171.1	5019	13.3	96.77	163.3
20:39:01	31	157	170.6	5018	13.3	98.05	165.5
20:39:02	32	157	170.3	5016	13.3	99.15	167.4
20:39:03	33	157	170.2	5014	13.2	100.2	169.1
20:39:04	34	157	170.2	5011	13.2	101.2	170.8
20:39:05	35	157	170.4	5008	13.2	102.1	172.3
20:39:06	36	157	170.3	5003	13.2	102.9	173.7
20:39:07	37	157	170.5	5000	13.2	103.7	175.0
20:39:08	38	157	170.6	4997	13.2	104.2	175.9
20:39:09	39	157	170.5	4995	13.2	104.7	176.8
20:39:10	40	157	170.6	4996	13.2	105.0	177.3
20:39:11	41	157	170.7	4998	13.2	105.2	177.5
20:39:12	42	157	170.8	5000	13.2	105.4	177.9
20:39:13	43	157	170.7	5002	13.2	105.6	178.2
20:39:14	44	157	170.6	5003	13.2	105.9	178.8
20:39:15	45	157	170.6	5004	13.2	106.2	179.2
20:39:16	46	157	170.7	5004	13.2	106.4	179.6
20:39:17	47	157	170.8	5004	13.2	106.6	180.0
20:39:18	48	157	170.9	5003	13.2	106.8	180.3

20:39:19	49	158	171.1	5003	13.2	107.5	181.4
20:39:20	50	158	171	5003	13.2	107.8	181.9
20:39:21	51	158	171.1	5003	13.2	107.9	182.1
20:39:22	52	158	171.1	5003	13.2	108.1	182.4
20:39:23	53	158	171.2	5004	13.2	108.2	182.7
20:39:24	54	158	171.1	5004	13.2	108.4	182.9
20:39:25	55	158	171.1	5005	13.2	108.5	183.0
20:39:26	56	158	171.2	5007	13.2	108.5	183.1
20:39:27	57	158	171.1	5007	13.2	108.6	183.3
20:39:28	58	158	171.2	5008	13.2	108.7	183.4
20:39:29	59	158	171.1	5009	13.2	108.8	183.6
20:39:30	60	158	171	5010	13.2	109.0	184.0
20:39:31	61	158	171.1	5010	13.2	109.2	184.2
20:39:32	62	158	171.2	5011	13.2	109.3	184.5
20:39:33	63	158	171.3	5011	13.2	109.4	184.7
20:39:34	64	158	171.4	5011	13.3	109.6	185.0
20:39:35	65	158	171.3	5011	13.3	109.8	185.4
20:39:36	66	158	171.2	5011	13.2	110.1	185.8
20:39:37	67	158	171.3	5011	13.2	110.3	186.2
20:39:38	68	158	171.4	5011	13.2	110.4	186.4
20:39:39	69	158	171.6	5011	13.2	110.5	186.5
20:39:40	70	158	171.5	5011	13.2	110.8	186.9
20:39:41	71	158	171.5	5012	13.2	110.9	187.2
20:39:42	72	158	171.6	5013	13.2	111.1	187.4
20:39:43	73	159	171.7	5014	13.2	111.6	188.4
20:39:44	74	159	171.8	5015	13.2	111.7	188.5

20:39:45	75	158	171.6	5016	13.3	111.5	188.1
20:39:46	76	158	171.5	5016	13.3	111.7	188.5
20:39:47	77	159	171.7	5017	13.3	112.2	189.4
20:39:48	78	159	171.5	5016	13.3	112.5	189.9
20:39:49	79	159	171.5	5016	13.3	112.7	190.2
20:39:50	80	159	171.6	5016	13.3	112.8	190.4
20:39:51	81	159	171.9	5016	13.3	112.8	190.3
20:39:52	82	159	171.9	5015	13.3	113.0	190.6
20:39:53	83	159	171.8	5015	13.3	113.1	190.9
20:39:54	84	159	171.8	5014	13.3	113.2	191.0
20:39:55	85	159	171.9	5013	13.2	113.3	191.3
20:39:56	86	159	172	5012	13.2	113.4	191.4
20:39:57	87	159	172	5012	13.2	113.5	191.5
20:39:58	88	159	172	5011	13.2	113.6	191.7
20:39:59	89	159	172.1	5011	13.2	113.5	191.6
20:40:00	90	159	172	5011	13.2	113.6	191.7
20:40:01	91	160	172.3	5012	13.2	113.9	192.2
20:40:02	92	159	172.5	5012	13.2	113.3	191.3
20:40:03	93	159	172.6	5011	13.2	113.4	191.3
20:40:04	94	160	172.9	5011	13.2	113.7	191.9
20:40:05	95	159	173.1	5009	13.2	113.1	190.9
20:40:06	96	159	173	5009	13.2	113.2	191.0
20:40:07	97	159	173.2	5009	13.2	113.1	190.9
20:40:08	98	159	173.1	5008	13.2	113.1	190.9
20:40:09	99	159	172.9	5008	13.3	113.2	191.1
20:40:10	100	159	172.9	5009	13.3	113.2	191.1

