GPU Based Monte Carlo Estimation of Eddy Current Losses in Electromagnetic Coil-Core System

by

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Abstract

Title: GPU Based Monte Carlo Estimation of Eddy Current Losses in Electromagnetic Coil-Core System

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A novel parallelizable probabilistic approach to model eddy currents in AC electromagnets is presented in this research. Consequently, power loss associated with the formation of these eddy currents is estimated and validated using experimental data. Furthermore, predicting the effect of ferromagnetic alternating field enhancement on power loss in the source excitation winding has been an active area of research. Unlike a stationary field, an alternating sinusoidal field diffuses partially into the ferromagnetic material leading to a predictably sub-optimal field enhancement. To model these physics, finite element techniques employ nonlinear iterative solvers which are time consuming. A novel method is developed in this research by packing the ferromagnetic domain with variable sized hard spheres. The interaction of these spherical sub-domains with the inducing background field results in a field enhancement that is theoretically identical to enhancement created by the solid ferromagnetic continuum. The evaluated losses in the winding with and without the ferromagnetic material are further employed to evaluate thermal distributions throughout the system. A holistic multiphysics tool has been realized as part of this research which could be employed to estimate field uniformity and engineer active cooling as part of the development and testing of an electromagnetic coil-core system.
Given the current trend in parallel computing, the probabilistic technique developed by the author has been implemented on General Purpose Graphical Processing Units consisting of 2688 processing cores, each of which handles solution of governing equations at an isolated location in the domain independent of one another. By employing the proposed method, a speed gain of 35% over conventional commercial finite element software is guaranteed for simulating commonly employed coil-core electromagnetic systems. In addition to this primary advantage, the resource intensive (∋ 20 GB RAM memory) matrix inversion techniques employed by these commercial finite element softwares are bypassed. Since probabilistic techniques intrinsically do not require extensive data storage, on-chip GPU memory of 8GB has been realized to be more than adequate for practical winding simulations.

Experimental validation of the probabilistic model is carried out by employing the Lock-in Amplifier loss determination procedure. The phase shift in voltage with respect to current could be precisely determined by the procedure which is key to evaluating winding losses. To isolate losses in the excitation winding due to ferromagnetic field enhancement, an existing inductive voltage cancellation technique using a compensation winding system has been adapted by the author. A sensitive J-type thermocouple and an IR imager are used to measure and compare temperature predictions made by an existing probabilistic model with the addition of the power loss source term for this study. It has been established that across a range of operating frequencies and drive current, the Monte Carlo probabilistic model results in an overall error of 0.7% in winding losses without the ferromagnetic core material and 2.6% with the core material.

Implementation of developed model to determine losses, field enhancement, field uniformity and multipole characteristics at the required reference radius in the aperture of a novel electromagnet to be used for carbon beam therapy is currently underway. Previously inconceivable simulation domains such as stranded litz wires and thin iron laminates have now become a possibility employing the developed model.
# Table of Contents

Abstract ........................................................................................................................................ iii

List of Figures .................................................................................................................................. viii

List of Tables ................................................................................................................................... xi

Acknowledgement ........................................................................................................................... xii

Chapter 1: Preface ............................................................................................................................ 1

1.1 Hypothesis: ................................................................................................................................... 1

1.2 Motivation: ................................................................................................................................... 2

1.3 Background and Literature Survey: ............................................................................................ 4

1.4 Dissertation Objective: ................................................................................................................ 10

1.5 Dissertation Structure: ............................................................................................................... 12

Chapter 2: A Parallelizable First Principles Approach to Estimate Eddy Current Loss ............... 14

2.1 Introduction: ............................................................................................................................... 14

2.2 Maxwell’s Equations for Magneto-Electrodynamics: ................................................................. 15

2.2.1 Eddy Currents and Lenz’s Law—A Brief Overview: .............................................................. 17

2.2.2 Amperes Law and Biot Savart Law—A Brief Overview: ....................................................... 19

2.3 Literature Review: ....................................................................................................................... 21

2.4 Existing 2D Parallelizable Analytical Approach to Estimate AC Current Density Distribution: .............................................................................................................................. 23

2.4.1 Altered Primary Current Density Due to Skin effect: ........................................................ 24

2.4.2 Proximity Effect and Resulting Induced Eddy Current: ..................................................... 24

2.4.3 Application of Existing Technique to Wire Bundle: ........................................................... 26
Chapter 3: A Novel Probabilistic Monte Carlo Technique for Estimating Eddy Current Loss

3.1 Introduction: .............................................................................................................41
3.2 Literature Review: Monte Carlo Floating Random Walk Technique ............41
3.3 Novel Probabilistic Mathematical Formulation of 3D Helmholtz Equation for Magnetic Vector Potential: .................................................................43
3.4 Standalone Implementation Procedure for Probabilistic Formulation: ..........51
3.5 Accelerated Implementation of Analytical and Probabilistic Model: ..........53
3.6 Experimental Validation: .........................................................................................55
   3.6.1 Preliminary Experimental Setup: .................................................................55
   3.6.2 Measurement of Winding Power Loss: .........................................................62
3.7 Conclusion: ..............................................................................................................67

Chapter 4: A Novel Parallelizable Model for Evaluating AC Field Diffusion and Magnetization in Ferromagnetic Material.................................................68

4.1 Introduction: .............................................................................................................68
4.2 A Novel Parallelizable Description of the Ferromagnetic Continuum: ..........68
   4.2.1 Discrete Ferromagnetic Spherical Vice-Domain Packing Algorithm ..........69
   4.2.2 Validation of Novel technique with Conventional Finite Element Discretization: ........................................................................................................73
4.3 Existing Technique to evaluate AC Field Diffusion in Ferromagnetic material: 76
4.4 Novel Parallelizable Estimation of AC Magnetization and Field Enhancement in Ferromagnetic material: .................................................................81
4.5 Preliminary Results: .......................................................... 84
4.6 Experimental Validation of Winding Power Loss with Ferromagnetic Core: .... 86
4.7 Conclusion: ........................................................................ 90

Chapter 5: Multiphysics Coupling for Winding Temperature Distribution .......... 91
5.1 Introduction: ...................................................................... 91
5.2 Existing 3D Probabilistic formulation for Steady State Heat Conduction: .... 91
5.3 Coupling of Heat conduction to Eddy Current Problem: ......................... 94
5.3.1 Implementation procedure: ............................................ 94
5.4 Experimental Validation of Winding Temperature: .................................. 96
5.4.1 Survey of Thermal Gradients across Winding by IR Imaging: ............... 96
5.4.2 Measurement of Winding Contact Point Temperature: ....................... 99
5.5 Conclusion: ....................................................................... 102

Chapter 6: CUDA Parallel Programming Techniques to Current Problem .......... 104
6.1 Introduction: ..................................................................... 104
6.2 CUDA Parallel Programming Model Basics: ........................................ 104
6.3 Implementation Hardware Benchmark Study: ......................................... 109
6.4 Recently Developed Dynamic Parallelism for Monte Carlo Problems: ...... 111
6.5 Application of GPU parallelization to current research: ......................... 112
6.6 Program Execution Time- Commercial FEM Software Vs CPU Vs GPU: .... 114
6.7 Advantages of Employing Monte Carlo simulations on GPU: .................. 115
6.8 Conclusion: ...................................................................... 116

Chapter 7: Final Remarks and Future Work ................................................. 117

References ............................................................................ 119
List of Figures

Figure 1-1: CPU Vs GPU Die Architecture showing core imparity ........................................3
Figure 1-2: High impact field of research for Monte Carlo Techniques- Stellarator design
(left) and Microelectronics (right) [25] [26] ........................................................................12
Figure 2-1: Eddy Currents induced in Conductors ..................................................................18
Figure 2-2: 2D Wahlstrom Coil Configuration with field vectors (left); Flux density
contour for 1000 A current (right) ..........................................................................................21
Figure 2-3: 2D Conductor Configuration for Eddy current Evaluation ........................................23
Figure 2-4: Coil Geometry under study-Real coil (left) and Simplified coil (right) ...............27
Figure 2-5: Current Density in single coil (top); dual coil (bottom) systems .......................28
Figure 2-6: Field Contour for Coil system with DC (top); AC (bottom) system ....................29
Figure 2-7: Magnetic Flux density variation along a reference radius R=25 mm ...............30
Figure 2-8 Conductor in Transverse Field (left) and Longitudinal Field (right) .............31
Figure 2-9 Model Geometric Setup .........................................................................................35
Figure 2-10 DC Flux density magnitude plotted on Reference Points ...............................37
Figure 2-11 DC Flux Density contour .....................................................................................38
Figure 2-12 Total current Density vector ...............................................................................38
Figure 2-13 Variation of Flux Density with Axial and Radial Distance .................................39
Figure 3-1: 2D Grid for fixed Random walk Monte Carlo approach ..................................42
Figure 3-2: Schematic showing random walks on a circular 2D domain .........................46
Figure 3-3: Unrolled View of Sample 3D Winding .................................................................54
Figure 3-4: Experimental Setup ..............................................................................................55
Figure 3-5: Primary and Secondary Coil + 3D printed Support Bobbin .............................57
Figure 3-6: Support Bobbin CAD Design ..............................................................................58
Figure 3-7: NI SCC-68 Connector Block + SCC TC02 Thermocouple Module ...............59
Figure 3-8: Magneto-resistive sensor power supply + Lock-in Amplifier .......................60
Figure 3-9: FAST-PST™ CAENels AC Power Supply .........................................................60
Figure 3-10: Power Resistors (left); Magneto-resistive sensor (right) for Current Measurement ................................................................. 61
Figure 3-11: Coil-Core Setup wound on 3D printed Bobbin support .......................................................... 62
Figure 3-12: Block Diagram for Coil Power Loss Measurement ................................................................. 64
Figure 3-13: Comparison for experiment coil AC power loss with COMSOL numerical .. 65
Figure 3-14: Data acquisition signals captured during experimentation for 2000 Hz (top)
and 50 Hz (bottom) drive current ................................................................................................................. 66
Figure 4-1: Custom Dynamic Sphere Packing algorithm result .......................................................... 71
Figure 4-2: 3D Sphere Packing ....................................................................................................................... 72
Figure 4-3: Discretization of a large iron sphere into sub spheres:
Packing factor 0.3 (top) 0.4 (middle) and 0.5 (bottom) .......................................................... 74
Figure 4-4: Radial Flux variation at reference radius=15 mm (top) and 25 mm (bottom) .. 75
Figure 4-5: DC (top); AC (bottom) ferromagnetic finite element domain discretization.... 75
Figure 4-6 DC (bottom left); AC (bottom right) spherical discretization .......................................... 76
Figure 4-7: Non-Linear B-H curve and Magnetic Co-energy (grey area) [18] .................. 80
Figure 4-8: B-H Curve for various ferromagnetic materials .......................................................... 82
Figure 4-9: Field variation around iron sphere ....................................................................................... 83
Figure 4-10: Diffused AC field in Iron at I=100 A and Fr=100 Hz FEM code (top) Monte
Dipole Domain Carlo Code (bottom) ........................................................................................................ 85
Figure 4-11: Block Diagram for Coil Power Loss Measurement ....................................................... 87
Figure 4-12: Comparison for experiment coil AC power loss (with iron core) with...... 88
Figure 5-1: Probability distribution functions for the angular positions \( \theta \) and \( \varphi \) .............. 93
Figure 5-2: Sample Thermal IR image ........................................................................................... 97
Figure 5-3: Thermal distribution .......................................................................................................... 97
Figure 5-4: Measurement Thermal imaging results for temperature distribution in coil
carrying current- 5 A (left) 10A(middle left) 15A (middle right) and 20 A (right) ............ 98
Figure 5-5: Thermal imaging results for temperature distribution in coil carrying current
(with iron core)- 5 A (left) 10A(middle left) 15A (middle right) and 20 A (right) ............ 99
Figure 5-6: Experiment setup to measure Coil temperature .......................................................... 100
Figure 5-7: Coil Temperature Variation with Frequency (without iron) .................... 101
Figure 5-8: Coil Temperature Variation with Frequency (with iron) .......................... 102
List of Tables

Table 1: Coil Geometric Parameters and Properties ................................................................. 26
Table 2: Utilized CPU Specifications .................................................. 110
Table 3: Utilized GPU device Specifications ........................................... 110
Table 4: Temporal Analysis ......................................................................................... 114
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Chapter 1: Preface

1.1 Hypothesis:

_Eddy currents are circular movement of fluid/electric current running contrary to the primary continuum flow/electric current._ This definition paints a simple yet illuminating picture for describing eddies. The consequential effects of these eddy currents could either be detrimental resulting in energy losses in case of electromagnetic coils or be harnessed for useful purposes such as fluid flow mixing, electromagnetic braking, and nondestructive testing. This dissertation aims to develop a novel parallelizable probabilistic model to determine power losses and thermal distribution in electrical machines consisting of coils and core materials subjected to the detrimental effects of induced eddy currents. Although parallelizable probabilistic models are not new, this research aims to overcome the widespread skepticism for commercial adoption of the technique owing to its practical applicability. Techniques are presented with the sole purpose of end consumer usability. The resource crunch associated with the traditional finite element models has been a huge impediment for innovation. GPUs with thousands of computational cores could be efficiently harnessed to overcome these setbacks. _The key hypothesis predicted by the author is a computational speed improvement of at least one order of magnitude achieved in implementing the proposed probabilistic model over conventional commercially available models in COMSOL multiphysics solver._ The author also hypothesizes that the error incurred in predicting coil losses with a ferromagnetic core material in vicinity should be much smaller for the proposed model in comparison to the traditional FEM approach. This is due to the inherently precise nature of the probabilistic Monte Carlo method to describe the ferromagnetic sub-domains which is physically more accurate than the continuum approach pursued in most FEM techniques. Optimizing ideal design configurations from a plethora of parametric variables and predicting losses in complex multiscale windings and core laminates is hence not beyond the realm of possibility. The author acknowledges the broad scope for this research owing to current trend in widespread adoption of electrification.
1.2 Motivation:

Maxwell’s equations govern the underlying physical phenomenon associated with all electromagnetic systems. One constituent correlation of significant consequence is Faraday’s Law of Induction. Any transient magnetic field experienced by a material induces an electromotive force in that material. This emf results in voltage-induced currents referred to as Eddy Currents aptly termed to describe their circular closed loop path in the material. Counteracting the inducing field these eddies increase the apparent Reluctance (magnetic resistance) of the circuit. Roberto et al [1] have shown analytically that the presence of eddies in the magnetic circuit of an electrical machine rotor increases the excitation winding AC-resistance and decreases both its Magnetization and AC-inductance (stored magnetic energy). This causes an apparent time lag/phase difference between the inducing magnetic field and the induced eddy currents which counteract this field. As a result of all the previously mentioned modifications, the classical formulation for power losses in AC circuits i.e., \( \text{Power Loss} \propto \text{Characteristic Dimension}^2 \) is no longer valid. Finite volume based numerical approaches that solve Maxwell’s equations on discretized domains are explicitly required to estimate heating \( (I^2R) \) and eddy losses. These techniques are inefficient in modelling multiscale domains with sharp edges such as laminates. An alternative more robust approach is needed.

The effects of field enhancement due to ferromagnetic materials for DC circuits have been well established. For AC circuits however, eddies induced in these materials have a detrimental effect on field enhancement. Some electromagnets such as those used in proton beam therapy operate using transient trapezoidal drive signals that contain a combination of both DC and AC components. Modifications such as laminations are explicitly required to counteract the effect of eddies in such systems. Simulation of these customized electromagnets is therefore vital to understand the tradeoff between field generated and power losses incurred for tailored drive signals. Conventional commercial software presents a major impediment to understanding and accomplishing unconventional innovations in the design process. Alternative techniques proposed in this research enable engineers to decouple electrical machine subsystems and apply probabilistic techniques to solve the underlying governing equations at an accelerated pace using GPUs. These GPUs offer
several magnitudes higher number of computational units when compared to traditional CPU-RAM architecture. Harnessing these resources aids in exploring millions of electrical machine architecture and optimally arriving at desired operations. The advent of high-speed General Purpose Graphic Processing Units (GPGPUs) capable of performing complex computations in addition to simple I/O bound tasks has broadened their scope to perform repeated numerical tasks several millions of times. These devices provide an ideal proving ground for Monte Carlo numerical techniques discussed in this study. The Tesla K20Xm GPGPU employed for this research has 2688 processing cores all of which could be parallelly used to run code asynchronously as opposed to code run on traditional quad core/octa core CPUs. *Innovative electromagnet design requires understanding tradeoffs associated with power losses to obtain required field uniformity and enhancement demanding need for rapid prototyping and simulation.*

Figure 1-1: CPU Vs GPU Die Architecture showing core imparity

Figure 1-1 above shows chip die architecture for a typical CPU (top) with four cores and the NVIDIA GK110 series GPU with 2880 cores organized inside 15 streaming multiprocessors (SMXs)
1.3 Background and Literature Survey:

Existing literature on estimating the redistribution of current density and consequent increase in power losses due to eddy currents based on first principles analytical approach is limited to two dimensional entities such as round planar windings as explored by Acero et al [2]. Moreover, the majority of such literature to evaluate 2D eddy current distribution assume that the two-dimensional geometry (cylinder or rectangular conductor) is axisymmetric and is under the influence of a uniform spatially invariant background field. In practice, however even the simplest coil configuration does not hold true to these assumptions making their application inaccurate. This publication however develops a novel technique to consider the interaction between successive turns of a planar induction coil by assuming filamentary windings thereby neglecting any possibility to tackle anisotropic current distribution in coil cross section mainly caused due to non-uniform field experienced at the same cross-sectional position. Jablonski et al [3] in their publication on *Analytical-numerical approach to the skin and proximity effect* discuss modelling the interaction between two parallel infinitely long conductors of circular cross section. In comparison to the previous technique the method discussed in this research considers the altered field around a conductor and its influence on inducing eddy currents in a neighboring conductor. Moreover, the authors also discuss a technique to account for the higher order eddy currents by performing a successive iteration. The drawback of this technique however is that it is limited to a 2D conductor and evaluates only the axial component of the induced eddy currents assuming that the inducing background field is never along the length of the conductor. The earliest accounts of evaluating eddy currents in conductor placed in both longitudinal and transverse fields has been found to be by Lammeraner et al in their book [4] solely dedicated to the topic of Eddy Currents. Furthermore, in this book the authors briefly provide hints to techniques that could be used for evaluating eddy current density in three-dimensional winding configurations by cleverly taking symmetry into account.
James R Nagel in [5] discusses variation in the direction of eddy currents due to changes in the orientation of the inducing field in simple cylindrical, rectangular, and spherical geometries. The underlying equations in all these resources however remain the same namely the Helmholtz magnetic vector potential equation which when solved assuming continuity along the conductor surface results in equations with Bessel functions. Based on the previous discussion it has been the consensus that a simple three-dimensional analytical model to evaluate eddy current density and consequently the AC power loss in winding configuration would be highly coveted. This problem is therefore tackled during the initial phase of this research. The type of field generated by any coil and the nature of the resulting eddy currents induced in it is a strong function of the coil geometry. It is therefore impossible to synthesize ahead of time the exact behavior (field uniformity, distribution, and power loss) by merely making use of proportionality equations and the knowledge of the underlying physics. Such an analytical model further helps to validate the probabilistic model developed in this research and would serve as a steppingstone to understand the underlying physics of the considered electromagnetic system.