20:40:11	101	159	172.6	5009	13.3	113.4	191.3
20:40:12	102	159	172.6	5010	13.3	113.4	191.3
20:40:13	103	158	172.4	5011	13.3	113.0	190.7
20:40:14	104	158	172.3	5011	13.3	113.0	190.7
20:40:15	105	158	172.4	5013	13.3	113.0	190.8
20:40:16	106	158	172.3	5013	13.3	113.2	191.0
20:40:17	107	158	172.3	5013	13.3	113.2	191.0
20:40:18	108	158	172.2	5012	13.3	113.3	191.3
20:40:19	109	158	172	5011	13.3	113.4	191.4
20:40:20	110	158	172.2	5011	13.3	113.3	191.3
20:40:21	111	158	172	5010	13.3	113.4	191.4
20:40:22	112	158	171.9	5010	13.3	113.5	191.5
20:40:23	113	158	172	5011	13.3	113.4	191.4
20:40:24	114	158	172	5011	13.3	113.3	191.3
20:40:25	115	158	172.1	5010	13.3	113.2	191.1
20:40:26	116	158	172	5009	13.3	113.3	191.1
20:40:27	117	158	172.1	5008	13.3	113.2	191.1
20:40:28	118	158	171.7	5007	13.3	113.4	191.4
20:40:29	119	158	171.8	5006	13.3	113.3	191.2
20:40:30	120	158	171.8	5006	13.3	113.3	191.2
20:40:31	121	158	172	5007	13.3	113.0	190.7
20:40:32	122	158	172.1	5007	13.3	112.9	190.5
20:40:33	123	158	172.3	5006	13.3	112.7	190.2

Table 40. Data Reduction Calculations for the 5000 ft Level Acceleration.

dv/dt	σ	V_T	W_T	$FHP_{inexcess}$	W_T/W_S	$(FHP_{inexcess})_{WC}$	ROC	P_s
Acceleration	Density Ratio	True Airspeed	Test Weight	Thrust Horsepower in Excess	Weight Ratio	Thrust Horsepower in Excess Corrected for Weight	Rate of Climb	Power in Excess Corrected for Weight
ft/sec^2	Unitless	ft/sec	lbs	Hp	unitless	Hp	ft/min	ft/min
2.901	0.8338	108.7	2260	40.24	0.8861	48.2	624	588
2.840	0.8338	112.1	2260	40.62	0.8861	48.7	630	593
2.780	0.8342	116.3	2259	41.25	0.8861	49.5	640	602
2.720	0.8343	120.3	2259	41.75	0.8861	50.1	648	610
2.661	0.8348	123.8	2259	42.03	0.8861	50.4	652	614
2.603	0.8349	127.2	2259	42.24	0.8861	50.6	655	617
2.545	0.8353	129.4	2259	41.99	0.8860	50.3	652	613
2.487	0.8354	131.5	2259	41.73	0.8860	50.0	647	609
2.431	0.8354	133.2	2259	41.30	0.8860	49.5	641	603
2.375	0.8356	135.1	2259	40.92	0.8860	49.1	635	598
2.319	0.8355	136.2	2259	40.29	0.8860	48.3	625	589
2.264	0.8354	138.1	2259	39.89	0.8860	47.8	619	583
2.210	0.8353	139.9	2259	39.44	0.8860	47.3	612	576
2.156	0.8355	141.2	2259	38.85	0.8860	46.6	603	567
2.103	0.8354	143.7	2259	38.54	0.8860	46.2	598	563
2.051	0.8353	145.4	2259	38.02	0.8860	45.6	590	555
1.999	0.8356	147.1	2259	37.51	0.8860	45.0	582	548
1.947	0.8355	148.7	2259	36.94	0.8860	44.3	573	540
1.897	0.8355	150.5	2259	36.42	0.8860	43.7	565	532
1.847	0.8355	152.9	2259	36.01	0.8860	43.2	559	526
1.797	0.8355	155.1	2259	35.55	0.8860	42.6	552	519

1.748	0.8358	157.7	2259	35.16	0.8859	42.2	546	514
1.700	0.8358	160.2	2259	34.74	0.8859	41.7	539	507
1.652	0.8358	162.8	2259	34.32	0.8859	41.2	533	501
1.605	0.8361	165.2	2259	33.83	0.8859	40.6	525	494
1.559	0.8361	166.9	2259	33.19	0.8859	39.8	515	485
1.513	0.8361	169.8	2259	32.76	0.8859	39.3	508	479
1.467	0.8361	172.2	2259	32.24	0.8859	38.7	500	471
1.423	0.8360	174.1	2259	31.59	0.8859	37.9	490	461
1.379	0.8363	176.0	2259	30.95	0.8859	37.1	480	452
1.335	0.8363	178.6	2259	30.41	0.8859	36.5	472	444
1.292	0.8364	180.9	2259	29.82	0.8859	35.8	463	436
1.250	0.8364	183.0	2259	29.17	0.8859	35.0	453	426
1.208	0.8365	184.9	2259	28.49	0.8859	34.2	442	416
1.167	0.8368	186.7	2259	27.79	0.8859	33.3	431	406
1.127	0.8369	188.3	2259	27.06	0.8859	32.5	420	395
1.087	0.8370	189.9	2259	26.32	0.8858	31.6	409	384
1.047	0.8372	191.2	2259	25.54	0.8858	30.6	396	373
1.009	0.8373	192.3	2259	24.74	0.8858	29.7	384	361
0.9705	0.8374	193.2	2259	23.91	0.8858	28.7	371	349
0.9330	0.8374	193.7	2259	23.05	0.8858	27.7	358	337
0.8961	0.8374	194.0	2259	22.17	0.8858	26.6	344	324
0.8598	0.8373	194.4	2259	21.32	0.8858	25.6	331	311
0.8241	0.8373	194.8	2259	20.48	0.8858	24.6	318	299
0.7890	0.8372	195.4	2259	19.66	0.8858	23.6	305	287
0.7545	0.8372	195.9	2259	18.85	0.8858	22.6	293	275
0.7206	0.8372	196.3	2259	18.04	0.8858	21.6	280	264
0.6873	0.8372	196.7	2259	17.24	0.8858	20.7	268	252
0.6546	0.8372	197.1	2259	16.46	0.8858	19.7	255	240