Field enhancing ferromagnetic materials are predominantly used in DC devices to improve both the magnitude and uniformity of the field generated by the electromagnet in the device. Francois Henrotte et al in their publication [6] present an in-depth account of the well-known and commonly employed Preisach’s model. As per this model, the magnetic hysteresis (delay imposed between the induced field and the inducing field inside a non-linear ferromagnetic material) is described as a network of small independently acting domains each magnetized to a binary state value i.e. either $h$ or $-h$. The net magnetic moment would therefore sum up to zero if all such domains were added together. The main complexity involved in modelling such ferromagnetic materials is the non-linear relation between the inducing field and the induced magnetization. Therefore, in the governing partial differential
equation for field strength in the ferromagnet this non-linearity introduces significant instabilities when solved numerically. In modelling ferromagnets placed in an AC background field limited resources exist however due to minimal use of such materials in AC circuits. With the advent of superconducting magnets that use pulsed triangular or trapezoidal waveforms, the input signal comprises of the core DC component and transient AC components. If field enhancing ferromagnet is used in such a system, its influence on the resulting transient field and delays introduced due to formation of eddy currents need to be thoroughly studied. Techniques to model such systems currently do not exist. The physics of magnetization of iron in alternating field is complicated and involves two major parts. Firstly, the inducing background field does not penetrate completely into the iron as in the DC case due to induction of eddy currents in the iron. Secondly, reorientation, interaction, and magnetization of magnetic domains inside iron due to the alternating field. Several publications exist that model either one of these effects individually. In [2] Acero et al discuss the influence of a field enhancing ferromagnetic iron pan in vicinity of the round planar induction winding and consequent increase in coil reactance. A complex analysis of the nonlinear diffusion problem in ferromagnetic materials under steady-state excitation is dealt in detail by Dimitris Labridis et al.[7] and Ali Nayfeh et al [8]. In the later publication, the authors present a method consisting of finding a linear solution for the diffusion equation governing the H field and then applying the method of variation of parameters to take the nonlinearity into account. As a result, both the radial field variation and the losses in a cylindrical domain are obtained. Dimitris et al however present a much simpler technique that has been used in the current research where the problem is solved by considering an equivalent fictitious material where the relative permeability is assumed to be constant in time by different from point to point and is related to the non-linear B-H curve with the help of a stored magnetic co-energy term. More details regarding this model is discussed in the further sections of this research. However, all these techniques
involve computational domains that are either two dimensional or axisymmetric. Therefore, the current research aims to fill the gaps by introducing a novel technique where the domain is subdivided into small dipole spheres of varying sizes and evaluating their resulting interactions. This model is not restricted to any geometric constraints and is easily amenable to be used for multiscale components of electronic devices that enhance fields such as iron laminates.

The principal objective of this research is to develop a probabilistic approach which is more amenable to parallelization to tackle the eddy current problem both in a coil and core domain of an electromagnetic system. Such methods are readily available in literature for wave propagation, thermal diffusion chemical kinetics and optics. Two-dimensional eddy current models [1], [2] and three dimensional electric potential models [9] provide an in-depth description of the techniques involved and serve as a starting point for further development. In most of these models, multidimensional integral formulations of the underlying governing partial differential equations are solved. By repeated random sampling in the solution space a technique commonly known as *Random Walk Process*, discretization and finite difference dissociations are completely avoided. For simple geometries and small solution space this technique is effective. Multiscale practical problems with anisotropic materials become intractable as the solution time is in the order of several hours. Therefore, the widespread skepticism surrounding these probabilistic *Monte Carlo* Random Walk techniques is genuinely warranted. K Chatterjee and J Poggie in their publication on developing a parallelized 3D floating random walk algorithm [9] have successfully made use of the two levels of parallelism which are inherent in an FRW algorithm First the solutions for different points in the domains are independent of each other. Second, for a given point of origin each random walk is independent and inter-processor communication is required only to sum up contributions of the walks. Although a GPU has not been used in this study the
authors have made use of a highly efficient COMPAQ SC-40 machine based on an 833 MHz EV 6.8 chip. This computer has a shared/distributed memory system with 4 processors per shared memory node which are easily amenable to parallelization. By increasing the number of used processors from 1 to 32 (hyperthreading) the total time required to calculate solution at 11 points decreased from 334 s to 13 s; a drop of 96%. Bahadori et al.[10]–[13] have developed a novel Effective Floating-Point Algorithm to solve transient non-homogenous three-dimensional heat transfer problems that overcome this setback. This technique eliminates the need for random walking or scattered interpolation in the solution space both of which constitute temporal bottlenecks. The authors describe a fixed Sphere of Influence surrounding the sink or point of interest to estimate all possible source points within the sphere contributing to energy transfer. This method is highly desirable to solve transient multiphysics problems. In dealing with steady state problems (infinite time) this technique cannot be employed since this sphere of influence needs to be extended to the domain boundaries and therefore no significant gain is obtained by pursuing this technique. A novel Monte Carlo technique is therefore developed as part of this research to solve the three-dimensional steady state Helmholtz equation for magnetic vector potential which governs all eddy current problems. The technique in theory could be used for both the coil and core domain where the formation of eddy currents due to alternating field primarily dictates the observed behavior of field diffusion and power loss. In the current research however, for the ferromagnetic core domain a novel and more efficient dipole domain Monte Carlo model is developed which is used in combination with the probabilistic Monte Carlo method for the coil domain.

Heat dissipation is an unavoidable consequence of power loss in AC electromagnetic systems. As noted in the experimental section of this research an appreciable thermal gradient exists along the winding configuration tested. The presence of a ferromagnetic core material clearly creates zones of elevated temperature close to the coil ends where the field enhancing effect is maximum. In
addition to the I2R DC joules heating losses conventionally dealt with in DC systems, eddy currents pose a serious threat to the coils thermal performance. Thermal cooling systems need to be designed a priori based on predicted peak temperature profiles in the system. Certain types of passive cooling techniques or convective air cooling become non-viable options when the operating conditions reach a certain critical value. Chenggong Zhao et al [14] in their extensive review article delve into the preparatory strategies of thermally conductive and electrically insulating polymetric materials and their application for thermal management to control heat dissipation in electronic devices. The effect of temperature on the performance of induction coil launcher is discussed in detail by Yadong Zhang et al [15]. Their study deals with the performance change in an pulsed induction coil launcher and how temperature increase affects the mechanical properties of the materials used. A multiphysics numerical analysis of coupled electromagnetic-thermal phenomena in power cable lines is performed by Cywinski et al in [16]. The authors have concluded that the conditions for heat exchange are significantly affected by the uneven current distributions in cable strands caused by proximity and skin effects. Modelling such thermal dependencies therefore becomes a vital task. Haaji Sheik et al [17], [18] present the earlier known probabilistic model for solving heat conduction problem with sources of heat distribution included in the problem formulation. They start with the integral formulation for temperature at the center of a sphere in terms of temperature along the sphere surface obtained for the Poisson governing differential equation for heat diffusion. The floating random walk technique is then applied to this formulation based on predetermined probability distribution functions that randomly pick the path of particle movement within the solution space. This simple yet effective technique is used in the current research and is coupled with the power loss model to predict the steady state temperature only at the desired coil locations. However a few minor modifications have been made to the algorithm by restricting particle walks only within the domain of interest namely the coil/core domain and
efficiently bypassing traversing the surrounding air domain which would prove to be computationally expensive. This research aims to advance existing probabilistic techniques and develop a pragmatic engineering eddy current model to design electromagnets.

1.4 Dissertation Objective:

A novel analytical technique based on first principles to estimate eddy current density and power loss in three-dimensional winding/coil configurations is to be developed as a foundation to understand the underlying physics of the electromagnetic system. Based on existing literature, a novel three-dimensional probabilistic integral formulation is derived mathematically to solve the governing Helmholtz Equation for Magnetic Vector Potential and evaluate the resulting redistribution in coil current density due to eddies. Propositions from Iterative Perturbation Theory [19] and Generalized Greens function in 3D Euclidian space [20] will be invoked to obtain this formulation. The well-established Random Walk Monte Carlo method will then be applied to this formulation. Following a brief introduction to GPU computing techniques, the performance of the probabilistic model is to be accelerated by implementing parallelization. A subsequent decrease in script runtime is to be demonstrated by utilizing the previously developed analytical model results as initial conditions.

Field enhancement due to ferromagnetic materials is significantly altered due to the formation of eddy currents in them. A novel Dipole Sphere Monte Carlo formulation of such a material is to be implemented to model this modification. Non-linear magnetization of these ferromagnetic materials adds to the complexity of this task although the underlying governing equations remain the same. Employing a custom dynamic size sphere packing algorithm, the ferromagnetic domain is packed up to 90% with constituent spheres. In contrast conventional optimization based hard
sphere packing algorithms offer only 64% packing density. The diffusion of the source field into the ferromagnetic material is to be modelled by an iterative approach following which the interaction and resulting field enhancements between neighboring sphere dipoles will be estimated. By modelling the eddy currents in both the winding/coil and ferromagnetic core domain, the power loss is easily found as ancillary quantity via suitable volume integration.

Thermal ramifications of the resulting power losses and the associated multiphysics coupling are to be modeled both in the coil and ferromagnetic core domain. Previously obtained loss quantities are to be used as source terms in resolving Fourier Heat Conduction equation in the solution space. Key parameters such as temperature distribution and required ambient heat transfer could be evaluated.

Adoption of any new techniques requires rigorous validation. Therefore, an appropriate experimental verification campaign is required to be carried out to this extent. Well established techniques are available in literature to isolate the AC resistance of inductor windings and evaluate power losses [21]–[23]. Using state of the art high voltage AC power supply, Lock-In Amplifier, and power resistors the winding AC losses over a wide range of operating current and frequency are measured. Experimentally isolating core ferromagnetic losses comprising of both eddy current and magnetization/hysteretic losses, however, is still an active area of research. By improving on a previously established Coil Compensation Technique developed by Jan Souc et al [24] core losses have been planned to be accurately measured in this research.

In combination both the avant-garde coil and core domain models present a compelling technique with two major advancements—de-coupled subsystem modelling without the need for simulation across the entire solution space and
utilization of parallel computing resources. Estimating required input supply power to overcome losses and necessary thermal cooling for a particular winding with ferromagnet design becomes a straightforward task. Complex multiscale systems such as those shown in Figure 1-2 could be modelled.

![Figure 1-2: High impact field of research for Monte Carlo Techniques-Stellarator design (left) and Microelectronics (right) [25] [26]](image)

### 1.5 Dissertation Structure:

The structure of this dissertation is highlighted as per the subsequent sequence of sections. In order to comprehend and deconstruct all underlying physical processes associated with eddy currents the three-dimensional model based on first principles is introduced. The aim of this section is to underline the significantly higher winding resistance associated with AC circuits due to eddy currents and show the failure of simple proportionality-based power loss estimates. This model therefore offers a quick alternative to numerical techniques with limitations in accuracy. Model predictions and the underlying rationale are presented in the same section to provide context for the upcoming novel probabilistic techniques. The seminal contribution of this research namely—3D Monte Carlo techniques for eddy current modelling in both coil and core domains are subsequently introduced. The mathematical derivation for the three-dimensional probabilistic formulation of the
governing partial differential equation is introduced in the next section. A detailed pseudo code for implementing this formulation using the random walk Monte Carlo scheme is further presented in the same section. The subsequent section is dedicated to discussing modelling of ferromagnetic field enhancement by both a traditional finite element method and the novel Monte Carlo dipole sphere method. The experimental test article along with the procedure followed to measure AC power losses in the coil and core is discussed in the following section. A brief section is dedicated to discussing GPU hardware specifications being used and benchmarked results to highlight the advantage of parallel computing in terms of simulation time. The penultimate section presents power loss predictions of the preliminary analytical model and the novel Monte Carlo model which are simultaneously validated against experimentally obtained data. Future applications of the developed tool such as Non-Destructive Testing and Subsurface Metal Detection are briefly presented towards the end for further progress.
Chapter 2: A Parallelizable First Principles Approach to Estimate Eddy Current Loss

2.1 Introduction:

The primary purpose of this chapter is to introduce all fundamental principles involved in modelling eddy currents and develop a simple analytical model that can be concurrently executed on the GPU. A contextual overview is provided as part of this process which could be leveraged for hindsight in the following chapters. An existing 2D analytical technique to evaluate the redistributed AC current density in two infinitely long parallel conductors is leveraged and applied for any 2D wire bundle configuration which could intern be directly translated into their equivalent axisymmetric configuration. This model is later used as an initial starting point to accelerate the 3D probabilistic model discussed in a future chapter. In addition to this preliminary analytical model, a separate standalone secondary 3D analytical model is developed purely based on first principles employing the concept of reference points immediately surrounding the domain of interest (coil/core). This secondary model serves two key roles:

- Provides a redundant method to compare with the probabilistic technique.
- The concept of using reference points as part of this procedure serves an important role to couple disjointed domains i.e., coupling between the coil and core domains is enforced via these reference points.
2.2 Maxwell’s Equations for Magneto-Electrodynamics:

All classical electromagnetic systems including the one discussed in this research strictly satisfy Maxwell’s equations. The preliminary partial differential equations that together constitute Maxwell’s equations for electric and magnetic field are given by:

\[ \text{Gauss’s Law} \quad \nabla \cdot E = 4\pi \rho \]  
\[ \text{(1)} \]
\[ \text{Gauss’s Law for magnetism} \quad \nabla \cdot B = 0 \]  
\[ \text{(2)} \]
\[ \text{Faraday’s Law of induction} \quad \nabla \times E = -\frac{\partial B}{\partial t} \]  
\[ \text{(3)} \]
\[ \text{Ampere’s Circuital law} \quad \nabla \times B = \mu_0 \left( J + \varepsilon_0 \frac{\partial B}{\partial t} \right) \]  
\[ \text{(4)} \]

For linear and homogenous materials (copper conductor in this study) subjected to sinusoidal AC excitation of fixed frequency, \( \partial / \partial t \) in the above equations could be replaced by \( j \omega \). Maxwells equations 3 and 4 then becomes,

\[ \nabla \times E = -j \omega B \]  
\[ \text{(5)} \]
\[ \nabla \times B = \mu_0 J + j \mu_0 \omega \varepsilon_0 E \]  
\[ \text{(6)} \]

Separating the electric field and magnetic field terms, substituting equation 6 in 5 for electric field \( E \), an expression for magnetic field \( B \) (second order differential equation) is obtained:

\[ \nabla \times \nabla \times B = \nabla (\nabla \cdot B) - \nabla^2 B = -(\sigma + j \omega \varepsilon_0) j \mu_0 \omega B \]  
\[ \text{(7)} \]

Since \( \nabla \cdot B = 0 \)

\[ \nabla^2 B = (\sigma + j \omega \varepsilon_0) j \mu_0 \omega B \]  
\[ \text{(8)} \]

The second term on the RHS of equation 8 describes the effect of displacement current which is negligible for all practical engineering purposes in conductors but is essential in modelling capacitive currents between connection leads and between adjacent turns in coils. The first term in the RHS is associated with moving charge and includes any eddy current effect.
Therefore Equation 8, which is called the Helmholtz parabolic second order partial differential equation constitutes the principal equation required to be solved numerically to obtain eddy current distribution.