0.6225	0.8372	198.3	2259	15.74	0.8858	18.9	244	230
0.5910	0.8372	198.8	2259	14.99	0.8858	18.0	233	219
0.5601	0.8372	199.1	2259	14.22	0.8857	17.1	221	208
0.5298	0.8372	199.4	2259	13.47	0.8857	16.2	209	197
0.5001	0.8372	199.6	2259	12.73	0.8857	15.3	198	186
0.4710	0.8372	199.9	2259	12.01	0.8857	14.4	186	175
0.4425	0.8372	200.1	2259	11.29	0.8857	13.5	175	165
0.4146	0.8371	200.1	2259	10.58	0.8857	12.7	164	155
0.3873	0.8371	200.4	2259	9.898	0.8857	11.9	154	145
0.3606	0.8371	200.5	2259	9.219	0.8857	11.1	143	135
0.3345	0.8370	200.7	2259	8.562	0.8857	10.3	133	125
0.3090	0.8370	201.1	2258	7.925	0.8857	9.51	123	116
0.2841	0.8370	201.4	2258	7.296	0.8857	8.75	113	107
0.2598	0.8370	201.6	2258	6.680	0.8857	8.01	104	97.6
0.2361	0.8369	201.9	2258	6.078	0.8857	7.29	94	88.8
0.2130	0.8369	202.2	2258	5.493	0.8857	6.59	85	80.3
0.1905	0.8366	202.7	2258	4.924	0.8857	5.91	76	71.9
0.1686	0.8366	203.1	2258	4.367	0.8856	5.24	68	63.8
0.1473	0.8369	203.5	2258	3.823	0.8856	4.59	59	55.9
0.1266	0.8369	203.8	2258	3.289	0.8856	3.95	51	48.1
0.1065	0.8369	203.9	2258	2.769	0.8856	3.32	43	40.5
0.08700	0.8369	204.3	2258	2.267	0.8856	2.72	35	33.1
0.06810	0.8369	204.7	2258	1.777	0.8856	2.13	28	26.0
0.04980	0.8369	204.9	2258	1.301	0.8856	1.56	20	19.0
0.03210	0.8369	206.0	2258	0.8431	0.8856	1.01	13	12.3
0.01500	0.8368	206.1	2258	0.3941	0.8856	0.473	6	5.76
-0.001500	0.8368	205.7	2258	-0.0393	0.8856	-0.0472	-1	-0.575
-0.01740	0.8365	206.1	2258	-0.4573	0.8856	-0.549	-7	-6.68

-0.03270	0.8365	207.1	2258	-0.8636	0.8856	-1.04	-13	-12.6
-0.04740	0.8365	207.6	2258	-1.255	0.8856	-1.51	-19	-18.3
-0.06150	0.8365	208.0	2258	-1.631	0.8856	-1.96	-25	-23.8
-0.07500	0.8365	208.2	2258	-1.991	0.8856	-2.39	-31	-29.1
-0.08790	0.8365	208.1	2258	-2.333	0.8855	-2.80	-36	-34.1
-0.1002	0.8365	208.5	2258	-2.663	0.8855	-3.20	-41	-38.9
-0.1119	0.8365	208.7	2258	-2.978	0.8855	-3.57	-46	-43.5
-0.1230	0.8365	208.9	2258	-3.276	0.8855	-3.93	-51	-47.9
-0.1335	0.8365	209.1	2258	-3.560	0.8855	-4.27	-55	-52.0
-0.1434	0.8369	209.2	2258	-3.825	0.8855	-4.59	-59	-55.9
-0.1527	0.8369	209.4	2258	-4.076	0.8855	-4.89	-63	-59.6
-0.1614	0.8369	209.5	2258	-4.312	0.8855	-5.17	-67	-63.0
-0.1695	0.8369	209.4	2258	-4.526	0.8855	-5.43	-70	-66.1
-0.1770	0.8369	209.5	2258	-4.728	0.8855	-5.67	-73	-69.1
-0.1839	0.8369	210.1	2258	-4.926	0.8855	-5.91	-77	-72.0
-0.1902	0.8369	209.1	2258	-5.070	0.8855	-6.09	-79	-74.1
-0.1959	0.8369	209.2	2258	-5.224	0.8855	-6.27	-81	-76.4
-0.2010	0.8369	209.8	2258	-5.375	0.8855	-6.45	-83	-78.6
-0.2055	0.8369	208.7	2258	-5.468	0.8854	-6.56	-85	-79.9
-0.2094	0.8370	208.8	2258	-5.574	0.8854	-6.69	-87	-81.5
-0.2127	0.8370	208.6	2258	-5.657	0.8854	-6.79	-88	-82.7
-0.2154	0.8370	208.7	2258	-5.732	0.8854	-6.88	-89	-83.8
-0.2175	0.8370	208.9	2258	-5.791	0.8854	-6.95	-90	-84.6
-0.2190	0.8367	208.9	2258	-5.832	0.8854	-7.00	-91	-85.2
-0.2199	0.8367	209.2	2258	-5.864	0.8854	-7.04	-91	-85.7
-0.2202	0.8367	209.2	2258	-5.872	0.8854	-7.05	-91	-85.8
-0.2199	0.8367	208.4	2258	-5.843	0.8854	-7.01	-91	-85.4
-0.2190	0.8366	208.5	2258	-5.822	0.8854	-6.99	-90	-85.1

-0.2175	0.8366	208.6	2258	-5.784	0.8854	-6.94	-90	-84.5
-0.2154	0.8366	208.9	2258	-5.735	0.8854	-6.88	-89	-83.8
-0.2127	0.8366	208.9	2258	-5.663	0.8854	-6.80	-88	-82.8
-0.2094	0.8366	209.1	2258	-5.582	0.8854	-6.70	-87	-81.6
-0.2055	0.8366	209.3	2258	-5.483	0.8854	-6.58	-85	-80.1
-0.2010	0.8366	209.1	2258	-5.358	0.8853	-6.43	-83	-78.3
-0.1959	0.8366	209.3	2258	-5.227	0.8853	-6.27	-81	-76.4
-0.1902	0.8367	209.4	2258	-5.077	0.8853	-6.09	-79	-74.2
-0.1839	0.8367	209.3	2258	-4.906	0.8853	-5.89	-76	-71.7
-0.1770	0.8366	209.1	2258	-4.719	0.8853	-5.66	-73	-69.0
-0.1695	0.8366	208.9	2258	-4.513	0.8853	-5.42	-70	-66.0
-0.1614	0.8367	209.0	2258	-4.299	0.8853	-5.16	-67	-62.8
-0.1527	0.8367	208.9	2258	-4.066	0.8853	-4.88	-63	-59.4
-0.1434	0.8367	209.2	2258	-3.825	0.8853	-4.59	-59	-55.9
-0.1335	0.8368	209.0	2257	-3.556	0.8853	-4.27	-55	-52.0
-0.1230	0.8368	209.0	2257	-3.277	0.8853	-3.93	-51	-47.9
-0.1119	0.8368	208.5	2257	-2.974	0.8853	-3.57	-46	-43.5
-0.1002	0.8368	208.2	2257	-2.659	0.8853	-3.19	-41	-38.9
-0.0879	0.8368	207.9	2257	-2.329	0.8853	-2.80	-36	-34.0

A.7 2000 ft Level Acceleration Data Reduction

Table 41. Raw Data for the 2000 ft Level Acceleration.

	t			H_{ic}	T_a	V_c	V_c
	Time	GPS Ground Track	Magnetic Heading	Calibrated Pressure Altitude	Calibrated Temperature	Calibrated Airspeed	Calibrated Airspeed
	sec	deg	deg	ft	deg C	kts	ft/sec
20:45:31	0	336	334.3	2044	21.8	59.02	99.61
20:45:32	1	336	334.7	2044	21.8	61.22	103.3
20:45:33	2	336	334.6	2040	21.7	63.90	107.8
20:45:34	3	335	334.4	2035	21.7	66.78	112.7
20:45:35	4	335	334.3	2030	21.6	69.36	117.1
20:45:36	5	334	334.3	2028	21.6	71.47	120.6
20:45:37	6	334	334.4	2028	21.5	73.19	123.5
20:45:38	7	333	334	2029	21.5	74.71	126.1
20:45:39	8	333	333.7	2031	21.4	76.06	128.4
20:45:40	9	332	333.4	2034	21.4	77.37	130.6
20:45:41	10	332	333.5	2038	21.4	78.60	132.7
20:45:42	11	332	333.8	2041	21.4	79.82	134.7
20:45:43	12	333	334.4	2044	21.3	81.18	137.0
20:45:44	13	333	335.2	2046	21.3	82.38	139.0
20:45:45	14	333	335.5	2048	21.3	83.60	141.1
20:45:46	15	334	335.8	2050	21.2	84.98	143.4
20:45:47	16	334	336.2	2052	21.2	86.20	145.5
20:45:48	17	334	336.6	2054	21.2	87.50	147.7
20:45:49	18	335	337	2056	21.2	88.77	149.8
20:45:50	19	335	337.6	2056	21.1	89.99	151.9
20:45:51	20	335	338.1	2057	21.1	91.19	153.9

20:45:52	21	336	338.3	2057	21.1	92.48	156.1
20:45:53	22	336	338.5	2058	21.1	93.61	158.0
20:45:54	23	336	338.8	2060	21	94.65	159.7
20:45:55	24	336	339	2061	21	95.58	161.3
20:45:56	25	336	338.8	2062	21	96.64	163.1
20:45:57	26	336	338.9	2065	21	97.49	164.5
20:45:58	27	337	339.1	2067	20.9	98.51	166.3
20:45:59	28	337	339.3	2068	20.9	99.45	167.9
20:46:00	29	337	339.5	2066	20.9	100.5	169.6
20:46:01	30	337	339.7	2064	20.9	101.6	171.5
20:46:02	31	337	339.9	2061	20.9	102.7	173.4
20:46:03	32	337	340	2059	20.9	103.8	175.2
20:46:04	33	337	340.1	2055	20.8	104.9	177.1
20:46:05	34	337	340.2	2051	20.8	106.0	178.9
20:46:06	35	337	340.3	2048	20.8	107.1	180.8
20:46:07	36	337	340.2	2044	20.8	108.1	182.5
20:46:08	37	337	340.2	2040	20.8	109.2	184.2
20:46:09	38	337	340.1	2034	20.7	110.2	185.9
20:46:10	39	337	340.4	2029	20.7	111.1	187.5
20:46:11	40	337	340.3	2024	20.7	112.0	189.0
20:46:12	41	337	340.3	2020	20.7	112.6	190.1
20:46:13	42	337	340.5	2017	20.7	113.1	190.9
20:46:14	43	337	340.5	2016	20.7	113.5	191.5
20:46:15	44	337	340.5	2016	20.7	113.8	192.0
20:46:16	45	337	340.7	2016	20.7	114.0	192.5
20:46:17	46	337	340.8	2015	20.7	114.3	192.9
20:46:18	47	337	340.6	2013	20.7	114.6	193.5
20:46:19	48	337	340.6	2012	20.7	114.9	193.9
20:46:20	49	337	340.6	2011	20.7	115.2	194.4