From a numerical standpoint it is easier to work with magnetic vector potential $A$ defined by $B = \nabla \times A$ and electric scalar potential $\varphi$ defined by $E = -\nabla \varphi - j\omega A$. This is because the only requirement that a potential description must comply with is that the function must be single valued. For instance, a constraint is placed that the divergence of a magnetic field vector has to be zero based on equation 6 while it follows automatically for the vector potential since $\nabla \cdot \nabla \times A = 0$. Therefore, substituting the definition of electric scalar potential and magnetic vector potential in equation 7:

$$
\nabla^2 A - \mu_0\omega^2 \varepsilon_0 A - \nabla(\nabla \cdot A + j\omega \varepsilon_0 \mu_0 \varphi) = -\mu_0 J
$$

The potential could be decoupled in equation 9 by applying Lorentz condition given by:

$$
\nabla \cdot A + j\omega \varepsilon_0 \mu_0 \varphi = 0
$$

As stated earlier, the $\nabla^2 A/\mu_0$ term in equation 11 signifies the applied current combined with eddy currents while the $\omega^2 \varepsilon_0 A$ term describes the magnetic induced displacement current. Displacement currents are generally ignored in the calculation of eddy currents making it a quasi-stationary effect which is realistic supposition provided the conductivity is high enough compared to the operating frequency. Therefore equation 11 could be further simplified as:

$$
\nabla^2 A = -\mu_0 J
$$

Equation 12 is primarily used to obtain eddy current distribution in linear conducting media such as copper windings in coil configurations.
2.2.1 Eddy Currents and Lenz’s Law– A Brief Overview:

An alternating current through a conductor creates an alternating time varying magnetic field surrounding the conductor based on the previously discussed Maxwell’s equations. This alternating field creates or induces secondary currents/emf in conductors at proximity. The magnitude of this emf is given by Faraday’s law of electromagnetic induction and their direction is given by Lenz’s law. Lenz’s Law states that the direction of electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field. In simpler terms the primary purpose of the back emf induced due to alternating fields is to oppose any rapid change in the field to begin with. Together Faraday’s and Lenz’s law could be summed up by the statement that induced emf directly opposes the rate of change in magnetic flux ($\varepsilon = -\frac{d\varphi_B}{dt}$). A brief overview of the physics governing eddy currents along with theory on their causes and consequences is discussed. Eddy currents play a key role in the diffusion of AC field in linear conducting materials (copper) and non-linear Ferromagnetic materials (iron) in which the effect is particularly pronounced. Figure 2-1 details the formation of eddy loops across the cross section of conductor. It is important to note that these eddies are induced due to primary current along the cross section of the same conductor. The direction of these loops relative to that of the primary drive current and the inducing field is therefore also indicated. These eddy currents could therefore be considered as Self Induced as opposed to Mutually Induced eddies due to field from neighboring conductors. The top figure therefore summarizes what is known in AC circuits as Skin effect which is the phenomenon of AC current redistribution closer to the conductor surface. The concept of field attenuation with conductor depth due to magnetic field shielding is also shown in the bottom figure. This effect plays a key role in ferromagnetic materials. The reason for such an effect is explained in subsequent paragraphs.[27]
Figure 2-1: Eddy Currents induced in Conductors

As can be seen from the figure above, eddy current component $i_1$ flows along primary current direction while eddy current component $i_2$ flows opposite to primary current direction. Although the eddy currents depicted in the figure are at only two radial positions, this phenomenon occurs at every radial location in the conductor. Moreover, the eddy currents circulate end to end in the conductor. Therefore, these eddy currents increase effective current along the conductor periphery while decrease it drastically along the axis. Based on the previous discussion, the thickness of a material required to reduce the internal magnetic field generated by the alternating current within the conductor by a factor of $1/e$ is called the Skin depth of the material. The skin depth [28] is given by the well-known formula:
\[ \delta = \sqrt{\frac{2}{\omega \mu \sigma}} \]  

(13)

Where \( \omega \) - operating angular frequency; \( \mu \) - conductor permeability; \( \sigma \) - conductor conductivity. In summary, skin depth determines both the extent to which an externally applied magnetic field penetrates the conductor and the internal primary current redistributes close to the surface. Larger eddy currents are generated in materials with low resistance and high magnetic permeability leading to magnified skin effect. As the operating frequency is increased beyond a certain threshold frequency \( \omega_s \), skin effect predominantly influences conductor behavior including its effect on overall circuit inductance. The concept of Skin-effect Inductance is therefore discussed next. At frequencies below \( \omega_s \) since skin effect does not dominate, current in a conductor redistributes to minimize resistance of circuit i.e., current traverses path of least resistance as conventionally defined. At frequencies above \( \omega_s \) however, redistribution occurs to minimize overall circuit inductance (\( Inductance \propto \frac{Voltage}{Current \ change} \)). The latter effect leads to redistribution of current close to the surface. The inductance of conductor in this case is called external inductance \( L_e \) because at such high frequencies, skin effect is so predominant that it expels all contributions of internal magnetic field generated by eddy currents within the conductor and considers only the influence of external field. Most 2D numerical solvers make use of this concept to assume surface current density at high operating frequencies. In contrast, at low frequencies since the influence of the internal magnetic field generated by the eddy currents inside the conductor is also included, the corresponding inductance is considerably higher than its counterpart whose difference with \( L_e \) is termed Internal Inductance \( L_i \). Eddy currents via circuit inductance affect the overall performance of AC electromagnetic windings.

2.2.2 Amperes Law and Biot Savart Law– A Brief Overview:

Ampere’s Circuital law relates the integrated magnetic field around a circular loop to the electric current passing through the loop which serves as the source of this magnetic field. Mathematically this law can be stated as the line integral of flux density over a current loop being proportional to the magnitude of current flowing in
the loop i.e., $\int Bdl = \mu_0 I$. Some of the applications of this law include determining the magnetic induction due to a long current carrying long wire, the magnetic field inside a toroid and the magnetic field created by a long current carrying conducting cylinder. Biot Savart Law is a generalized approximation to the Amperes Law for magnetostatic systems that is not constrained by the symmetry of the problem unlike Amperes law. In other words, Amperes law is restricted to cases where the magnetic field around an Amperian loop is constant. The Biot Savart law can be formally defined by the following integral equation:

$$\vec{B}(p) = \frac{\mu_0}{4\pi} \int \frac{\left(\vec{J}(q)dl\right) \times \vec{r}_{pq}}{|\vec{r}_{pq}|} \, ds$$

(14)

In equation 14, $p$ refers to the point at which flux is to be evaluated and $q$ is source point within the current carrying conductor associated with a portion of the source current density. The flux density $\vec{B}$ at point $p$ is obtained based on a filamentary current element carrying a current density $\vec{J}$ location at point $q$. The vector $\vec{r}_{pq}$ indicates the distance vector from point $p$ to $q$. Equation 14 could therefore be effectively used to divide a circular current loop for instance into infinitesimal length elements $dl$ each of which is associated with a fraction of the current density $\vec{J}$. As could be observed from equation 14 the magnitude and direction of flux density at a point depends on both the location of point $p$ and the orientation of the current element $dl$ with respect to point $p$. The resulting flux density field is therefore a strong function of the overall geometric orientation of the current carrying coil. Figure 2-2 shows the flux density field in the region surrounding a specialized Wahlstrom winding configuration because of applying the previous formulation. The blue dots indicate individual infinitely long conductors carrying current into the plane of the page and the red dots correspond to conductors carrying current out of the page. Consequently, this is a 1D representation of the real winding with each turn of the winding individually represented by a filamentary point. Alternatively, a 2D representation of each coil cross section could be made by randomly distributing
several such filaments across the cross-sectional area. Equation 14 can then be normally applied with the current element now replaced by a surface area element $ds$. For windings of radius less than 4 mm the 1D method has been found to produce sufficiently accurate results on par with the 2D method.

![Figure 2-2: 2D Wahlstrom Coil Configuration with field vectors (left); Flux density contour for 1000 A current (right)](image)

### 2.3 Literature Review:

Analytical estimation of Eddy current distribution in 2D conductors of various geometry has been well established. James R Nagel [5] in his publication presents analytical formulations in terms of Bessel functions to evaluate induced eddy current density distributions along the cross section of simple geometries such as rectangular, cylindrical and spherical conductors. These eddy currents are, however, induced in the conductor due to a uniform background field that is isotropic i.e., a magnetic field that does not vary in the space surrounding the conductor. Jiri Lammeraner and Milos Stafl in their [4] premier book dedicated to Eddy Currents published in 1996 first derived the power losses associated with these fundamental cases. Since the conductor is assumed to be infinitely long the eddy current distribution is purely two dimensional and the associated current flows in one direction. Although these are important cases to understand the concept of eddy currents, they provide minor insight into real process associated with real interaction of between
conductor configurations and their resulting anisotropic background field distribution. The earliest attempt to tackle this problem for real planar winding used in induction heating was made by Acero et al. [29]. In their publication, the authors explore an iterative approach to include the alteration in field caused due to nearby windings. However, this does not include the 2D effects within the winding cross section itself and assumes filamentary nature of these windings. This is a good approach for windings of small radius relative to the overall configuration characteristic dimension. However, in conductors with large wire radius or at large operating frequencies when skin effect dominates this model breaks down. The author of the current research therefore explores techniques to address this drawback. The next section covers a simple 2D model that addresses anisotropic background field effect introduced when two conductors interact, and the subsequent section is dedicated to employing a novel Reference point scheme to implement the technique for 3D winding configurations.
2.4 Existing 2D Parallelizable Analytical Approach to Estimate AC Current Density Distribution:

A simple analytical two-dimensional model is introduced in this section to evaluate the modification in field uniformity due to the induced eddy currents. Based on the work by Jablonski et al [10] the skin and proximity effects in a system of two parallel conductors of circular cross section are evaluated in a segregated manner and superimposed together. It is important to note that this simple, yet powerful technique could be extended to three-dimensional axisymmetric winding configurations with multiple coil components that exist as closed loops. By employing analytical approximations, the Laplace and Helmholtz equations are evaluated in the domain of interest. The considered simplified geometry for the purpose of initial derivation is shown in figure 2-3.

Figure 2-3: 2D Conductor Configuration for Eddy current Evaluation

In the above figure, two conductors of radius \( R_1 \) and \( R_2 \) are placed at a distance of “d” apart. Conductor 1 carries primary AC sinusoidal drive current \( I_1 \) into the plane of the paper of angular frequency \( \omega \) which induces first order eddy currents \( J_{2,1}^{(1)} \) in conductor 2 due to a time harmonic magnetic field \( H_{1,1}^{(1)} \). These eddy currents in conductor 2 intern induce second order eddy currents \( J_{1,2}^{(2)} \) in conductor 1. The sequence of successive reactions between source eddy currents and the consequential field generated can be continued but in practice the next
reactions are often neglected[10]. Based on the nomenclature introduced, the second number in the subscript denotes the conductor associated with the source current and the first number in the subscript denotes the conductor associated with the consequential induced eddy currents component. The superscript denotes the order of approximation of the induced eddy currents. The origin for the global coordinate system is assumed to be at the center of conductor 2 with its axis aligned along the z axis. This enables the assumption of invariant field strength along the z axis due to the infinitely long constant cross section conductors. To evaluate the first order eddy current density at various locations \( X(r,\theta) \) in conductor 2, all the primary current \( I_1 \) in conductor 1 is assumed to flow along the axis i.e., filamentary current assumption. The solution procedure to obtain analytical equations for eddy current density inside a conductor consist of expressing the field generated by the primary current outside the conductor using Laplace equation taking into account the reverse reactions of eddy current density within the considered conductor. A combination of Helmholtz equation and continuity conditions along the boundary are then employed inside the conductor in which the altered current density distribution is to be evaluated.

### 2.4.1 Altered Primary Current Density Due to Skin effect:

Based on the solution for Helmholtz equation inside an infinitely long cylindrical conductor carrying sinusoidal drive current given by Lammeraner et al. [11] the redistributed current density due to the isolated skin effect in conductor 1 carrying primary drive current is given by:

\[
j'^{(0)}_1(\rho, \varphi) = \frac{I_1}{\pi R_1^2} \frac{I_1 R_1}{2 J_1(I_1 \rho)} \]

(15)

### 2.4.2 Proximity Effect and Resulting Induced Eddy Current:

Equation 15 in the previous section evaluates the altered current density distribution inside a conductor carrying alternating current due to the skin effect. As discussed in earlier sections the skin effect could be thought of as a redistribution of current to minimize overall...
inductance of the current flow path and is the consequence of microscopic effects. The proximity effect [30] [31] as the name suggests is the redistribution in conductor current density due to induced eddy currents generated by alternating field formed resulting from current in neighboring conductors. Two higher orders of corrections to equation 15 are needed to model this process. The first correction due to proximity effect for the conductor system discussed in this section results from eddy currents induced in conductor 2 due to field created by primary current in conductor 1. This correction in current density is denoted by \( J_{2,1}^{(1)} \) and is given by equation:

\[
J_{2,1}^{(1)}(r, \theta) = \frac{I_2 R_2}{\pi R_2^2} \sum_{n=1}^{\infty} \left( \frac{R_2}{d} \right)^n \frac{J_n(I_2 r)}{J_{n-1}(I_2 R_2)} \cos n\theta
\]  

(16)

If both conductors carry a primary current a similar equation for conductor 1 i.e., \( J_{1,2}^{(1)} \) can be written. Typically, two cases could be considered. Same currents \( (I_1 = I_2 = I) \) and opposing currents \( (-I_1 = I_2 = -I) \) in the two conductors. Therefore, two equations with different initial sign value as shown in equation 16 could therefore be obtained. In equation 16, the upper and lower signs are for same and opposing currents respectively.

Eddy currents \( J_{2,1}^{(1)} \) and \( J_{1,2}^{(1)} \) generate their own magnetic field which induced additional eddy currents in neighboring conductor. It is important to note that to obtain equation 16 all the source current density (primary current) was summarily assumed to flow along a single filamentary winding along the conductor axis. This assumption cannot be further extended to evaluate the second order correction since now the eddy currents are induced in a non-localized manner along the conductor cross section. Therefore, a combination of Laplace equation in the air domain and Helmholtz equation for vector potential in the conductor domain need to be solved. Continuity conditions also need to be imposed on the boundary. The first step involves dividing conductor 2 into discrete differential areas and representing each area with a point given by \( X(r, \theta) \). These points could either be in a rigid geometric grid in fixed distances apart or randomly distributed across the cross section based on a uniform probability distribution. The first order eddy current density \( J_{2,1}^{(1)} \) is then evaluated in each
of these distinct points. Conductor 2 could therefore be considered an aggregated sum of these differential elements that each represent a filament carrying a portion of the total current density. Each of these filaments therefore creates its own vector potential $A(r, \theta)$ in space which induces eddy currents in conductor 1. The magnitude of this eddy current density in conductor 1 is given by $j_{1,2}^{(2)}$:

$$
j_{1,2}^{(2)}(\rho, \varphi) = -\frac{I_1 R_1}{\pi R_1^2} \sum_{s=1}^{S} \sum_{s=1}^{n} \frac{(R_1)}{\xi_s} \frac{J_n(I_1 \rho)}{J_n(I_1 R_1)} \cos n(\pi - \varphi - \psi_s) \tag{17}
$$

Whereby law of cosines $\xi_s = \sqrt{r_s^2 + d^2 - 2r_s d \cos \theta_s}$ and $\psi_s = \arcsin \frac{r_s \sin \theta_s}{\xi_s}$. In a similar manner the second order eddy current density in conductor 2 due to vector potential in conductor 1 $j_{2,1}^{(2)}$ is obtained.

### 2.4.3 Application of Existing Technique to Wire Bundle:

The previously discussed analytical approach is now applied to the case of a helical copper coil with all geometric parameters as per the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major Radius of copper wire</td>
<td>31.3 mm</td>
</tr>
<tr>
<td>Minor Radius of copper wire</td>
<td>1.3065 mm</td>
</tr>
<tr>
<td>Coil pitch</td>
<td>14 mm</td>
</tr>
<tr>
<td>Coil inclination</td>
<td>90°</td>
</tr>
<tr>
<td>No of turns</td>
<td>17</td>
</tr>
<tr>
<td>Relative Permeability of Copper</td>
<td>0.999994</td>
</tr>
<tr>
<td>Resistivity of Copper</td>
<td>1.787e-08 Ωm</td>
</tr>
<tr>
<td>Conductivity of Copper</td>
<td>5.5950e+07 S/m</td>
</tr>
</tbody>
</table>

The coil geometry for analysis is shown in the following figure 2-4. It consists of a dual coil system comprising of a primary excitation winding for AC drive current and a secondary winding which is an artifact of the experimental setup used to measure induced eddy currents. As can be seen the cross section of each coil is distributed with randomly distributed
Monte Carlo data points in which the current density could be evaluated applying equations 15, 16 and 17.

Figure 2-4: Coil Geometry under study-Real coil (left) and Simplified coil (right)

As could be further observed from figure 2-4, the real helical coil system could be replaced by a system of disjointed closed current loops. Each loop represents a single turn of the real helical coil system separated by a distance equal to the coil pitch. Such a simplification enables evaluating the redistributed AC current density based on the previously discussed analytical correlations assuming axis symmetry. The results could then be compared to those obtained from numerical, experimental and the novel Monte Carlo method developed as part of this research. At this juncture, it is important to point out the secondary winding serves to
isolate, and measure induced eddy currents which are completely decoupled from the primary drive current of significantly larger magnitude. The following figure 2-5 shows variation of current density within conductor cross section applying equations 15, 16 and 17 previously discussed.

![Current Density in single coil (top); dual coil (bottom) systems](image)

Figure 2-5: Current Density in single coil (top); dual coil (bottom) systems

The effect of introducing a secondary coil not carrying any drive current is shown in the figure. Figure 2-6 shows the distribution of magnetic field due to DC vs AC current.
Figure 2-6: Field Contour for Coil system with DC (top); AC (bottom) system

Figure 2-5 and 2-6 have been made for AC current of 10 A at 5000 Hz frequency. Without
the secondary winding (top figure of 2-5), significantly larger eddy currents are induced
along the direction of the drive current which leads to a higher resultant current density in
the windings close to the ends. This translated to a spike in field near coil ends. Figure 2-6
below shows this redistribution of energy in changing drive current from DC to AC. In addition to this effect, along the central windings eddy currents are induced in a direction opposing the primary drive current direction. This leads to a significant drop in field in this region which translated to coil power loss associated with the constituent AC signal due to these eddy currents. It is important to note that Figure 2-6 is made for field variation along a reference line at radius of 25 mm from the axis. The presence of secondary winding further decreases the overall field since induced eddies in this winding flow in the opposite direction relative to the primary drive current and cause a field that counters the primary field.

Figure 2-7: Magnetic Flux density variation along a reference radius R=25 mm

AC current results in an overall reduction in field in the winding bore region and further leads to regions of localized high field close to the coil as can be clearly seen in the bottom figure 2-6.

2.5 Proposed Novel 3D Analytical Approach to evaluate Eddy Current Density and Loss in Windings:

A rudimentary cylindrical conductor of circular cross-section and infinite length is first considered to evaluate induced eddy current density. This could then be
extended to complete electromagnetic coil systems. In the considered simple starting point problem, a cylindrical conductor is placed in an external magnetic field which varies alternatingly. Figure 2-8 shows the simplified geometry considered. The orientation of the conductor with respect to the field is vital in determining both the type and magnitude of induced currents in the conductor. This has been clearly discussed in documents 1, 2 and 3.

Lammeraner et al. (Lammeraner & Stafl, 1966) have derived an analytical expression to evaluate current density at various locations along the conductor cross-section as a function of skin depth, coil permeability, and magnitude of external field strength. Such an expression for a cylindrical conductor placed in a transverse alternating magnetic field is given by:

$$\sigma = 4\mu_c H_o j^{\frac{3}{2}} k \frac{J_1 \left( j^{\frac{3}{2}} k r \right)}{F \left( j^{\frac{3}{2}} k r_o \right)} \sin \varphi$$

In the above Equation $\mu_c$ represents the permeability of the conductor material, $r_o$ the conductor radius and $H_o$ is the magnitude of the external field, which is applied perpendicular to the direction of the conductor axis. $r$ and $\varphi$ represent the
corresponding polar coordinates. The constant $k$ inside the cylindrical conductor is given by:

$$ k^2 = \omega \gamma \mu_c \tag{19} $$

Where $\omega$ is the frequency at which the external field varies, $\gamma$ and $\mu$ are the conductivity and permeability of the conductor material. For the region outside the cylinder:

$$ k^2 = 0 \tag{20} $$

The term $j^3k$ with the imaginary operator is therefore given by:

$$ j^3k = e^{(3\pi/4)} j \left( \frac{\sqrt{2}}{\delta} \right) = \left( \cos \left( \frac{3\pi}{4} \right) + j \sin \left( \frac{3\pi}{4} \right) \right) \frac{\sqrt{2}}{\delta} \tag{21} $$

Where $\delta$ is the theoretical skin depth of the conductor. This is an important parameter that determines the depth up to which the applied external field penetrates inside the conductor, which further influences the induced eddy current to accumulate close to the surface at high frequencies. $\delta$ is given by:

$$ \delta = \sqrt{\frac{2}{\omega \gamma \mu_c}} \tag{22} $$

Further, the first-order Bessel function ($J_1$) with the imaginary parameters could be evaluated using Kelvin functions expressed by:

$$ J_n \left( j^{2} x \right) = \text{ber}_n x + j \text{bei}_n x \tag{23} $$

Equation six shows that the Kelvin functions represent the real and imaginary parts of the Bessel function with imaginary arguments. The symbol $F$ in the denominator of equation one is given by:
Where $\mu_a$ is the permeability of the material surrounding the conductor. Similar to Equation (1) an expression for current density in a cylindrical conductor placed longitudinally along the direction of an alternating magnetic field is given by:

$$F \left( j^{\frac{3}{2}}k r_o \right) = (\mu_a + \mu_c) J_0 \left( j^{\frac{3}{2}}k r_o \right) + (\mu_a - \mu_c) J_2 \left( j^{\frac{3}{2}}k r_o \right)$$  \hspace{1cm} (24)

$$\sigma = H_o j^{\frac{3}{2}}k \frac{J_1 \left(j^{\frac{3}{2}}kr\right)}{J_0 \left(j^{\frac{3}{2}}kr_o\right)}$$  \hspace{1cm} (25)

Note than in the above expression in contrast to Equation (1), the current density distribution is independent of the permeability and the tangential coordinate direction. As is evident from Equation (1), both the magnitude of induced eddy current (real part) and its phase (imaginary part) could be estimated at every location inside the conductor. The eddy current power losses could then be evaluated (Lammeraner & Stafl, 1966) by integrating the obtained current densities across the conductor volume given by:

$$P = \frac{1}{2\gamma} \int_0^{r_o} \int_0^{2\pi} |\sigma|^2 r \, dr \, d\varphi$$  \hspace{1cm} (26)

Note that the complex conjugate of the current density is used within the integral in the above expression. The above expression could be simplified by substituting Equation (1) or (8) (Lammeraner & Stafl, 1966) and applying Lommel integrals to the resulting integration over radius, giving the final expression for eddy current power loss per unit length for the transverse field case:

$$P = \frac{4\pi\mu_c^2}{\gamma} (kr_o)^2 H_o^2 \frac{ber(\nu \nu) \, ber' \nu \nu - ber' \nu \nu \, ber(\nu \nu)}{[\nu \nu + \nu \nu ber(\nu \nu) + (\nu \nu - \nu \nu) ber_2(\nu \nu)]^2}$$  \hspace{1cm} (27)
Similarly for the longitudinal field case eddy current power loss per unit length is given by:

\[ P = \frac{\pi}{\gamma} kr_0 H_0 \frac{ber(kr_o) bei'(kr_o) - ber'(kr_o) bei(kr_o)}{ber^2(kr_o) + bei^2(kr_o)} \]  

Equations (1), (9), (10) and (11) could be employed for any operating condition since they have not been simplified and represent expressions in their complete form.

The entire electromagnetic coil geometry is divided into cylindrical segments, as shown in Figure 2-8. Such an operation facilitates the use of equations from the previous section. Data points are then randomly distributed within the coil domain now comprising adjacent cylindrical segments. Each coil segment is surrounded by a fixed number of reference points close to the coil surface midway along its length, as shown in Figure 2-8. These reference points significantly reduce the overall computational effort as will be demonstrated further. The magnitude of field strength is first calculated using Biot-Savart law in all reference points with respect to data points distributed inside the coil. Each data point is assigned a portion of the total primary current. This factored value of primary current remains the same for all data points. If the applied AC current frequency is significantly high so that it results in skin effect (redistribution of current close to coil surface) then the altered distribution of current density based on the skin depth is to be associated with every corresponding Monte Carlo data point based on its position in the coil. Therefore, in this case data points close to the surface within the calculated skin depth would have high values of factored primary current density while data points in the interior would have values close to zero. The factored value of primary current density is obtained by multiplying the primary current by the total coil length and dividing it further by the total number of coil data points within the skin depth which take part in current conduction at the specified frequency. As can be seen from figure 2-9, each reference point represents a sector of the corresponding coil segment. Therefore, all data points
within a sector are assumed to be exposed to a field represented by the reference point. Based on the direction of field strength at each reference point (below 45° angle between field vector and tangent vector at reference point location is considered a longitudinal field for the corresponding segment and an angle above 45° is considered a transverse field) either Equation (1) or (8) is used. In employing these equations to estimate current density in the data points, it is essential to determine the radial distance of the data points from the coil axis and the angle of incidence of the data point with respect to the corresponding reference point apriori. The first order induced eddy current density is thus evaluated for all data points in the coil domain.

The field generated in all the reference points by this induced eddy current is then evaluated. The procedure is repeated to estimate higher-order eddy current densities until a certain threshold is reached, beyond which the subsequent eddy current density is negligibly small. Therefore, successive approximation is used to evaluate the total eddy current density in all coil data points.

Power loss per unit length per sector represented at each reference point is estimated in a similar manner as current density using either Equation (10) or (11) along with (9) modified with the inner integral evaluated between 0 and 2π divided by the number of reference points per segment. Power loss per unit length for each segment is further obtained by performing integration over all sectors across the coil domain.

Figure 2-9 Model Geometric Setup
corresponding segment and multiplying this by the sector arc length. Finally, the Total Coil Power loss in Watts is calculated by integrating the previously obtained values across all segments and multiplying the result by the segment length.

2.6 Preliminary Results:

This section presents important preliminary results associated with the novel analytical model discussed in the previous section. In this regard, Figure 2-10 shows the DC flux density at reference points immediately surrounding the coil domain. These values could be considered as boundary conditions for the induced eddy current density in each cylindrical segment associated with the corresponding reference points. Based on the previously discussed model and the following figure, each cylindrical segment is surrounded by a minimum of 10 reference points at different angular locations. As could be seen, the flux density significantly varies from the inner surface of the coil winding to outer surface. This is due to the geometry of helical winding which creates a higher field in the bore region and a significantly lower field outside which decays as the square of coil radius. An additional complication in the field distribution is introduced due to the presence of the secondary winding. Reference points at different angular locations therefore help to capture this difference in field and the consequent difference in eddy currents induced within the cylindrical segment associated with constituent reference points. The three-dimensional field influence is therefore accounted for by the introduction of these reference points although the constituent formulation for eddy currents was originally developed for the two-dimensional case.
Following Figure 2-11 shows the cross-sectional contours of flux density in the region surrounding the copper winding. As previously mentioned, this figure clearly highlights the extant of exponential drop in field outside the coil configuration which is reflected in the huge difference in values associated with reference points in different angular locations. At this juncture it is important to point out the fact that the Biot-Savart integral previously introduced is extensively employed in evaluating these field distributions.
Figure 2-11 DC Flux Density contour

Figure 2-12 below shows eddy currents induced within the winding. Figure 2-11 could be thought as the cause and Figure 2-12 the consequence of the causal background field.

Figure 2-12 Total current Density vector
Figure 2-12 highlights the difference in magnitude associated with the eddy currents induced in both the primary and secondary winding. Since the majority of these eddy currents flow in a direction opposing the primary current direction, they work to reduce the overall current density in the primary winding. At this juncture it is important to point out that this novel model accounts for eddy currents induced both along the direction of coil and along the angular direction of the coil cross section which are induced due to longitudinal fields outside the winding based on equations 18 and 24. This significantly improves accuracy of the model at locations where the coil suddenly changes orientation where longitudinal fields dominate.

Figure 2-13 below shows a parametric study performed for different cases of winding geometry by altering the coil pitch i.e. distance between two coil turns.

![Figure 2-13 Variation of Flux Density with Axial and Radial Distance](image)

It could be clearly concluded that as the coil pitch increases variation in field along the radial direction significantly decreases. This translated to the fact that the flux density or the total number of field lines cross a cut cross sectional area significantly increases as the coil pitch is decreased which is an expected behavior. A coil with a larger pitch or greater distance between two subsequent turns creates a field in the bore region that is more smeared out and therefore sees a lower deviation in field variation in the radial direction. Variation in field along the axial direction however does not significantly differ with coil pitch as could be seen from the bottom figure.
2.7 Conclusion:

Based on the preliminary results presented in the previous sections, it could be reasonable concluded that the novel analytical model performs as expected based on characteristic field variables. Validation of this novel model is performed in the next section where these results are compared with experimental data. Although this model could be employed for simple winding configurations as shown in this section its limitations lie in the fact that it is restricted to conductors of circular cross section and windings without any abrupt change in orientation or sharp turns. Multiscale windings also cannot be implemented using this model. The Novel probabilistic scheme developed in the next section therefore provides an important alternative capable of being employed on a broader universal range of problems.
Chapter 3: A Novel Probabilistic Monte Carlo Technique for Estimating Eddy Current Loss

3.1 Introduction:

Probabilistic techniques offer a unique alternative to the previously discussed disadvantages to analytical modelling of 3D winding configurations. Furthermore, these methods offer a significantly improved alternative to existing widely employed finite element schemes in commercial softwares such as ANSYS/COMSOL in terms of performance and duration of simulation. With recent advances made in GPU parallel computing, the concurrent probabilistic solution associated with each domain data point independently of neighboring points offers a unique potential for implementation in the thousands of available computational GPU cores. Further advances made in recent artificial intelligence techniques enable convenient integration of these probabilistic methods for rapid optimization or exploring previously not considered parameter space of electromagnet design. Monte Carlo techniques are one class of such probabilistic methods where solution of a governing equation is obtained by repeated random sampling of the integral form of that governing equation. The underlying principle for this method is the fact that as the process of random sampling is repeated numerous times (limit approaching infinity), a solution nearing the real value is obtained.

3.2 Literature Review: Monte Carlo Floating Random Walk Technique

Several probabilistic Monte Carlo techniques exist in literature to model two-dimensional eddy current problems. Davey et al. [12] proposed a random walking technique on a fixed 2d grid for a conductor with Neuman boundary condition. The technique comprises of casting the governing Helmholtz equation for magnetic vector potential i.e., \( \nabla^2 A - k^2 A = -\mu J \) in the form given by:
In this equation \( r \) represents the point location where the solution is to be computed; \( J_s \) is the source current at a specific mesh node. \( D = \frac{4 + k^2h^2}{h^2} \) and \( p_t = \frac{4}{h^2D} \), represents the transition probability. The solution procedure for a specified point in the considered 2D domain comprises of taking steps of fixed length along the 2D grid representing the domain. The direction of these steps at each instant is randomly chosen based on the obtained transition probability. When a Dirichlet boundary is reached the steps are stopped and the accumulation of the heuristic vector potential values associated with each step is aggregated together to obtain the solution based on equation 31. This represents the solution for a single walk. By taking the average solution of several such walks the actual solution is obtained. Figure 3-1 below shows a simple 2D grid for a rectangular conductor. This figure also shows the steps taken based on the transition probability from a starting point of interest to the boundary of the conductor domain. In practice the air domain surrounding the conductor also needs to be meshed in a similar manner and the random walking continues till the boundary of this domain. This commonly employed technique to referred to in literature of the Fixed Random Walking Monte Carlo Technique.

Figure 3-1: 2D Grid for fixed Random walk Monte Carlo approach

As could be observed this technique has the same limitations as conventional finite element-based approaches in that accuracy of the solution is limited by ability to discretize the domain
into regular grids. Discretization of multiscale domains also becomes an inconceivable notion. Therefore, a newer technique called the FLOATING RANDOM WALK MONTE CARLO TECHNIQUE was first developed by Haaji Sheik et al[13]. This technique eliminates the need for a regular structured grid by constructing circles in the case of a 2d domain or spheres in the case of a 3d domain. The detail of this technique is presented in the next section.

### 3.3 Novel Probabilistic Mathematical Formulation of 3D Helmholtz Equation for Magnetic Vector Potential:

The starting point for the stochastic Monte-Carlo method is the non-homogenous form of the Helmholtz equation for magnetic vector potential $A$ given by equation 32

$$\nabla^2 A - \gamma^2 A = -\mu_0 J_s$$  \hspace{1cm} (30)

Where $J_s$ is the non-homogenous source current density in the coil. The wave number $\gamma$ is given by:

$$\gamma^2 = -\mu_0 \varepsilon \omega^2 + j \mu_0 \sigma \omega$$  \hspace{1cm} (31)

Where $\varepsilon$ is the permittivity, $\sigma$ is the conductivity and $\omega$ is the angular frequency of the excitation current. The first term in equation 4 corresponds to the solution applicable for a dielectric medium while the second term corresponds to the solution in a conducting medium.

As with several non-homogenous equations the solution procedure consists of employing the Greens function equation. Given a differential equation with a differential operator $L$,
\[ L[U(r)] = f(r) \]  

(32)

The solution \( U(r) \) is a function of the three-dimensional position vector \( r \). The function \( f(r) \) is a source term. The Greens function for equation 35 are the solutions of the differential equation

\[ L[G(r|r_o)] = \delta(r - r_o) \]  

(33)

Subject to boundary conditions specific to the problem of interest. We assume the operator \( L \) is the Strum-Liouville form \( L = \nabla \cdot [p(r)\nabla] + q(r) \) where \( p(r) \) and \( q(r) \) are known functions of \( r \). Using Greens integral representation \( U(r) \) can be written as

\[
U(r_o) = \iiint dv \, G(r|r_o)q(r) - \oint [ds \cdot \nabla_r U(r)]p(r) [G(r|r_o)] \\
+ \oint [ds \cdot \nabla_r G(r|r_o)]p(r)U(r)
\]  

(34)

The first term on the right-hand side of equation 7 is a volume integral involving the source term in the entire volume of interest. The second and third terms are vector surface integrals over the surface \( S \) enclosing volume \( V \) where \( ds \) is a vector whose magnitude is equal to that of an infinitesimally small area unit on the surface \( S \) and directed normally outward from the center of the unit area. The term \( G(r|r_o) \) is often referred to as the volume Greens function. The second integral corresponds to the Neumann boundary condition while the third integral corresponds to the Dirichlet boundary condition. Equation 7 corresponds to the basis of the stochastic Monte Carlo method called the Floating Random walk method discussed here.

In a summary, the Random walk method to evaluate the solution for equation 3 at a particular point in the domain (coil), consists of considering maximally sized spheres whose radius equals the maximum possible distance from the point to the domain boundary. A random “hop” is then designed to the surface of this imaginary sphere
based on predefined probability density. The angle on the surface of this sphere to which the hop proceeds depend on random probability density determined as part of the problem. Suitable “weights” are designed for each of these hops by sampling the various applicable integrals of equation 36 based on the boundary condition type and presence of source terms. Therefore, each hop is assigned a weight factor and the final solution of the Monte Carlo technique consists of evaluating an infinite-dimensional integral as given by the equation below[14]

\[
U(r_o) = \oint_{S_1} dS_1 K(r_o|r_1) \oint_{S_2} dS_2 K(r_1|r_2) \ldots \oint_{S_n} dS_n K(r_{n-1}|r_n)U(r_n)
\]

\[
K(r_{n-1}|r_n) = p(r_n)|G(r_{n-1}|r_n)|\cos(\gamma_{n-1,n})
\]

Where \(\gamma_{n-1,n}\) is the angle between \(\nabla_r G(r_{n-1}|r_n)\) and \(dS_n\). The successive surface integrals of equation 38 correspond to successive random hops across the problem domain and the weight factors of the form \(K(r_{n-1}|r_n)\) are derived for the third integral term on the right-hand side of equation 7. A particular random walk is terminated at the boundary of the domain considered and the value of magnetic vector potential at the boundary multiplied by the product of the successive weight factors gives the solution to the problem at the required data point. A solution is arrived by averaging over many such random walks each consisting of several hops. Figure 3-2 below shows a schematic of the random walk procedure carried out.
Figure 3-2: Schematic showing random walks on a circular 2D domain

The Greens function at the point position coordinate \( r(x,y,z) \) due to a 3D Dirac delta function source at \( r_o \) for the Helmholtz vector potential equation 39 is given by:

\[
\nabla^2 G(r|r_o) - \gamma^2 G(r|r_o) = \delta(r - r_o)
\]

Replacing \( \hat{r} = r/R \) and \( \hat{r}_o = r_o/R \) i.e. non dimensionalizing the above equation with respect to radius of the spherical domain

\[
\nabla_{\hat{r}}^2 G(\hat{r}|\hat{r}_o) - \gamma^2 R^2 G(\hat{r}|\hat{r}_o) = \delta(\hat{r} - \hat{r}_o)
\]

Therefore, the solution to the above equation is found with \( G(\hat{r}|\hat{r}_o) = 0 \) at the surface of a unit sphere. In order to arrive at the solution for equation 11 two factors need to be considered. All traditional solutions for such integral equations do not consider the effect of non-homogeneity introduced by the source current density term and non-linearity introduced by the wave number \( \gamma \) in equation 40 which is a function of spatial location or domain coordinates (inside or outside the conductor). Therefore, a new technique first proposed by Chatterjee et al. [15] called the iterative perturbation theory is employed as a work around to this problem. As part of this procedure two successive order approximations are performed to arrive at the final
solution. The zeroth order approximate is obtained by solving the Greens function equation

$$\nabla^2 G^0 (\hat{r} | \hat{r}_o) = \delta(\hat{r} - \hat{r}_o)$$

(39)

The solution to the above equation is given by [14]:

$$G^0 (\hat{r} | \hat{r}_o) = \frac{1}{4\pi} \left[ \frac{1}{\{1 + \hat{r}^2 \hat{r}_o^2 - 2\hat{r}\hat{r}_o C\}^{1/2}} - \frac{1}{\{\hat{r}_o^2 + \hat{r}_o^2 - 2\hat{r}\hat{r}_o C\}^{1/2}} \right]$$