20:46:21	50	337	340.7	2010	20.7	115.4	194.7
20:46:22	51	337	340.4	2009	20.7	115.7	195.3
20:46:23	52	337	340.3	2009	20.7	115.9	195.6
20:46:24	53	337	340.4	2008	20.7	116.1	195.9
20:46:25	54	337	340.3	2006	20.7	116.4	196.4
20:46:26	55	337	340.2	2004	20.7	116.6	196.8
20:46:27	56	337	340.1	2002	20.7	116.8	197.1
20:46:28	57	337	340.2	2002	20.7	116.8	197.1
20:46:29	58	336	340	2002	20.7	116.6	196.8
20:46:30	59	336	339.9	2005	20.7	116.4	196.5
20:46:31	60	336	339.9	2010	20.8	116.1	195.9
20:46:32	61	336	339.8	2015	20.8	115.8	195.5
20:46:33	62	336	339.8	2020	20.8	115.5	195.0
20:46:34	63	336	339.8	2024	20.8	115.3	194.6
20:46:35	64	336	339.8	2028	20.8	115.1	194.3
20:46:36	65	336	339.8	2031	20.8	115.0	194.1
20:46:37	66	336	339.8	2033	20.8	115.0	194.1
20:46:38	67	336	339.8	2034	20.8	115.1	194.3
20:46:39	68	336	339.9	2035	20.8	115.3	194.6
20:46:40	69	336	339.9	2034	20.8	115.6	195.0
20:46:41	70	336	340.1	2034	20.8	115.8	195.5
20:46:42	71	336	340.1	2032	20.8	116.1	196.0
20:46:43	72	336	340.2	2029	20.8	116.5	196.6
20:46:44	73	336	340	2026	20.8	116.8	197.1
20:46:45	74	336	339.9	2024	20.8	117.2	197.8
20:46:46	75	336	340.1	2023	20.8	117.5	198.2
20:46:47	76	336	340	2021	20.8	117.7	198.6
20:46:48	77	336	340.1	2019	20.8	117.8	198.9
20:46:49	78	336	340	2019	20.8	118.0	199.1

20:46:50	79	336	339.9	2019	20.8	118.0	199.1
20:46:51	80	336	340.5	2021	20.8	117.9	199.0
20:46:52	81	336	340.1	2022	20.8	117.9	199.1
20:46:53	82	336	340.2	2023	20.8	117.9	199.0
20:46:54	83	336	339.8	2024	20.8	118.0	199.1
20:46:55	84	336	340.3	2025	20.8	117.9	199.0
20:46:56	85	336	340.1	2025	20.8	118.0	199.2
20:46:57	86	336	340.1	2025	20.8	118.1	199.4
20:46:58	87	336	340.1	2025	20.8	118.1	199.4
20:46:59	88	336	340.1	2025	20.8	118.3	199.7
20:47:00	89	336	339.9	2025	20.8	118.4	199.9
20:47:01	90	336	340.1	2024	20.8	118.5	200.0
20:47:02	91	336	340	2023	20.8	118.6	200.2
20:47:03	92	336	340.2	2023	20.8	118.7	200.3
20:47:04	93	336	340.4	2023	20.8	118.7	200.4
20:47:05	94	336	340	2022	20.8	118.9	200.7
20:47:06	95	336	340.1	2022	20.8	119.0	200.8
20:47:07	96	336	340.1	2022	20.8	119.1	201.0
20:47:08	97	336	340.3	2022	20.8	119.0	200.9
20:47:09	98	336	340.4	2022	20.8	119.0	200.9
20:47:10	99	336	340.2	2021	20.8	119.0	200.8
20:47:11	100	336	340.4	2021	20.8	118.8	200.6
20:47:12	101	336	340.1	2021	20.9	118.7	200.3
20:47:13	102	336	340.1	2021	20.9	118.4	199.8

Table 42. Data Reduction Calculations for the 2000 ft Level Acceleration.

dv/dt	σ	V_T	W_T	$FHP_{inexcess}$	W_T/W_S	$(FHP_{inexcess})_{WC}$	ROC	P_s
Acceleration	Density Ratio	True Airspeed	Test Weight	Thrust Horsepower in Excess	Weight Ratio	Thrust Horsepower in Excess Corrected for Weight	Rate of Climb	Power in Excess Corrected for Weight
ft/sec^2	Unitless	ft/sec	lbs	Hp	unitless	Hp	ft/min	ft/min
3.595	0.9069	104.6	2252	47.82	0.8833	57.6	746	701
3.507	0.9069	108.5	2252	48.39	0.8833	58.3	754	709
3.421	0.9073	113.2	2252	49.25	0.8832	59.3	768	722
3.335	0.9075	118.3	2252	50.18	0.8832	60.4	782	735
3.250	0.9080	122.9	2252	50.79	0.8832	61.2	792	744
3.167	0.9081	126.6	2252	50.98	0.8832	61.4	795	747
3.085	0.9084	129.6	2252	50.85	0.8832	61.3	793	745
3.004	0.9083	132.3	2252	50.53	0.8832	60.9	788	740
2.923	0.9086	134.7	2252	50.07	0.8832	60.3	781	734
2.844	0.9085	137.0	2252	49.56	0.8832	59.7	773	726
2.766	0.9083	139.2	2252	48.97	0.8832	59.0	764	718
2.690	0.9082	141.4	2252	48.35	0.8832	58.3	754	708
2.614	0.9084	143.8	2252	47.78	0.8832	57.6	745	700
2.539	0.9084	145.9	2252	47.11	0.8832	56.8	734	690
2.466	0.9083	148.0	2252	46.42	0.8832	55.9	724	680

2.393	0.9086	150.5	2252	45.79	0.8832	55.2	714	671
2.322	0.9085	152.6	2252	45.06	0.8832	54.3	703	660
2.251	0.9084	154.9	2252	44.36	0.8831	53.4	692	650
2.182	0.9084	157.2	2252	43.62	0.8831	52.6	680	639
2.114	0.9087	159.3	2252	42.83	0.8831	51.6	668	628
2.047	0.9086	161.5	2252	42.03	0.8831	50.6	655	616
1.981	0.9086	163.8	2252	41.25	0.8831	49.7	643	604
1.916	0.9086	165.8	2252	40.39	0.8831	48.7	630	592
1.852	0.9088	167.6	2252	39.47	0.8831	47.6	616	578
1.790	0.9088	169.2	2252	38.51	0.8831	46.4	601	564
1.728	0.9088	171.1	2252	37.60	0.8831	45.3	586	551
1.668	0.9087	172.6	2252	36.60	0.8831	44.1	571	536
1.608	0.9089	174.4	2252	35.67	0.8831	43.0	556	523
1.550	0.9089	176.1	2252	34.70	0.8831	41.8	541	509
1.493	0.9090	177.9	2252	33.77	0.8831	40.7	527	495
1.437	0.9090	179.9	2252	32.86	0.8831	39.6	513	482
1.382	0.9091	181.9	2252	31.95	0.8831	38.5	498	468
1.328	0.9092	183.7	2252	31.02	0.8830	37.4	484	455
1.275	0.9096	185.7	2252	30.11	0.8830	36.3	470	441
1.223	0.9098	187.6	2252	29.17	0.8830	35.2	455	428
1.173	0.9099	189.6	2252	28.26	0.8830	34.1	441	414
1.123	0.9100	191.3	2252	27.31	0.8830	32.9	426	400
1.075	0.9101	193.1	2252	26.39	0.8830	31.8	412	387
1.027	0.9106	194.8	2252	25.45	0.8830	30.7	397	373
0.981	0.9108	196.5	2252	24.50	0.8830	29.5	382	359

0.936	0.9110	198.0	2252	23.56	0.8830	28.4	367	345
0.892	0.9111	199.2	2252	22.58	0.8830	27.2	352	331
0.849	0.9112	200.0	2252	21.58	0.8830	26.0	337	316
0.807	0.9112	200.6	2252	20.58	0.8830	24.8	321	302
0.766	0.9112	201.1	2252	19.59	0.8830	23.6	306	287
0.726	0.9112	201.6	2252	18.61	0.8830	22.4	290	273
0.688	0.9113	202.1	2252	17.67	0.8829	21.3	276	259
0.650	0.9113	202.6	2252	16.75	0.8829	20.2	261	245
0.614	0.9114	203.1	2251	15.85	0.8829	19.1	247	232
0.578	0.9114	203.7	2251	14.97	0.8829	18.0	234	219
0.544	0.9114	204.0	2251	14.11	0.8829	17.0	220	207
0.511	0.9115	204.5	2251	13.28	0.8829	16.0	207	195
0.479	0.9115	204.9	2251	12.47	0.8829	15.0	195	183
0.448	0.9115	205.2	2251	11.68	0.8829	14.1	182	171
0.418	0.9116	205.7	2251	10.93	0.8829	13.2	170	160
0.389	0.9116	206.1	2251	10.19	0.8829	12.3	159	149
0.361	0.9117	206.5	2251	9.48	0.8829	11.4	148	139
0.335	0.9117	206.4	2251	8.78	0.8829	10.6	137	129
0.309	0.9117	206.1	2251	8.10	0.8829	9.8	126	119
0.285	0.9116	205.8	2251	7.45	0.8829	9.0	116	109
0.261	0.9111	205.3	2251	6.82	0.8829	8.2	106	100
0.239	0.9110	204.8	2251	6.23	0.8828	7.5	97	91.3
0.218	0.9108	204.3	2251	5.66	0.8828	6.8	88	83.0
0.198	0.9107	203.9	2251	5.13	0.8828	6.2	80	75.2
0.179	0.9105	203.6	2251	4.63	0.8828	5.6	72	67.9

0.161	0.9104	203.4	2251	4.165	0.8828	5.0	65	61.1
0.144	0.9104	203.4	2251	3.730	0.8828	4.5	58	54.7
0.129	0.9103	203.6	2251	3.327	0.8828	4.0	52	48.8
0.114	0.9103	203.9	2251	2.953	0.8828	3.6	46	43.3
0.100	0.9103	204.4	2251	2.610	0.8828	3.1	41	38.3
0.088	0.9103	204.9	2251	2.292	0.8828	2.8	36	33.6
0.077	0.9104	205.4	2251	2.001	0.8828	2.4	31	29.3
0.066	0.9105	206.0	2251	1.740	0.8828	2.1	27	25.5
0.057	0.9106	206.6	2251	1.504	0.8828	1.8	23	22.0
0.049	0.9107	207.3	2251	1.297	0.8828	1.6	20	19.0
0.042	0.9107	207.7	2251	1.116	0.8828	1.3	17	16.4
0.036	0.9108	208.1	2251	0.962	0.8827	1.2	15	14.1
0.032	0.9108	208.4	2251	0.837	0.8827	1.0	13	12.3
0.028	0.9108	208.6	2251	0.740	0.8827	0.9	12	10.8
0.025	0.9108	208.6	2251	0.671	0.8827	0.8	10	9.84
0.024	0.9108	208.5	2251	0.631	0.8827	0.8	10	9.25
0.023	0.9107	208.6	2251	0.620	0.8827	0.7	10	9.09
0.024	0.9107	208.6	2251	0.64	0.8827	0.8	10	9.35
0.026	0.9107	208.7	2251	0.69	0.8827	0.8	11	10.0
0.029	0.9106	208.5	2251	0.76	0.8827	0.9	12	11.2
0.033	0.9106	208.8	2251	0.87	0.8827	1.0	14	12.7

A.8 Cost Analysis Calculations

Table 43. For Sale Values of All of the Aircraft from Several Market Places.