(40)

Where $C = \cos \hat{\theta} \cos \hat{\theta}_o + \sin \hat{\theta} \sin \hat{\theta}_o \cos (\hat{\phi} - \hat{\phi}_o)$. The first order approximation is then given by:

$$\nabla^2 G^1 (\hat{r} | \hat{r}_o) = \delta(\hat{r} - \hat{r}_o) + \gamma^2 R^2 G^0 (\hat{r} | \hat{r}_o)$$

(41)

Solution of the above first order approximation equation obtained by considering the Greens theorem given by equation 44 by applying the correct boundary conditions is given as:

$$G^1 (\hat{r} | \hat{r}_o) = G^0 (\hat{r} | \hat{r}_o)
+ \gamma^2 R^2 \int_0^1 \int_0^{2\pi} \int_0^{\pi} \sin \hat{\theta}' d\hat{\theta}' d\hat{\phi}' d\hat{r}'(\hat{r}')^2 [G^0 (\hat{r} | \hat{r}') G^0 (\hat{r}' | \hat{r}_o)]$$

(42)

For suitable representation the first part of the RHS of equation 44 will be henceforth called Term A while the second term within the 3-dimensional integral is called Term B. The final solution of the magnetic vector potential is therefore obtained by considering Greens theorem based on equation 47 and above equation 55 given by:
\[
\hat{A}(\hat{r}_o) = \int_0^{2\pi} \int_0^\pi d\hat{\theta} \sin \hat{\theta} d\hat{\phi} \left[ \frac{dG^1}{d\hat{r}} \right]_{r=1} \times \hat{A}(1, \hat{\theta}, \hat{\phi}) + \int_0^1 \int_0^{2\pi} \int_0^\pi [-\mu_0 j_s](\hat{r} | \hat{r}_o) \sin \hat{\theta} d\hat{\theta} d\hat{\phi} \hat{r}^2 d\hat{r}
\]

For suitable ease of representation, the first term in equation 56 within the surface integral is referred to as TERM I and the second term within the volume integral is referred to as TERM II. The solution consists of substituting the obtained first order and second order greens function and its derivative in equations 53 and 55 as a result of which Weight functions are obtained which can be then directly applied to the Monte Carlo random walk as discussed earlier. The derivative of the first order green’s function is given by:

\[
\frac{dG^0}{d\hat{r}} = \frac{1}{4\pi} \left[ \hat{r} - C \hat{r}_o - \frac{\hat{r}_o^2 - C \hat{r}_o}{(\hat{r}^2 + \hat{r}_o^2 - 2\hat{r}_o C)^{3/2}} \right] \quad (44)
\]

Since the derivative of the first order green’s function is to be evaluated at \( \hat{r} = 1 \) as it is involved in equation 56 the final value of the derivative is given by:

\[
\left[ \frac{dG^0}{d\hat{r}} \right]_{r=1} = \frac{1}{4\pi} \left[ \frac{1 - \hat{r}_o^2}{(1 + \hat{r}_o^2 - 2\hat{r}_o C)^{3/2}} \right] \quad (45)
\]

Each of the individual terms are next evaluated. Term I now comprise of two parts given by:

\[
TERM I = \int_0^{2\pi} \int_0^\pi d\hat{\theta} \sin \hat{\theta} d\hat{\phi} \hat{A}(1, \hat{\theta}, \hat{\phi})[IA + IB] \quad (46)
\]

\[
TERM IA = \frac{1}{4\pi} \left[ \frac{1 - \hat{r}_o^2}{(1 + \hat{r}_o - 2\hat{r}_o C)^{3/2}} \right] \quad (47)
\]

With zero centered notation of Greens function at \( \hat{r}_o = 0 \) above equation reduces to:
\[ TERM \, IA = \frac{1}{4\pi} \]  

\[ TERM \, IB = \frac{y^2 R^2}{4\pi} \int_0^1 d\hat{\theta}' \int_0^{2\pi} d\phi' \int_0^{\pi} \left[ \frac{1}{1 - (\hat{r}')^2} \right] \left[ \frac{(1 - (\hat{r}')^2)((\hat{r}')^2 - \hat{r}')}{(1 + (\hat{r}')^2 - 2\hat{r}' C)^{3/2}} \right] \]  

The first weight factor employing TERM 1 in equation 56 is called the surface weight factor since it considers effect of boundary condition on sphere surface at each hop of random walk and given by:

\[ K_S = \frac{1}{4\pi} + \frac{y^2 R^2}{4\pi} \int_0^1 d\hat{\theta}' \int_0^{2\pi} d\phi' \int_0^{\pi} \left[ \frac{(1 - (\hat{r}')^2)((\hat{r}')^2 - \hat{r}')}{(1 + (\hat{r}')^2 - 2\hat{r}' C)^{3/2}} \right] \]  

Term II comprises of two parts are given by:

\[ TERM \, II = [\mu_0 J_s] \int_0^1 d\hat{\theta}' \int_0^{2\pi} d\phi' \int_0^{\pi} \left[ \frac{(1 - (\hat{r}')^2)((\hat{r}')^2 - \hat{r}')}{(1 + (\hat{r}')^2 - 2\hat{r}' C)^{3/2}} \right] [IIA + IIB] \]  

Term II A is given by:

\[ TERM \, IIA = \frac{1}{4\pi} \left( 1 - \frac{1}{\hat{r}} \right) \]  

Term II B is given by:

\[ TERM \, IIB = \frac{y^2 R^2}{4\pi} \int_0^1 \int_0^{2\pi} d\hat{\theta}' \int_0^{\pi} \left[ \frac{1}{(1 + \hat{r}^2((\hat{r}')^2 - \hat{r}' C)^{1/2}]} \right] \left( 1 - \frac{1}{\hat{r}'} \right) \]  

49
\[ \text{TERM IIIB} = \frac{\gamma^2 R^2}{4\pi} \int_0^1 \int_0^{2\pi} d\hat{r}' d\hat{\theta}' d\hat{\phi}' \sin \hat{\theta}' \left[ \frac{1}{\{1 + \hat{r}' (\hat{r}')^2 - 2\hat{r}' c\}^{1/2}} - \frac{1}{\{\hat{r}'^2 + (\hat{r}')^2 - 2\hat{r}' c\}^{1/2}} \right] ((\hat{r}')^2 - \hat{r}') \] 

(55)

The second weight factor can now be defined as a volume weight factor since this considers the effect of the source current density term and is given by:

\[ K_V = \frac{1}{4\pi} \left( 1 - \frac{\gamma R}{\hat{r}} \right) + \frac{\gamma^2 R^2}{4\pi} \int_0^1 \int_0^{2\pi} d\hat{\theta}' \sin \hat{\theta}' d\hat{\phi}' d\hat{r}' \left[ \left( \frac{(\hat{r}')^2 - \hat{r}'}{\{1 + \hat{r}' (\hat{r}')^2 - 2\hat{r}' c\}^{1/2}} - \frac{(\hat{r}')^2 - \hat{r}'}{\{\hat{r}'^2 + (\hat{r}')^2 - 2\hat{r}' c\}^{1/2}} \right) \right] \] 

(56)

Combining two weight factors solution for vector potential \( A \) at point \( \hat{r}_o \) is given by:

\[ \hat{A}(\hat{r}_o) = \int_0^{2\pi} \int_0^{\pi} \sin \hat{\theta} d\hat{\theta} d\hat{\phi} \ K_S(\hat{\theta}, \hat{\phi}) \hat{A}(1, \hat{\theta}, \hat{\phi}) \]

\[ + \int_0^1 \int_0^{2\pi} \int_0^{\pi} \sin \hat{\theta} d\hat{\theta} d\hat{\phi'} d\hat{r}' \left[ -\mu_0 J_s \right] K_V(\hat{r}, \hat{\theta}, \hat{\phi}) \]

(57)

Rewriting equation 70 in terms of probability distribution functions \( F(\hat{\phi}) = \hat{\phi}/2\pi; \ G(\hat{\theta}) = \frac{1}{2} (1 - \cos \hat{\theta}) \) and \( H(\hat{r}) = \hat{r}^2 \)

\[ \hat{A}(\hat{r}_o) = 4\pi \int_0^1 \int_0^1 dF(\hat{\phi}) dG(\hat{\theta}) \ K_S(\hat{\theta}, \hat{\phi}) \hat{A}(1, \hat{\theta}, \hat{\phi}) \]

\[ + 4\pi \int_0^1 \int_0^1 dH(\hat{r}) dF(\hat{\phi}) dG(\hat{\theta}) \left[ -\mu_0 J_s \right] K_V(\hat{r}, \hat{\theta}, \hat{\phi}) \]

(58)

As mentioned earlier, point of interest within the coil conductor domain is first chosen. The maximum radius from this point to the boundary of the domain surrounding the coil (chosen to be a spherical volume in this case) is calculated. Based on a predefined probability density, a random angle for \( \theta \) and \( \phi \) are chosen and a hop is made to this location on the surface of the sphere. Based on equations 25 and 29 \( K_c \) and \( K_s \) are evaluated for this hop. The cumulative product of \( K_c \) until
the present hop is found out and denoted as $K_c, cl$. The product of $K_c, cl$ and $K_s$ is found out and denoted as $K_p$. After every hop in the domain the value of $K_p$ is successively summed up and updated. This procedure is repeated until the boundary of the domain is reached. The solution for the current walk is given by $K_p + K_c, cl \times BC$ (boundary condition at the domain in boundary). This procedure is repeated for $N$ walks and the average of the solution across each of the walks is declared as the solution of magnetic vector potential at the desired location. Using the calculated value of magnetic vector potential at the data point, the current density is evaluated using the equation:

$$J = -j\sigma\omega A - \sigma J_s$$  \hspace{1cm} (59)

In the above equation the first term could be considered as the eddy current component contribution to the current density and the second term is the contribution of the primary current density which acts as the source term. Once the redistributed value of current density is evaluated in the coil and core domains respectively, the volumetric power loss is obtained by the following integral equation given by:

$$P = \int \frac{1}{\sigma} J \cdot J^* \, dV$$  \hspace{1cm} (60)

### 3.4 Standalone Implementation Procedure for Probabilistic Formulation:

This section provides a summary of the algorithm for implementing the previously discussed mathematical formulation of the Random walking technique associated with the Monte Carlo method.

1. Choose the point of interest/ data point within the coil domain at which the solution for the Helmholtz equation is to be evaluated.
2. Find the maximum radius of a sphere from the point to the domain boundary (includes air domain surrounding the coil). It is important to make sure that the chosen maximally sized sphere does not exceed the volume of the air domain sphere.

3. Choose the angular coordinates on this maximally sized sphere based on probability distribution functions associated with the two angular coordinates theta and phi. These probability distribution functions are obtained because of the integral form of the governing equation.

4. Hop to the new point on the circumference of this sphere.

5. Find the surface weight factor $K_S$ (surface greens function) which accounts for the boundary condition. If the hop is made to a location within the domain where no boundary condition is known value of $K_S$ based on equation 64 with the value of boundary condition is used.

6. Find the volume weight factor $K_V$ (volume greens function) which accounts for the source current density within the coil domain. If the hop occurs to a point that lies inside the coil, the value of wave number associated with copper is used and the value of source current density associated with this new location is used in equation 69. However, if the hop to the new location is to a point in the surrounding air domain, the value of wave number associated with air is used along with the value of zero source current density.

7. This hopping procedure is repeated until a location close to the air domain boundary is reached that is within a prefixed tolerance. It is now assumed that the air domain boundary is reached and therefore the value of boundary condition of vector potential associated with this boundary is substituted in the surface weight factor term for this specific hop.

8. Find cumulative product of all the surface weight factors $K_S$ associated with all the hops till the boundary location is reached. This is term $KC$.

9. Find value of $KP$ at the end of each hop which is given by the product of $KC$ and the value of surface weight factor $K_S$ of the current hop.
10. This sequence of hops until the domain boundary is reached is called a walk. The solution for the chosen data point associated with the current walk is obtained by finding the cumulative sum of KP associated with all the hops belong to the specific walk.

11. The solution associated with a certain random walk is therefore given by the formula

\[ KP + KC \times BC \text{ (at final hop)} \]

12. The procedure is repeated for N number of random walks.

13. Average value of solution for these N walks gives the desired solution for vector potential at that point in the coil domain.

### 3.5 Accelerated Implementation of Analytical and Probabilistic Model:

This section summarizes the procedure for the combined implementation of the 2D analytical model with the 3D probabilistic model presented in the previous section. Although the standalone probabilistic technique has a low simulation duration for simple geometries as seen in this research, the performance could quickly deteriorate for complicated geometries with multiscale elements such as filamentary Litz wires and thin laminates where the duration of implementation is in the order of a few hours. In such cases, the first step to improving performance is to make use of any symmetry in the coil geometry thereby preventing the needs for any redundant calculations that are inherently not necessary. For all the resulting necessary computational domains however, the following simple procedure could be used to implement a hybrid analytical-probabilistic technique. For this procedure, as mentioned previously the continuous 3D winding configuration is modified by representing each turn as a separate closed loop. For most modern windings this equivalency holds to a reasonable degree of accuracy in the resulting axial field generated in the central bore region of interest. Turn transitions mostly occur near the ends of the windings where higher order multipole effects dominate the field distribution. To apply the simple 2D model, these coil loops need to be replaced by their 2D axisymmetric representation. For simple configurations such as helical windings this could be easily carried out. However, for
complicated windings such as that shown in the following figure 3-3 it is impossible to obtain such an equivalent 2D representation. This is due to the orientation of the looped winding along the same plane this is not axisymmetric.

![Unrolled View: Azimuth versus X](image)

**Figure 3-3: Unrolled View of Sample 3D Winding**

For such a configuration the location where the coil changes orientation is ignored, and the 2D winding is assumed to be in the form of a rectangular racetrack with four straight segments. Therefore, in evaluating the 2D equivalent configuration, two sets calculations associated with the two pairs of parallel wire bundles with each wire in the bundle representing a single turn have to be implemented. Once the 2D calculations are completed employing the analytical model, they can be converted into their corresponding 3D form associated with the looped coils. The analytical value of eddy current density in each of the data points associated with the looped winding is then used as an initial condition for the 3D real winding configuration in the Monte Carlo probabilistic method. In regions where a significant deviation in initial value is noticed a higher number of Monte Carlo random walks are enforced for accuracy while in regions with minimum deviance this number of random walks is reduced thereby significantly decreasing the computation time. Therefore the initial values of eddy current density obtained using the analytical model is used to provide control
thresholds associated with the random walking number of each solution point and its associated convergence.

3.6 Experimental Validation:

3.6.1 Preliminary Experimental Setup:

The experimental setup to measure AC power losses in both the copper coil and ferromagnetic core material is shown in Figure 3-5. The main components include a high voltage AC FAST-PS CAENels™ power supply, Lock-in amplifier, NI DAQ 6024E card and IR thermal imager.

![Experimental Setup](image)

Figure 3-4: Experimental Setup

As noted in the figure, output data is recorded at a pre-fixed sampling rate. State-of-the-art driver software enables direct link via ethernet connection to the power supply. A feedback loop can therefore be set up to control operating drive current based on measured coil voltage drop. This feature also enables measurement of the system’s frequency response. Accurate evaluation of experimental coil power loss primarily depends on measurement of the in-phase component of drive current with

55
respect to the coil voltage drop which is sensitive. Factors like parasitic capacitance introduced by connection cables strongly influence current measurement. Therefore, several redundancies have been employed to measure current as noticed in the figure. Primarily, the RMS value of drive current is indicated by a Multimeter with an AMP clamp. This is used for visual verification. Two other sources of current measurement are employed for the purpose of data recording. A high-power, low resistance resistor capable of withstanding high temperatures and a Magneto-resistive sensor with dedicated power supply constitute these sources. The measured current signal from these two sources is used as input for a Lock-in amplifier to measure the phase introduced relative to the voltage signal across the coil and finally evaluated power loss. Such a redundancy helps to avoid any parasitic phase shifts introduced in the measurement process which are not inherently introduced by the coil. More details regarding this will be provided in subsequent discussions. A thermal IR imager calibrated for emissivity of copper is used to measure temperature distributions across the entire system. The coil support bobbin is 3D printed using a dark black ABS plastic material with the aim of mimicking an ideal black body with emissivity of 1 and reflectivity of 0. A thermocouple is also employed to measure coil temperature at a single point for reference. Since multiple voltage signals are to be recorded from a host of sensors an NI SCC-68 I/O connector block with SCC-TC02 thermocouple modules have been used to measure and record data at high bandwidth. LabView and CAEN-els proprietary software have been used to this extent.

Figure 3-6 shows the Test Article comprising the primary coil excited with drive current a secondary pickup coil used for measuring the induced eddy currents. Both coils are held in place as per design configuration using a 3D printed support bobbin with grooves that precisely lock them in position to generate a predetermined bore AC field. A provision is made in the support bobbin to locate a J-type thermocouple at a fixed coordinate location adjacent to the coil.
As could be noticed from figure 3-6, to avoid imperfections in the narrow groove during 3D printing, two segments of the bobbin are individually printed at 45° inclination and joined together. A high temperature resistant ABS plastic filament is used to this extent. The melting point of ABS plastic is roughly 200° C (392° F) and the deflection temperature of plastic deformation temperature is 161 to 198° C (323 to 390° F) which is well above the calculated coil temperature under the operating conditions considered in this study. From a 3D printing standpoint, the printing head is maintained close to ABS melting point unlike conventional prints and is run at higher speeds than normal. This enables a structurally rigid test article with ability to withstand high hoop stress caused by the tightly wound coil. The use of polar 3D printer fitted with stepper motors with higher precision enables near circular cross section of the support bobbin. A highly precise support bobbin that configures the coil to match its intended design helps to achieve a highly uniform AC field in the bore region with minimal fringe fields perpendicular to the direction of the coil axis. Since the operating current does not exceed 15 A at maximum frequency of 2000 Hz the coil power loss per unit length is sensitive to small changes in coil geometry. Figure 3-7 below shows the CAD diagram of the support bobbin structure with all the functional elements explicitly noted.
Figure 3-6: Support Bobbin CAD Design
Since voltage signals from more than three different sources have to be recorded, an I/O connector block attached to the NI 6024E DAQ card is used. This connector block also features SCC TC02 thermocouple modules with cold junction coupling that enables the J-type thermocouple to be easily calibrated. As previously noted, all signal inputs are voltage signals that are connected in differential mode to the DAQ to minimize interference and avoid measuring closed loop.

Figure 3-7: NI SCC-68 Connector Block + SCC TC02 Thermocouple Module

Figure 3-9 in the next page shows the Lock in Amplifier (bottom) and the isolated power supply for the Magneto resistive Current sensor. As previously noted, the lock in amplifier is a device to accurately measure phase difference between two input signals. In this case the phase difference to be measured is that between the input drive current across the entire circuit and the resulting potential drop across the inductor. The inductor introduces a phase shift in the voltage dropped across it. The power lost in the inductor is directly proportional to the current through it and the voltage across it that is in phase with this current. Therefore, it is vital to accurately obtain the voltage drop which is compensated for this phase shift.
The FAST-PS CAENels™ power supply shown in figure 3-10 can deliver output current in the range of ±30A at a voltage range of ±40V. The maximum output power is up to about 600 W. As could be observed from the figure below, using an ethernet connection operating conditions could be directly set from a computer with LabView software.

Figure 3-9: FAST-PS™ CAENels AC Power Supply

Figure 3-11 shows the various types of power resistors used to measure current in the circuit. In choosing the correct power resistors care should be taken in considering the tradeoff that exists between sensitivity of the current measured across the resistor and the ability of the resistor to withstand a higher load without overheating. For smaller values of current in the circuit are smaller frequencies, the 0.1 Ohm 5 W power resistor is used; for very high values
of drive current above 20 A and above 1500 Hz the large 0.35 Ohm power resistor is used and for all most of all moderate ranges the green 0.5 Ohm 20 W ceramic coated power resistor is employed. This ceramic coated power resistor is the best available in the market to withstand high temperature without the need for active cooling and does not compensate in the accuracy of signal measured across it. The left of this figure also shows a Magneto-resistive current sensor which works on the principle of differential magnetic field measurement with compensation [16]. The primary current is fed through a U-shaped conductor creating a magnetic field gradient between two sides of the conductor. A compensating current is then passed through another conductor. The resulting field created by this compensating current exactly compensates for the field created by the primary current. By measuring the voltage difference across a measurement resistor placed in series with the compensating circuit the value of primary current in the main circuit could be easily determined. Such a sensor has a wide measurement range with nominal currents of up to 50 A. Both DC and AC impulses could be measured using these NT-15 MR sensors.

Figure 3-10: Power Resistors (left); Magneto-resistive sensor (right) for Current Measurement

The drive circuit for measuring the induced eddy currents in the ferromagnetic iron core is slightly more complicated than that used to measure induced current in the coil alone. This consists of two windings, a primary and a secondary compensation winding as shown in figure 3-12. The purpose of the secondary winding is to compensate for the induced eddy current signal corresponding to the coil itself. More details of the compensating system is provided in the next sections.
As could be noted in figure 33, it is very important to isolate the compensating coil system from the primary coil system. During the experimental run the two setups shown above as far apart from one another and remain isolated by a suitable medium in between. The picture above does not reflect this, and the two systems have been placed together for convenience to capture the combined image. Isolating the two systems ensures that no parasitic capacitance exists in the secondary system and prevents the system from behaving like a transformer due to the resulting electric field created between them.

### 3.6.2 Measurement of Winding Power Loss:

Experimental measurement of AC losses in winding configuration is a well-established area of research. Numerous literatures exists on especially measuring coil losses in high temperature superconducting materials. Amaro et al [21] proposed a Contactless Loop electrical/electromagnetic method for loss measurement wherein a single loop of conductive wire is wound close to a turn of HTS coil. The voltage induced across this loop is proportional to the loss induced in the primary winding whose value could be obtained using a lock in amplifier. Such contactless methods avoid risks associated with traditional methods that employ voltage taps across the coil carrying current requiring the insultation to be removed. An alternative to this is the Calorimetric method as shown by Yuan et al [23] wherein a superconducting pancake coil is placed in a vacuum sealed container with liquid nitrogen. The boil-
off rate of the nitrogen is proportional to the power loss in the coil. This technique, however, requires the use of expensive equipment and sensitive instruments to accurately capture this boil-off. Most of the electrical methods available in literature as discussed by Yuan et al in [23], Pei Xiaonze et al in [22] and Pin-Yu Huang et al in [32] rely on using a compensation coil system to remove the primary inductive component of the coil voltage which dominates the total voltage across the coil but does not contribute to the power loss due to induced eddy currents. They also rely on using a phase determination device such as a Lock-In amplifier which measures the phase difference between the measured voltage and the current since power loss is the product of the in-phase component of the current and the voltage. The current research combines techniques from all the mentioned resources on electrical measurement and presents the following schematic summary for coil loss measurement which has been implemented. Figure 3-13 shows this schematic. Since the operating AC drive current is reasonably low below 25 A at a maximum of 2000 Hz voltage taps and a secondary coil have been used to measure the developed potential across the coil. These two methods could therefore be validated against each other. One important note that the author wishes to bring forward because of conducting this experiment is the importance of measuring the AC drive current in the circuit which is devoid of any inherent phase shifts introduced in the circuit due to parasitic capacitances. This is especially important since the current signal is primarily used as a reference signal in the Lock-In amplifier to measure the phase shift in the voltage signal across the coil. Therefore, as could be seen in the figure below, a high-power resistor capable of withstanding 5-10W is used in tandem with a Magneto-resistive sensor to measure current for redundancy. Since this circuit involves redundancies for measuring both voltage and current a DAQ card is used to acquire all signals and process the power loss values produced by each of these combinations.
Thermal measurements are made using a T-type thermocouple located at a reference turn of the coil. It is important to note that during all the experimental cases run as part of this research at high frequencies it was noticed that a significant thermal deviation was noticed between the center and end portions of the coil. It is therefore vital to pick this representative turn of the coil with care and that it is representative of the mean value and not the local extrema. Such thermal profiles along the coil length were able to be picked up early on during the trials by means of a thermal imager using Infra-Red imaging.

This section covers the Monte Carlo model results and its validation with results predicted by COMSOL software and experimental measurements. At the onset it is important to note that two important variables are of particle significance in this research namely: power loss in coil for both cases with and without a field enhancing ferromagnetic material and coil temperature. The following figure 3-14 shows the variation of coil power loss per unit length in mW/m for the case without iron core. As could be observed the difference/ percentage error of COMSOL and
the Monte CARLO methods with respect to experimental data is negligible in the order less than 0.5%. The general trend of increase in power loss with frequency is due to higher magnitude of eddy currents induced in the copper coil which significantly increases the power loss in comparison to the DC I2R loss. This increase in power loss is noticeable especially at frequencies higher than 1000 Hz since at this frequency considering the wire radius of 1.3 mm skin effect starts to set in and the penetration depth gradually becomes lesser than this value thereby driving most of the current closer to the surface region starting from this frequency. For instance, at I=10 A, at 100 Hz power loss is 550 mW/m and as the frequency is increased to 2000 Hz the power loss increases to 2300 mW/m; a 1750 mW/m increase. The Monte Carlo method inherently captures this transition from low frequency where skin effect is insignificant to the onset of skin effect at 1000 Hz with smooth transition.

As the current increases from 1A to 10A at 100 Hz the power loss increases from 5.74 to 532.4 mW/m and at 1000 Hz this increase is from 12.26 to 1233.2 mW/m therefore increasing current by 10 times increases power loss/unit length by nearly 100 times. This is a major conclusion and is significant since this directly relates to

![Figure 3-13: Comparison for experiment coil AC power loss with COMSOL numerical simulation and previously discussed Monte Carlo method at various drive currents](image)
the influence between the magnitude of current and its frequency. Figure 3-15 below shows the voltage signals captured during the experimental procedure. The top image corresponds to an operating frequency of 2000 Hz and the bottom figure corresponds to 50 Hz drive current. As could be observed, a high-fidelity data acquisition procedure has been set up employing suitable high pass/low pass filters that enable sufficient removal of background noise due to any inherent parasitic capacitances in the system.

Figure 3-14: Data acquisition signals captured during experimentation for 2000 Hz (top) and 50 Hz (bottom) drive current
3.7 Conclusion:

The accelerated analytical-probabilistic model is thereby validated to be in reasonable agreement with experimental data for coil power loss for frequencies ranging from 50 Hz to 2000 Hz and drive current between 1 A to 10 A. The general expression for eddy current power loss indicates a direct proportionality with the square of both frequency and maximum flux density. This dependence could be directly observed based on the data displayed in Figure 3-14. In addition to this dependence, as the current is increased from 5 A to 10 A at 1000 Hz, power loss increases from ~300 mw/m to ~1100 mw/m. This translates to roughly 4 times increase in coil power loss when the magnitude of drive current is doubled.
Chapter 4: A Novel Parallelizable Model for Evaluating AC Field Diffusion and Magnetization in Ferromagnetic Material

4.1 Introduction:

This chapter introduces a novel method to evaluate field enhancement of ferromagnetic materials in vicinity of a source field generated by a coil carrying AC current. Existing literature is used to initially develop a model for DC field. Eddy currents induced in the ferromagnetic material prevent the complete penetration of the source field into the material which needs to be accounted for in modelling AC field enhancement. This is incorporated based on the work by Dimitris Labridis et.al [18]. The contribution of this research however has been to apply this technique developed originally for a 2D vertically symmetric slab to any 3D object without the use of finite element discretization by packing the 3D domain with finite number of ferromagnetic spheres and evaluating their resulting interactions in the background field.

4.2 A Novel Parallelizable Description of the Ferromagnetic Continuum:

To proceed further implementing the previously discussed DC magnetization of any ferromagnetic 3D geometry a sufficiently robust procedure to pack the domain with constituent spheres needs to be developed. Traditional hard sphere packing algorithms like the one developed by MIT and Argonne National Laboratory constituting the OPENMCC python package use optimization procedures to fill simple geometries like cube or cylinder with hard spheres of fixed size. The maximum packing factor that could be achieved by this procedure stands to be 0.65. The resulting unpacked void space results in a significant error for application in this research since the permeability of this void air is significantly lower than the iron matrix through which the background field traverses and causes magnetization. Therefore, a novel Dynamic Sphere Packing algorithm has been developed by the author of
This research consists of initially hard packing the domain with spheres of fixed radius followed by a secondary packing in the resulting void regions with dynamically sized spheres of radius within a certain predefined range. As can be seen in the following figure 4-2, the initial attempt (left) and a pre-final version (right) of the packing algorithm result is shown in a 2D domain. This can be easily implemented in a similar manner to a 3D domain. Lozano et al [33] in their research publication discuss an efficient scheme to generate random sphere packs in irregular geometries however limited to single sized sphere distributed throughout the domain. A brief report on existing methods of sphere packing is provided by Henry Cohen [34] in this report. Details of the underlying pseudo-codes are also provided for implementation and future improvements.

4.2.1 Discrete Ferromagnetic Spherical Vice-Domain Packing Algorithm

The following procedure presents a pseudo-code for implementation of the dynamic sphere packing algorithm used in this research to pack any ferromagnetic domain with iron sphere. It is important to note that most conventional sphere packing algorithms limit the packing factor to attain a maximum value of 0.64 or 64%. This means that the remaining 36% of the domain volume remains empty with voids. Pertaining to utility, in the current context of ferromagnetic diffusion and magnetization these air voids significantly alter the overall permeability of the constituent material since magnetization is predominantly a volume effect. It is therefore of prime importance to develop an alternative packing scheme capable of reaching values close to 100%. A custom packing scheme is therefore introduced where the size of the packed spheres is dynamically varied based on the volume of void located between bounding spheres.

- The Argonne-MIT OPENMCC python package is used to fill the ferromagnetic domain initially with hard spheres of fixed radius equal to 25% of the characteristic dimension i.e. for an annular cylinder of thickness 10 mm radius of the hard spheres is ~ 2.5 mm. This algorithm uses a sophisticated optimization scheme to pack any domain with the constituent spheres overlapping one another.

- The next step consists of a dynamic packing scheme developed by the author.
• The total number of passes of the algorithm is to be preset.
• Each pass of the algorithm consists of starting at any random location in the ferromagnetic domain.
• Hops are then made from this location to subsequent locations within the domain.
• If the new location is in the void/air domain, a sphere is generated at this location. The radius of this sphere is determined based on the distance to all neighboring hard spheres that are located within a constituent zone of influence.
• If the new location is inside a hard sphere the process proceeds to a new location in the domain.
• Algorithmically the previous two steps are implemented by storing the location and radius of hard spheres in a buffer matrix. By restricting the nearest hard sphere check to within a zone of influence surrounding the hop location the size of this buffer matrix is ensured to be of manageable size instead of storing the entire history of all previously generated dynamic spheres and the hard spheres throughout the domain.
• This process is repeated until the boundary of the domain is reached when the pass of the algorithm is terminated.
• It is however important to note that the hop size associated with each pass should be carefully chosen based on the dimensions of the geometry. For the current ferromagnetic cylindrical domain, the hop size is fixed to be about 50% of the length of the cylinder which roughly estimates to be about 5 mm.
• The radius of the resulting spheres is also limited to be within a certain range between a maximum value equal to the radius of the hard spheres and a minimum value of about 10% of the hard sphere radius to prevent localized domain loitering.
• Figure 4-2 shown later on in this section highlights the effect of two different hop size chosen to dynamically pack the 3D domain. The figure on the left is for a hop size of 10% of cylinder length and the figure on the right corresponds to a hop size of 25% of cylinder length which results in a much robust packing of the entire domain.
• It is also important to note that each pass of the algorithm is capable of concurrent execution on the GPU which significantly decreases the time taken to completely pack the domain.
- Figure 4-1 below shows the algorithm implemented for a 2D domain using only about 1000 algorithmic passes.

The novel random walk Monte Carlo filling procedure previously discussed is used for the above process. The following figure 4-2 show the same algorithm implemented for a 3D cylindrical annular shell. It is important to note that in the following figure a non-uniform biased filling is purposely introduced to contrast the distribution of the spheres. Employing this custom algorithm enables filling the iron domain with spheres having a packing factor up to 0.95. Moreover, having such a high value of packing factor enables the capability to resolve small edges and the complete surface of the 3D domain.
To further understand the extent to which sphere packing factor influences the resulting enhanced field generated by the aggregate of iron spheres, a series of successive numerical simulations are carried out. As shown in the following figure 4-3, a large spherical domain is filled with smaller iron spheres of fixed radius. Three different cases corresponding to packing factor = 0.3, 0.4 and 0.5 are made. As can be clearly seen, a higher packing factor results in a greater resolution of the larger spherical domain surface. The OPEN MONTE CARLO python package is used to generate the sphere locations given the fixed radius of the larger and smaller spheres as input. A background field of 1 T is then imposed on the resulting composite geometry and solved for the static field distribution using the COMSOL MULTIPHYSICS commercial numerical solver. The results for radial field variation around the sphere at two different reference radii namely \( R_{\text{ref}} = 15 \text{ mm} \) (top) and \( R_{\text{ref}} = 25 \text{ mm} \) (bottom) is also shown in the following figure 4-4. For comparison, the field variation around a solid iron sphere without the constituent smaller spheres is also shown. For a packing factor of 0.5, the maximum error in radial field occurs at \( \theta = 180 \) of 25% at a reference radius of 15 mm. However, when the reference radius is increased to 25 mm from the surface of the spherical domain, the error at the same angular location decreases to <5%. It can be therefore concluded that for a reasonable packing factor of 0.5-0.8 this model could only be used to calculate field enhancement at reference location far away from the ferromagnetic domain.
If this setback is to be overcome, then a much higher packing factor using the dynamic sphere packing algorithm developed must be used.

4.2.2 Validation of Novel technique with Conventional Finite Element Discretization:

In addition to packing the ferromagnetic domain with constituent spheres it is important to validate the accuracy of the field generated by the constituent packed domain with a solid continuum. The following COMSOL simulations are therefore performed to this extent. They consist of packing a large sphere with constituent hard spheres of differing radius and associated packing factors and comparing the variation in field along a certain reference radius surrounding the packed media. Figure 4-3 shows there different cases for three packing factors of 30%, 40% and 50% volume packing.
Figure 4-3: Discretization of a large iron sphere into sub spheres:
Packing factor 0.3 (top) 0.4 (middle) and 0.5 (bottom)

Finite element meshes are generated uniformly for each constituent sphere by employing custom scripts in COMSOL to prevent exorbitantly high simulation time. The results of these simulations are shown in the following figure 4-4. These figures show the variation in Radial field density along two different reference radii surrounding the media. It can be clearly seen that as the location of the reference circle decreases or gets closer to the surface of the constituent sphere the deviation between the field generated by the solid continuum vs that generated by the packed sphere increases. It can be therefore reasonable concluded that if
the region of interest is sufficiently far away from the surface of ferromagnetic material, field found by spherical discretization result in reasonable estimates.

Figures 4-5 and 4-6 show the finite element mesh with inflation and the newly developed spherical discretization for the cylindrical ferromagnetic domain employed in this research to validate the probabilistic monte Carlo scheme.

It is important to note that although an iron cylinder of radius 22 mm is used, since the inducing background field is AC this field does not penetrate all the way into the iron domain due to skin effect which is discussed in the next section. Therefore, it is sufficient to represent only an annular portion of the iron cylinder that is involved in the actual physics of magnetization which would be approximately equal to the skin depth of the iron at the
operating frequency. This assumption leads to significantly reducing the computational time in both the finite element and Monte Carlo procedures.

4.3 Existing Technique to evaluate AC Field Diffusion in Ferromagnetic material:

Having established the previous fundamental concepts, the phenomenon of field diffusion in non-linear materials is next considered. The study of nonlinear diffusion of electromagnetic fields in ferromagnetic materials is of vital interest in the design of electrical machines ranging from transformers to actuators. “Indeed, analysis of electromagnetic field diffusion
in magnetically non-linear conductors is in a way analysis of eddy currents in those conductors” [19]. As previously established, the Inductance of any electrical circuit is a function of magnetic field established by the current flowing through the conductor, but this field as a whole is divided into two parts: one being wholly external to the conductor and the other being wholly internal to the conductor; only the latter can be regarded as corresponding to the magnetic properties of the conducting material itself. “The resistance and the internal inductance are solely dependent on the material properties and change in effective values when the current changes with varying frequencies which is termed skin effect” [20].