Cessna 172N (unmodified)	Cessna 172N (modified)	DA40	M20	AA5B Tiger
$139,900	$175,000	$295,000	$59,900	$164,900
$87,500	$99,950	$399,900	$66,000	159,900
$149,000	$142,950		$68,000	78000
$85,000	$139,500		$76,000	79,950
$125,000	$139,000		$79,900	119,999
$109,900			$92,000	159000
$167,500			$129,000	167000
$99,900				119,950
$139,000				140000
$109,900				180000
$121,260	$139,280	$347,450	$80,257	$136,870

A.9 Statistical Error Data Calculations

Table 44. Raw Data Values and Statistical Calculations for the 65 knot, 80 knot, and 95 knot Steady Climbs.

SSC 65 knots				SSC 80 knots				SSC 95 knots			
Magnetic Heading	$(x_i\text{-mean})^2$	Indicated Airspeed	$(x_i\text{-mean})^2$	Magnetic Heading	$(x_i\text{-mean})^2$	Indicated Airspeed	$(x_i\text{-mean})^2$	Magnetic Heading	$(x_i\text{-mean})^2$	Indicated Airspeed	$(x_i\text{-mean})^2$
138.7	47.61	65.9	0.3211	339.2	32.00	80.9	0.9437	161.3	52.05	93.6	0.9437
143.6	4	65.5	0.9344	337.3	14.12	79.5	0.1837	166.9	2.606	94.9	0.1080
146.7	1.21	69.2	7.471	335.1	2.425	79.7	0.05224	168.3	0.04592	94.4	0.02939
148.2	6.76	67	0.2844	333	0.2947	79.6	0.1080	169.5	0.9716	94.1	0.2222
148.4	7.84	65.6	0.7511	331.5	4.173	79.6	0.1080	171	6.179	94.2	0.1380
148	5.76	65.6	0.7511	329.2	18.86	80.9	0.9437	171.1	6.686	95.7	1.274
				329.5	16.34	79.3	0.3951	171.5	8.914	95.1	0.2794

Table 45. Raw Data Values and Statistical Calculations for the 2000 ft and 5000 ft Level Acceleration.

2000 ft LA				**5000 ft LA**			
Magnetic Heading	$(x_i\text{-mean})^2$	Indicated Altitude	$(x_i\text{-mean})^2$	Magnetic Heading	$(x_i\text{-mean})^2$	Indicated Altitude	$(x_i\text{-mean})^2$
334.3	20.03	2044	49.48	175.1	5.810	5017	48.21
334.7	16.61	2044	49.48	175.3	6.815	5015	24.44
334.6	17.44	2040	9.21	175.5	7.899	5011	0.8903
334.4	19.15	2035	3.86	175.3	6.815	5005	25.57
334.3	20.03	2030	48.52	175.5	7.899	5000	101.1
334.3	20.03	2028	80.38	176.2	12.32	4997	170.5
334.4	19.15	2028	80.38	176.7	16.08	4995	226.7
334.0	22.81	2029	63.45	176.4	13.77	4995	226.7
333.7	25.76	2031	35.59	176.5	14.52	4997	170.5
333.4	28.90	2034	8.794	176.9	17.73	5001	82.02
333.5	27.83	2038	1.070	177.1	19.45	5004	36.68
333.8	24.76	2041	16.28	177.5	23.14	5008	4.229
334.4	19.15	2044	49.48	177.3	21.26	5011	0.8903
335.2	12.785	2046	81.62	177.8	26.12	5014	15.55
335.5	10.729	2048	121.8	178.0	28.20	5016	35.33
335.8	8.854	2050	169.9	178.3	31.48	5017	48.21
336.2	6.634	2052	226.0	178.3	31.48	5018	63.10
336.6	4.733	2054	290.2	178.3	31.48	5018	63.10
337.0	3.153	2056	362.3	178.4	32.61	5019	79.99
337.6	1.3820	2056	362.3	178.1	29.27	5018	63.10
338.1	0.456410	2057	401.4	177.0	18.58	5018	63.10
338.3	0.22618	2057	401.4	176.4	13.77	5019	79.99
338.5	0.07595	2058	442.4	175.9	10.31	5020	98.87
338.8	0.0006	2060	530.6	175.3	6.815	5020	98.87
339.0	0.0504	2061	577.7	174.9	4.886	5020	98.87
338.8	0.0006	2062	626.7	174.4	2.926	5020	98.87
338.9	0.0155	2065	785.9	173.5	0.65688	5020	98.87
339.1	0.1052	2067	902.1	172.9	0.04430	5021	119.8
339.3	0.2750	2068	963.1	172.1	0.3475	5021	119.8
339.5	0.5248	2066	843.0	171.9	0.6233	5021	119.8
339.7	0.8545	2064	730.9	171.1	2.527	5019	79.99