In non-linear conducting media equation 10 is used neglecting the displacement current term and replacing flux density with field strength since boundary conditions are usually given in field strength. The altered equation 10 is given by:

\[ \nabla^2 \mathbf{H} = j \sigma \omega \mathbf{H} \]  

As could be observed the permeability in the above equation is not a constant in the case of a nonlinear material such as iron. This non-linear permeability is given by the B-H curve and is characterized by the knee region of the curve. A suitable technique is therefore required to solve for the field strength based on the above governing equation taking the non-linearity of permeability into account. Before proceeding to this technique, it is important to note the parallel between equation 15 to evaluate field strength H and the vector potential equation 13. The only difference being the inclusion of the source current density \( \mu J \) in equation 13 and 14 which are used to solve for the vector potential and consequently the current density in the current carrying conductor. This is in contrast with equation 15 in which the variation of field strength inside the ferromagnetic material due to eddy currents induced in it is evaluated.

Dimitris Labridis et.al [18] in their publication “Calculation of Eddy Current Losses in Non-Linear Ferromagnetic Materials” elaborate a simple analytical technique based on iteratively solving for the non-linearity in magnetic permeability in the material based on the B-H curve. In the publication the authors solve equation 15 shown previously in cartesian coordinates assuming only the z component of magnetic field and constrained only to be a
spatial function of $y$. This equation is however adopted to cylindrical coordinates to be used in this study. Therefore, in relation to this study, the iron cylindrical core is assumed to be under the influence of a background field strength only along the axial direction i.e. $Z$ direction and the field strength inside the iron is varied only in the radial direction. This is a good approximation pertaining to the problem considered in this study and this enables use of the technique used in the discussed publication provided suitable alterations are made to the coordinate system (cartesian to cylindrical change). The primary equation is given based on equation 15 in cylindrical coordinates as:

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial H_z}{\partial r} \right) = j\omega \sigma \mu_0 \mu_r H_z
$$

(62)

The $B$-$H$ curve of the ferromagnetic material could be approximated based on the Frohlich representation [21] given by:

$$
B = H \frac{\alpha + \beta |H|}{\alpha + \beta |H|}
$$

(63)

The field strength at boundary/surface of the ferromagnetic cylinder which would be the maximum value of field strength induced by the AC current carrying coil is given by $H_m \sin \omega t$. To further simplify the problem, the permeability in equation 16 is assumed to be a constant with time rate of change of the sinusoidal wave of period $T$.

A fictitious material with constant temporal permeability and varying only as a function of cylinder radius $r$ is therefore defined. This permeability is related to the $B$-$H$ curve through the unknown values of magnetic field strength at every point. The constraint on this considered fictitious material is that eddy current loss density should be the same as the non-linear material at every point. Equation 16 could be rewritten for the fictitious material as

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial H'_z}{\partial r} \right) = j\omega \sigma \mu_0 \mu'_r (H'_z)(H'_z)
$$

(64)

Where $f$ represents the fictitious material with constant temporal permeability. The presence of two unknowns i.e. both $\mu'_r$ and $H'_z$ leads to an iterative solution. Thus, in order to close the problem (no of variables = no of unknowns) a secondary equation must be established.
between these two variables and the nonlinear B-H curve. At this juncture it is important to point out two important concepts of magnetic energy and co energy density in the material. The magnetic energy by definition is \( w_f = \int_0^{\psi} I d\psi \) or \( w_f = \int_{B_{max}}^{\psi} h db \) in the case of using the B-H curve while the magnetic co-energy which is a non-physical quantity measured in energy units (used to derive force and torque) is given by \( w'_f = \int_0^{I} \psi dI \) or \( w'_f = \int_{H_{max}}^{I} b dH \).

As could be implied, both quantities represent the area under the \( \psi - I \) or B-H curves with respect to the corresponding axis. For linear materials it could be further implied that these two quantities are equal. The concept of magnetic co-energy density however could only be applied to linear conditions. Therefore, it is necessary to linearize the governing pde in this study.

Three different magnetic co-energy densities are defined henceforth. A quarter period of AC sinusoid with field strength varying from 0 to \( H_{max} \) in the real non-linear material is first considered. Employing the integral definition of magnetic co-energy density discussed earlier and utilizing the Frohlich equation\(^1\)\(^7\), the following first equation is obtained and is represented by figure 4-7 below:

\[
w_1 = \frac{H_{max}^f}{\beta} - \frac{\alpha}{\beta} \ln \frac{\alpha + \beta H_{max}^f}{\alpha}
\]

Where \( H_{max}^f = H_{max} \) is the maximum field strength at peak sinusoid and is equal for both the non-linear and fictitious materials. \( \alpha \) and \( \beta \) represent Frohlich parameters. The second equation of magnetic co energy density in the real non-linear material is related to the average value of slope of the B-H curve as shown in figure 4-7 and is given by:

\[
w_2 = \frac{1}{2} \frac{(H_{max}^f)^2}{\alpha + \beta H_{max}^f}
\]

It is important to state the goal at this juncture which is to evaluate both the upper and lower bounds of eddy current power loss in the fictitious material which would most closely approximate the real non-linear material. A fictitious material which has a constant temporal permeability \( \mu_r^f \) and same \( H_{max}^f \) as the real material is related to a magnetic co-energy density given by:
\[ w^f = \frac{1}{2} \mu_0 \mu_r^f (H_{\text{max}}^f)^2 \]  

(67)

If this co-energy is set equal to that of the real material of co-energy density equal to \( w_2 \) then the value of fictitious permeability at each radial location of the fictitious material could be obtained and given by:

\[ \mu_r^f = \frac{1}{\mu_0 (\alpha + \beta H_{\text{max}}^f)} \]  

(68)

Since power loss is directly proportional to the relative permeability of the material, \( \mu_r^f \) which is obtained by equating with \( w_2 \) (represented by figure 4-7) clearly would result in a minimum value of power loss thereby representing the lower limit for power loss. The upper bound for power loss would therefore be given by \( w_1 \) given by figure 4-7 since this takes saturation at \( H_{\text{max}}^f \) into account. Based on figure 4-7 one reasonable approximation for the non-linear eddy current power loss would that obtained based on the average of the \( w_1 \) and \( w_2 \) co-energy density values.

![Figure 4-7: Non-Linear B-H curve and Magnetic Co-energy (grey area) [18]](image)

The procedure consists of using the boundary value of maximum field strength \( H_{\text{max}}^f \) to solve for the non-linear complex radial diffusion equation given by 18 to get \( H_{\text{max}}^f \) in each radial location of the non-linear material. In this step the relative permeability in this equation is assumed to be constant and is set equal to the initial slope of the Frohlich curve given by:
\[ \mu' = \frac{1}{\mu_o \alpha} \]  
\( (69) \)

Which has the same value in every radial point. Using this value of \( H^f \) maximum flux density in each of these locations is obtained based on the linear relationship between B and H. Equation 21 is then used to obtain \( w^f \) in each point. The average co-energy density based on \( w_1 \) and \( w_2 \) in each point is then evaluated. This value is then compared with \( w^f \) to see the difference in each radial location. If this difference \( w_{diff} \) is greater than a set threshold, a new permeability given by equation 22 is obtained. Using this value, the diffusion equation is solved again and the procedure is iteratively repeated until the threshold is achieved in every radial location.

4.4 Novel Parallelizable Estimation of AC Magnetization and Field Enhancement in Ferromagnetic material:

Considering the problem of a uniformly magnetized iron sphere in an external DC magnetic field (magnetostatic problem), the magnetic field inside the sphere are given by the equations:

\[ B = B_o + \frac{2\mu_o M}{3} \]  
\( (70) \)
\[ H = \frac{1}{\mu_o} B_o - \frac{1}{3} M \]  
\( (71) \)

For non-linear ferromagnetic materials, the direct proportionality between B and H i.e. \( B = \mu H \) cannot be used. Therefore, to obtain field values inside these non-linear materials a third equation is required to complete the system of equations with three variables namely B, H and M. This third equation is provided by the characteristic B-H curve of the corresponding material. Eliminating M from 18 and 19 gives:

\[ B = -2\mu_o H + B_o \]  
\( (72) \)

This equation along with the following B-H curves are to be simultaneously used to obtain the induced field values. It is important to note that geometrically equation 20 represents a
straight line with slope \( m = -2\mu_0 \) and y intercept=\( B_0 \). Therefore, the intersection coordinates of this line on the B-H curve gives the internal B and H values of the ferromagnetic material. The magnetization M is then obtained using either equation 18 or 19 with known value of inducing background field \( B_0 \).

Figure 4-8: B-H Curve for various ferromagnetic materials

Because of the previously outlined procedure, the value of Magnetization associated with each ferromagnetic sphere packed inside the 3D volume is estimated. The next step comprises of modelling the interaction between each magnetized sphere. As a result of such an interaction assuming, each sphere behaves like a dipole, which undergoes reorientation along the direction of the inducing background field. Because of such a superposition, field magnification results along the direction of the applied magnetic field. An iterative procedure is carried out by repeating the previous steps and updating the total field experienced by each iron sphere. This total field comprises of the field generated by the source coil and the field generated at each sphere location due to all the other neighboring spheres which is found using the equation:

\[
H_m(p) = - \frac{1}{4\pi} \iiint \frac{(\vec{M}, \vec{r})}{|r|^3} dV
\]  

(73)
Where \( p \) is the location where the field due to a neighboring magnetized iron sphere of magnetization \( M \) is to be evaluated. At the end of a fixed number of iterations, the converged value of magnetization associated with each ferromagnetic sphere inside the domain is stored. Using these values of magnetization and equation 21 the enhanced additive field with the background field at any reference point in the surrounding space can be evaluated.

At this juncture it is important to highlight a few underlying phenomena associated with the DC magnetization of ferromagnetic spheres and the interactions between several such identical spheres. To this extent figure 4-9 below shows the variation of field strength with distance from the sphere surface evaluated using equation 21. The plot is made for 2 iron sphere sizes of 2 mm and 4 mm. Since these spheres behave like dipoles in a background field, this variation is plotted at two different angular locations.

![Figure 4-9: Field variation around iron sphere](image)

In the previous figure \( \theta_{fe} \) corresponds to angular location associated with the iron sphere and \( \theta_{coil} \) corresponds to angular location in the bore region associated with the 2D Wahlstrom coil geometry shown earlier in figure 2-1. Two cases are considered with the iron sphere located at different \( \theta_{coil} \). The source field strength generated by the Wahlstrom coil is highest at \( \theta_{coil}=0 \) and lowest at \( \theta_{coil}=90 \). One important conclusion from figure 4-9 of major consequence is the fact that as the iron sphere radius is reduced by half (4 mm → 2
mm), the distance at which the magnetized field generated by the iron sphere reduces to 10% of the inducing source field decreases from 17.5 mm to 10 mm; a decrease of 42%. This is an important result when considering the size range of iron spheres to be packed inside the domain and the sphere of influence surrounding each magnetized iron sphere that determines the cutoff value of distance from the sphere surface. Choosing this cutoff value carefully helps reduce code runtime significantly during the iteration procedure to evaluate field interaction.

4.5 Preliminary Results:

In this section sample quantitative results for the enhanced additive field created by the presence of a ferromagnetic material such as an iron solid cylinder in vicinity of a helical coil carrying both DC and AC current is presented. To isolate and validate the enhanced field, the current carrying coil is assumed to be filamentary. Due to this assumption, eddy currents induced in the coil are eliminated from model results. The following figure 4-10 shows the discretized ferromagnetic cylinder using a custom meshing script developed in python. Both the traditional finite element grid and the novel dipole domain discretization have been illustrated. To reduce computational time, the domain for the AC case could be significantly reduced since field penetration in most ferromagnetic materials is significantly smaller than the characteristic dimension by at least an order of magnitude. Since both techniques for domain discretization are custom built, flexibility to alter grid/sphere density close to regions of interest is easily enforced. The filamentary coil used to generate the DC and AC source field is also shown in the figure. For the AC current case at frequency of 100 Hz the skin depth or maximum penetration depth of the source field for 1020 Low carbon steel is only 1.34 mm from the surface. As could be noted from the FEM mesh in figure 4-10, two elements are present within this skin depth by design and radial thickness of elements outside this skin depth are exponentially increased. Overall domain thickness is also reduced by a factor of two to reduce computational time. Figure 4-6 previously shown indicates the same domain distributed with Monte Carlo dipole spheres which are packed up to a factor of 0.95 using the custom algorithm discussed earlier. Just like the FEM grid, the density and
size range of spheres could be dynamically changed for different annular cylinders as seen in the same figure (bottom right image).

Figure 4-10 shows the plot of Magnetization in A/m overlayed on both the FEM tetrahedral mesh and the dipole domains and evaluated based on the two contrasting methodologies. Both methods involve iterative solution of the governing equations since the effect of interaction between different domains of the iron needs to be accounted for in the model. Pertaining to such an interaction the Monte Carlo method is better suited to naturally model random ferromagnetic domains in the iron and is therefore the more apt method for modelling ferromagnetic field enhancement. To overcome the setbacks introduced in not modelling the random reorientation of these domains the FEM method involves a few assumptions that inherently introduce modelling errors. Such errors are clearly brought out when the power loss in the copper coil is plotted for the case with iron. Once the magnetization value in each composite iron domain (FEM element or Monte Carlo dipole domain) these values can be stored along with both the size and centroid of these domains. If the enhanced iron field is to be evaluated at a particular location, then equation 21 for field strength
in terms of magnetization could be used. This field is then to be superposed with the field due to the coil to obtain the total field at that specific location.

4.6 Experimental Validation of Winding Power Loss with Ferromagnetic Core:

Measurement of losses in ferromagnetic materials in vicinity of a field generating coil due to eddy currents induced in the ferromagnet is still, however, an active area of research with significant scope for innovation. The technique developed in this research is directly derived from the work by Jan Souc et al. in [6] wherein a calibration free method that does not require extremely sensitive instruments to measure the relatively small value of induced voltage in the ferromagnet. The idea is based on the measurement of the part of the power supplied by the AC source winding which generates the background field that results in dipole domain alignment and field enhancement in the ferromagnetic material in its vicinity. This technique used a coil wound in parallel to the AC source winding as the measurement winding. To increase the sensitivity of measurement two identical systems as shown in the figure 4-11 below one of them containing the ferromagnetic material is used. As previously noted, the purpose of the secondary measurement coil is to isolate and remove the dominant inductive component of voltage in the primary winding and the purpose of the second identical system without the ferromagnetic sample is to isolate only the required voltage signal induced in the measurement coil as a consequence of eddy currents only in the ferromagnetic material and remove any other signal due to eddy currents in the coil itself. This method has been shown to be effective without the use of any calibration constant and therefore been implemented with a few changes in the research. It has been suggested by the author that the use of an electrically insulating material between the two coil systems is vital to prevent any inductive components from creeping into the system. Furthermore, this prevents the system from behaving like a transformer. All the other features such as the thermal measurement system and the data acquisition system remain the same as the previous case for measuring coil loss.
By performing the current compensation technique suggested in the previous section the power loss in the coil in vicinity of the iron core is next validated. Figure 4-12 shows this result. It could be clearly observed that the variation in power loss is not linear like the previous plot for coil loss without iron. This non-linear variation in power loss with frequency could be associated due to two specific reasons:

- At a frequency of about 1000 Hz a counterintuitive effect of ferromagnetic field interaction starts to set in.
- At this frequency the skin depth or penetration depth of background field within the surface of iron is lesser than 1 mm
- Therefore, instead of enhancing the field due to limitation imposed by the formation of eddy currents in the ferromagnetic material, a drop in the total field
(coil field + iron field) occurs i.e. the iron field component slowly starts to decrease resulting in a lesser overall total field.

- The power loss however increases with frequency since power is still being supplied by the coil to the iron which induces eddy currents in the iron which are the main source for the drop in its field enhancing properties at higher AC frequencies.

- This result is particularly significant since most literature only considers the DC case when it pertains to ferromagnetic materials. This is because only at DC the ferromagnetic material act to purely enhance the background source field due to the lack of eddy currents in this case.

Therefore, is could be reasonable concluded that pertaining to an Alternating background source field the presence of a ferromagnetic material is of no practical use in terms of field enhancement. Moreover, the presence of iron in an AC field also introduced complex multipolar components along the axial of the bore region which is undesirable in the design of electromagnets.

Figure 4-12: Comparison for experiment coil AC power loss (with iron core) with COMSOL numerical simulation at various drive currents
In comparison to the case without iron, power loss for 10 A drive current at 1000 Hz increases to about 1758 mW/m from 395 mW/m an increase of 77%. As mentioned earlier, this significant increase in power loss is due to the transfer of magnetic energy in the form of AC magnetic field which is transferred to the ferromagnetic material and is loss in it by the formation of eddy currents. The eddy currents which are formed in iron are also significantly higher than those formed in copper due to the significantly higher electrical conductivity and magnetic permeability of iron in comparison to copper. As mentioned in the previous section a special treatment for AC field penetration in iron is required to be modelled prior to magnetization. The effect of applying such a magnetic diffusion model is shown in the following figure 40 which clearly shows the depth tow which the background field generated by the coil penetrates inside the iron. For clarity a special case pertaining to a high current of I=100A at frequency of 100 Hz is simulated here. The following figure also shows the comparison between the two methods i.e. FEM code developed by the author and the novel dipole domain Monte Carlo method shown below the former. For clarity and purpose of graphical processing the total number of dipole spheres inside the iron domain is capped with a filling factor of 0.8 in the figure. This results in the figure with clearly observable air gaps which are not desirable in the finalized model. In practice a filling factor of about 0.95 is used with tightly packed iron spheres that prevent any permeability leakage of the inducing background field. Once the value of diffused field in the iron is obtained the next step comprises of applying the non-linear magnetization solvers for both the cases.
4.7 Conclusion:

The Monte Carlo model predictions for coil power loss with iron could therefore be validated within a reasonable degree of accuracy in comparison to experimental data. The newly developed spherical discretization scheme for ferromagnetic material is also compared with sample COMSOL simulations for validity and found to be in reasonable agreement. At this juncture it is important to point out that the B-H curve associated with the ferromagnetic material plays an important role in the accuracy and sensitivity of the resulting magnetization model of the ferromagnetic material. It is therefore important to choose the correct B-H curve that most accurately captures the real performance of the material, and which has sufficient data points especially in the non-linear region of the curve. Based on the comparison plot for coil loss and the error bars associated with the experimental data, it cannot be stated that although a significant deviation between COMSOL results and experimental data is observed especially at high frequencies, the result predicted by COMSOL is incorrect since it still lies within the error tolerances of the data. Furthermore, based on the deviation observed between model predictions and experiment data especially in the region between 500 Hz to 1500 Hz future works needs to be pursued so as to understand the cause of this deviation i.e. weather this is purely a result of experimental/model error or due to the onset of skin depth dominance starting from 500 Hz onwards.
Chapter 5: Multiphysics Coupling for Winding Temperature Distribution

5.1 Introduction:

The following section provides the coupling model to use the power loss obtained because of the previously developed Monte Carlo model and obtain the temperature distribution in the coil/core domain. It is important to obtain steady state coil temperature associated with losses since this enables the informative design of suitable active/passive cooling systems for the electromagnet prior to its design and implementation. Utilizing an existing Monte Carlo scheme for thermal predictions enables evaluating temperature specifically at the desired coil location independently without any requirement for calculations at any other location due to the inherent nature of the model and thereby saving tremendously on computation time in comparison to FEM models. Moreover, the scheme developed henceforth is capable of execution within the coil domain alone without requirement for solution to proceed inside the surrounding air domain which further improves model performance.