339.9	1.2643	2061	577.7	170.6	4.366	5018	63.10
340.0	1.4992	2059	485.5	170.3	5.710	5016	35.33
340.1	1.7541	2055	325.2	170.2	6.198	5014	15.55
340.2	2.0290	2051	197.0	170.2	6.198	5011	0.8903
340.3	2.3239	2048	121.8	170.4	5.242	5008	4.229
340.2	2.0290	2044	49.48	170.3	5.710	5003	49.79
340.2	2.0290	2040	9.208	170.5	4.794	5000	101.1
340.1	1.7541	2034	8.794	170.6	4.366	4997	170.5
340.4	2.6387	2029	63.45	170.5	4.794	4995	226.7
340.3	2.3239	2024	168.1	170.6	4.366	4996	197.6
340.3	2.3239	2020	287.8	170.7	3.958	4998	145.4
340.5	2.9736	2017	398.6	170.8	3.570	5000	101.1
340.5	2.9736	2016	439.6	170.7	3.958	5002	64.91
340.5	2.9736	2016	439.6	170.6	4.366	5003	49.79
340.7	3.7034	2016	439.6	170.6	4.366	5004	36.68
340.8	4.0983	2015	482.5	170.7	3.958	5004	36.68
340.6	3.3285	2013	574.3	170.8	3.570	5004	36.68
340.6	3.3285	2012	623.3	170.9	3.202	5003	49.79
340.6	3.3285	2011	674.2	171.1	2.527	5003	49.79
340.7	3.7034	2010	727.1	171.0	2.854	5003	49.79
340.4	2.6387	2009	782.1	171.1	2.527	5003	49.79
340.3	2.3239	2009	782.1	171.1	2.527	5003	49.79
340.4	2.6387	2008	839.0	171.2	2.219	5004	36.68
340.3	2.3239	2006	958.9	171.1	2.527	5004	36.68
340.2	2.0290	2004	1087	171.1	2.527	5005	25.57
340.1	1.7541	2002	1223	171.2	2.219	5007	9.342
340.2	2.0290	2002	1223	171.1	2.527	5007	9.342
340.0	1.4992	2003	1154	171.2	2.219	5008	4.229
339.9	1.2643	2004	1087	171.1	2.527	5009	1.116
339.9	1.2643	2005	1022	171.0	2.854	5010	0.003187
339.8	1.0494	2006	958.9	171.1	2.527	5010	0.003187
339.8	1.0494	2007	897.9	171.2	2.219	5011	0.8903
339.8	1.0494	2008	839.0	171.3	1.931	5011	0.8903
339.8	1.0494	2009	782.1	171.4	1.663	5011	0.8903
339.8	1.0494	2010	727.1	171.3	1.931	5011	0.8903
339.8	1.0494	2011	674.2	171.2	2.219	5011	0.8903
339.8	1.0494	2012	623.3	171.3	1.931	5011	0.8903

339.9	1.2643	2013	574.3	171.4	1.663	5011	0.8903
339.9	1.2643	2014	527.4	171.6	1.187	5011	0.8903
340.1	1.7541	2015	482.5	171.5	1.415	5011	0.8903
340.1	1.7541	2016	439.6	171.5	1.415	5012	3.777
340.2	2.0290	2017	398.6	171.6	1.187	5013	8.664
340.0	1.4992	2018	359.7	171.7	0.9791	5014	15.55
339.9	1.2643	2019	322.8	171.8	0.7912	5015	24.44
340.1	1.7541	2020	287.8	171.6	1.187	5016	35.33
340.0	1.4992	2021	254.9	171.5	1.415	5016	35.33
340.1	1.7541	2022	224.0	171.7	0.9791	5017	48.21
340.0	1.4992	2023	195.0	171.5	1.415	5016	35.33
339.9	1.2643	2024	168.1	171.5	1.415	5016	35.33
340.5	2.9736	2025	143.2	171.6	1.187	5016	35.33
340.1	1.7541	2026	120.2	171.9	0.6233	5016	35.33
340.2	2.0290	2027	99.31	171.9	0.6233	5015	24.44
339.8	1.0494	2028	80.38	171.8	0.7912	5015	24.44
340.3	2.3239	2029	63.45	171.8	0.7912	5014	15.55
340.1	1.7541	2030	48.52	171.9	0.6233	5013	8.664
				172.0	0.4754	5012	3.777
				172.0	0.4754	5012	3.777
				172.0	0.4754	5011	0.8903
				172.1	0.3475	5011	0.8903
				172.0	0.4754	5011	0.8903
				172.3	0.15172	5012	3.777
				172.5	0.03592	5012	3.777
				172.6	0.008013	5011	0.8903
				172.9	0.04430	5011	0.8903
				173.1	0.1685	5009	1.116
				173.0	0.09640	5009	1.116
				173.2	0.2606	5009	1.116
				173.1	0.1685	5008	4.229
				172.9	0.04430	5008	4.229
				172.9	0.04430	5009	1.116
				172.6	0.008013	5009	1.116
				172.6	0.008013	5010	0.003187
				172.4	0.08382	5011	0.8903
				172.3	0.1517	5011	0.8903

	172.4	0.08382	5013	8.664
	172.3	0.1517	5013	8.664
	172.3	0.1517	5013	8.664
	172.2	0.2396	5012	3.777
	172.0	0.4754	5011	0.8903
	172.2	0.2396	5011	0.8903
	172.0	0.4754	5010	0.003187
	171.9	0.6233	5010	0.003187
	172.0	0.4754	5011	0.8903
	172.0	0.4754	5011	0.8903
	172.1	0.3475	5010	0.003187
	172.0	0.4754	5009	1.116
	172.1	0.3475	5008	4.229
	171.7	0.9791	5007	9.342
	171.8	0.7912	5006	16.45
	171.8	0.7912	5006	16.45
	172.0	0.4754	5007	9.342
	172.1	0.3475	5007	9.342