5.2 Existing 3D Probabilistic formulation for Steady State Heat Conduction:

A previously developed Monte Carlo formulation for the three-dimensional steady state Poisson’s heat conduction equation by Haaji Sheik et al [13], [22] is implemented. Although originally developed for an isolated heat conduction problem, its use in the currently discussed coil-core system and novel technique to couple with the previously developed electromagnetism formulation in Chapter 4 is introduced in this research. The volumetric loss density obtained previously is used as the source term in the Poisson equation to evaluate the steady state temperature values in query data points. The Poisson differential equation is given by:
\[
\left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q \nu}{k} = 0
\]

(74)

Where \( k \) is the material thermal conductivity and \( q \nu \) is the rate at which energy is generated per unit volume in units of \( \text{W/m}^3 \). The above equation could be used to obtain the temperature distribution \( T \) in a domain given the boundary conditions in the surface of said domain. To obtain an integral Monte Carlo formulation for equation 61, the unsteady time dependent version of equation 61 is first considered in spherical coordinates without the heat generation term \( q \nu/k \) and is given by:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) = \frac{\partial T}{\partial t}
\]

(75)

By applying Laplace transform, the separation of variables method and continuity conditions at the surface of the spherical domain, the temperature at the center of the sphere \( T(r = 0, \theta = 0, \phi = 0, t) \) in terms of the temperatures at the surface of the spherical domain \( T(r = 1, \theta, \phi, t) \) is given by:

\[
T(0, t) = -\frac{\pi}{2} \int_0^t \int_0^{2\pi} \int_{-1}^1 T_w(\theta, \phi, -\eta) \left[ \sum_{k=1}^{\infty} (-1)^k k^2 e^{-k^2 \pi^2 \eta} \right] d(\cos \phi) d\theta d\eta
\]

(76)

The steady state approximation of the above equation can be simplified to:

\[
T(x_i, y_i, z_i) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} T_w(r, \theta, \phi) \sin \phi d\phi d\theta
\]

(77)

Which can be non dimensionalized as:

\[
T(x_i, y_i, z_i) = \int_0^1 \int_0^1 T_w(r, \theta, \phi) dF(\theta) dG(\phi)
\]

(78)

Where \( F(\theta) = \theta/2\pi \) and \( G(\phi) = \frac{1}{2} (1 - \cos \phi) \) in which \( \phi \) is the angular coordinate with respect to the z axis and \( \theta \) is the angular coordinate with respect to the z axis. A probabilistic interpretation of equation 65 to employ the Monte Carlo random walk
technique previously discussed results in the use of the two probability distribution functions for $\theta$ and $\varphi$ shown in the following figure 5-1. Using these functions and equations 65, the procedure for computing the solution at a required query point within the domain $X(x_i, y_i, z_i)$ consists of making random hops starting from $X$ as previously discussed in chapter 4. One major contrasting difference with the fixed random walk is now the length of each hop is not fixed based on a domain mesh size. Instead, the hop length is equal to the radius of the largest sphere connecting the point and the domain boundary. Once the radius of the hop is so determined, the angular coordinates for each hop are determined based on the probability distribution functions.

![Figure 5-1: Probability distribution functions for the angular positions $\theta$ and $\varphi$](image)

As could be observed from figure 5-1, the probability density $dF/d\theta$ is a constant which means every $\theta$ location on the sphere is equally probable whereas $dG/d\varphi = \frac{1}{2} \sin \varphi$ and non-linear function which implies certain $\varphi$ locations closer to the equator of the sphere are more probable than locations closer to the poles. Therefore, a point from $X$ hops to another point on the surface of the sphere given by:
$$x_{i+1} = x_i + r_i \sin \varphi \cos \theta$$  \hspace{1cm} (79)

$$y_{i+1} = y_i + r_i \sin \varphi \cos \theta$$  \hspace{1cm} (80)

$$z_{i+1} = z_i + r_i \cos \varphi$$  \hspace{1cm} (81)

A variation of equation 65 that could be applied for the current research is shown below with the additional heat source term:

$$T(x_i, y_i, z_i) = \int_0^1 \int_0^1 \left[ T_w(r, \theta, \varphi) + \frac{q_0 r^2}{6K} \right] dF(\theta) dG(\varphi)$$  \hspace{1cm} (82)

### 5.3 Coupling of Heat conduction to Eddy Current Problem

#### 5.3.1 Implementation procedure:

In summary, the procedure applied to the current system pertaining to this research consists of the following steps:

1. Choose the query point \(X(x_o, y_o, z_o)\) inside the coil/core domain where the steady state value of temperature is required to be found out.

2. The boundary of the coil/core domain is assumed to be a Dirichlet boundary with constant fixed temperature. This is a reasonable assumption since the coil is predominantly insulated. Therefore, the fixed temperature at the boundary of the coil/core domain is assumed to be that of the ambient air temperature \(T_B = 28^\circ \text{C}\).

3. Random hops are made from \(X\) towards the domain boundary with the length of each hop equal to the radius of maximally sized spheres to the domain boundary.

4. A score is recorded with each hop until the culmination of the walk when the boundary is reached.

5. Since the temperature in the interior of the domain is not known, the score recorded with each hop is equal to the heat generation term i.e., the volumetric
loss density due to the induced eddy currents at which location which is previously calculated using the Helmholtz equation for vector potential.

6. Let $S_q$ be the term accumulating the heat source at each hop location of the random walk.

7. When the boundary is reached, this accumulated term is then summed up with the boundary temperature at this location and the walk is terminated.

8. For instance, if a random walking particle originating at the point $X_1$ follows a path $X_1 \rightarrow X_2 \rightarrow XB_3$, the overall score recorded for this path is given by:

$$T(x_0, y_0, z_0) = \frac{1}{6K} [q_1 r_1^2 + q_2 r_2^2] + T(XB_3)$$

(83)

Where $S_q = \frac{1}{6K} [q_1 r_1^2 + q_2 r_2^2]$ and $T(XB_3)$ is the temperature at the boundary point $X_3$.

The final solution of temperature at the sampling point $X$ is obtained by repeated this process $N$ times i.e. carrying out $N$ successive/parallel walks in the domain starting from the same point and averaging all these values. As $N \rightarrow \infty$ error in solution

$$ERROR[T_{MC}(x_o, y_o, z_o) - T_{Rea}(x_o, y_o, z_o)] \rightarrow 0.$$

One important characteristic of the method discussed in this section is the fact that the random walk Monte Carlo method for solving the heat conduction does not need to be implemented in the air domain surrounding the coil/core domain unlike the electromagnetism problem discussed earlier. The main reason for this difference is the fact that in solving for the Helmholtz equation the underlying physics of eddy currents is based on wave propagation in the air domain which means that primary currents flowing in one part of the coil influence induced eddy currents in other part of the coil which are sufficiently farther apart. In contrast, in solving for the Poisson heat conduction equation, energy transfer could be assumed to be in the form of particles/packets of energy transferred from sources situated either along the boundary of the domain or within the domain in the case of volumetric heat generation terms towards a sink generally based on the gradient
in energy difference between the source and the sink location. However, it is important to point out one setback of using such a floating Random walk method as compared to the fixed random walk method on a meshed fixed domain discussed earlier. The floating random walk method employs the use of floating-point operations as opposed to the fixed random walk method which employs only integer operations which is much less time consuming computationally.

5.4 Experimental Validation of Winding Temperature:

5.4.1 Survey of Thermal Gradients across Winding by IR Imaging:
This section presents results of thermal distribution captured by the IR camera during the experiment. At this juncture it is important to point out the purpose of performing these IR imaging. In order to compare model results of temperature predictions with experimental data it is vital to measure the temperature using a suitable thermocouple at the correct location of the winding. This location along the winding should be carefully chosen based on the performed IR images avoiding regions of high thermal gradients or localized hot spots that are not representative of the mean winding temperature. Based on the previous discussions of coil power loss such localized hot spots are unavoidable especially since AC current through the winding results in eddy currents that redistribute resulting current density along the coil leading to higher power loss in certain sections of the winding. This further translate to zones of high temperature corresponding to these locations of high volumetric power loss. Figure 5-2 below shows the global thermal IR image of the entire experimental setup. In addition to showing any localized hot spots in the winding itself, these images present useful summary of devices used in the setup that generate heat like the power resistors and the current sensors. Suitable cooling mechanisms could thus be designed based on these images for safety reasons and prevent any possible fire hazards.
Figure 5-3 shows the temperature gradient associated with both cases of coil with and without a ferromagnetic core material. It could be clearly noted that a significant difference in temperature distribution in the winding is observed between the two cases.

Figure 5-4 shows images processed using the IR camera. It is important to note that to notice an appreciable thermal gradient across the coil a high frequency of 3000 Hz is chosen for the following test. Moreover, the temperature scale for these IR images

97
is restricted between 48 and 51 C. This implies that any temperature below or above these values is effectively cut off and represented by these limit values i.e., a temperature of 32 C is therefore displayed as 48 C in the image and a temperature of 55 C as 51 C. This small range is chosen based on trial and error to highlight the thermal gradient occurring across the coil. The intention of the following images is to not obtain the magnitude of temperature at a particular coil location but to simply identify any underlying trends in thermal gradients across the coil. As mentioned previously, with the help of these thermal gradients the ideal location of the thermocouple could be chosen that best represents a mean value of coil temperature that can intern be compared with model data. As could be observed, for the case of coil without iron core, a clear spike in temperature is noticed closer to the central turns of the winding. This spike in temperature increases with an increase in coil current. Therefore, based on these images it is important to choose a turn further away from the winding center and not close to the winding ends where lead inter connection to the power supply is made. Hence the 10th turn is chosen to connect the thermocouple.

Figure 5-4: Measurement Thermal imaging results for temperature distribution in coil carrying current- 5 A (left) 10A(middle left) 15A (middle right) and 20 A (right)
Figure 5-5 shows a very important distinction with the previous case. For the case with iron core a significant thermal spike is observed close to the ends of the winding as could be clearly discerned from these images. This spike is due to the fact that the field enhancement of iron is higher close to its ends and therefore results in higher eddy currents to be induced in the copper coil at this location. Therefore for the case with iron the location of the thermocouple is switched to the 5th turn far away from the ends.

5.4.2 Measurement of Winding Contact Point Temperature:

The current research focuses on developing a multiphysics Monte Carlo model which includes modelling the thermal behavior of the coil-core system under test. Therefore, it is vital to validate the thermal predictions of the model against accurate experimental data. Although a sensitive thermocouple is used to measure temperature at a reference location of the coil it is important to understand the overall thermal behavior of the system prior to this being performed. Therefore, as shown below an IR thermal imager has been used to understand any bottlenecks in the system. Figure 5-2 clearly shows one such setback of using power resistors to measure the current signal. Although more accurate at measuring current signals than magneto resistive sensors especially at high frequency the figure shows the thermal spike at the location of the resistor. This has enabled the author to predict the power
resistor failure ahead of time at high operating frequencies and drive current. Figure below shows experimental setup to measure temperature using thermocouples and IR imager.

![Experimental Setup](image)

Figure 5-6: Experiment setup to measure Coil temperature

Other locations of thermal spikes such as at interconnecting points between power cables and test specimen, input and output cables of the magneto resistive sensor could also be observed in the figure. These thermal observations have therefore enabled operating the designed system with abundant caution and prevent any thermal or current leakages. A simple yet effective floating random walk Monte Carlo model is implemented to determine heat dissipation and temperature distribution in the copper coil because of the developed power loss due to eddy currents. The result of this model is presented below. Since the operating current ranges from 1 A to 20 A for this research no significant temperature variation has been observed for the low current cases of 1A, 2A and 5A. Therefore, these cases have been ignored in the following results. Figure 5-6 below shows the variation in temperature at the 10th turn of coil measured using the T-type thermocouple. This location is picked based on thermal images obtained from the IR camera which revealed significant thermal gradients along coil
ends and at the coil central locations that fluctuate significantly with drive current and frequency. The temperature at the 10th turn was identified to be the most stable and there it was picked. The experimental values shown below are obtained based on time average values obtained from the thermocouple. In order to obtain the Monte Carlo temperature, a specific data point along the 10th coil turn is picked and solution at this point is obtained.

![Figure 5-7: Coil Temperature Variation with Frequency (without iron)](image)

Having previously established that solution at a specific data point is independent of the solution at all the other points the process to obtain the Monte Carlo temperature variation with frequency at a specific point is simple and quick. It is important to note that Monte Carlo results for figures 5-6 and 5-7 have been obtained by performing linear regression of results obtained at frequency sample points are large sample locations apart from one another. For instance, at 20 A drive currently three Monte Carlo cases pertaining to frequencies of 50 Hz, 200 Hz and 2000 Hz are performed. By this technique the overall trend in temperature variation is captured between different values of frequency belonging to a range with noticeable change in slope based on experimental data. A custom scrip is written in python to this
extent to perform such a linear regression based on input data obtained from experiment and employing the evaluated Monte Carlo result. It is however the strongest intention of the author to perform a series of tests at higher frequency ranges with a higher variation in temperature and compare with Monte Carlo results. The following figure shows the variation in temperature for the case of coil with iron core.

![Figure 5-8: Coil Temperature Variation with Frequency (with iron)](image)

One noticeable difference between figures 5-6 and 5-7 is the fact that at I=10 A and 15 A no noticeable difference in temperature change is observed by adding iron. However, at 20 A drive current the presence of iron significantly increases the coil temperature to 32°C from 28°C for the case without iron.

### 5.5 Conclusion:

At the operating drive current and frequency range it could be clearly observed from experimentally measured thermocouple values of coil temperature that variation in temperature is not appreciably high. This is reflected in the high value of error tolerance bars in both figures 5-6 and 5-7. The Monte Carlo model predictions are
well within these error tolerances of experimental data and overall result in the common trend of temperature rise with the same slope of change on temperature associated with different zones of frequency ranges of operation.
Chapter 6: CUDA Parallel Programming Techniques to Current Problem

6.1 Introduction:

This section presents a broad overview of the parallel programming techniques implemented as part of this research. It is important to point out at this juncture that no prebuilt python package has been used as part of this research and all parallel programming schemes have been inherently implemented natively in CUDA C++ programming languages to make use of the performance associated with machine level programming to its fullest extent. Furthermore, coding using the native programming language associated with NVIDIA GPUs broadens the scope to use custom developed functions for register control, memory allocation and data transfer tailored for the problem of interest of this research.

6.2 CUDA Parallel Programming Model Basics:

The Monte Carlo techniques discussed earlier do not require discretizing the domain into a finite number of mesh elements. Moreover, as could be observed, the solution at every query point could be independently obtained without the need for the solution at neighboring point locations within the domain. Therefore, if the solution is required for a group of points situated in the simulation space the solution associated with each of these could be parallel carried out. This fact makes Monte Carlo techniques ideal to be simulated on General Purpose Graphical Processing units with several thousands of processing cores as compared to the limited number of cores on the CPU. CUDA (Compute Unified Device Architecture) is a relatively new programming model for NVIDIA compatible GPU devices, which fundamentally builds on top of traditional C++ programming models with GPU device specific commands for allocating memory. Although superior to CPUs in the
number of available processing cores, the ability of each core to execute complex
arithmetic and floating-point operations is limited. Therefore, adequate attention is
required to model the script around the underlying equations in a manner so as to
hide this limit. One commonly employed technique is to overload the GPU with
sufficient data to hide the underlying computational latency. Another major
bottleneck is the large overhead involved in transferring information from the CPU
to the GPU prior and post the simulation process. Therefore, a sufficiently large data
set that takes a relatively large time for processing in comparison to this transfer time
threshold is the only justification for running simulations on the GPU.

The solution of each query point in the domain is allocated to be executed by
one processing unit known as a thread. As per the article cited in [35] and the book
titled Introduction to Embedded Systems [36] “A thread is an execution context
which is all the information a CPU/GPU processor needs to execute a stream of
information”. A computer program becomes what is known as a process when it is
loaded from some store into the computer’s memory (RAM) and begins execution.
Such a process can be executed by a processing core. A process description contains
vital information such as program counter (current execution location in the
program), registers, etc. on the other hand is a bunch of resources allocated to execute
the stream of information. Threads are within the same process address space that
share much of this information which can execute segments of the program
independently. As an example, “When an operating system starts executing a process
to open a Webpage individual threads within this process are spawned to execute
sub tasks such as loading images, fonts, audio etc. which are a part of the main
Webpage process”. Figure 6-1 below shows the major difference in architecture
between the CPU and the GPU. A typical CPU consists of three primary units- A
control unit (CU) that extracts information from memory which it decodes and
executes; An arithmetic and logical unit (ALU) that handles the namesake operations
and Process registers (PR) that gives the ALU the needed data or operands and stores it. The figure below shows a single CPU processing execution core with four processing execution units (ALU+CU+PR). In contrast the figure to the right shows a single GPU multiprocessing core with several execution units. Due to the drastic increase in the number of execution units, the CACHE (faster more expensive local memory) is broken up to be easily accessed by a subset of the execution units. The global memory however remains the same between the two.

![Figure 6-1: CPU Vs GPU device architecture [25]](image)

Therefore, a GPU has thousands of cores whereas a CPU usually contains 2-8 cores. A typical CPU core can run two threads concurrently/parallel. Therefore a 4-core processor could run a maximum of 8 threads with independent tasks in parallel. A GPU core, however, can process a maximum of up to 4-10 threads. Consequently, 10s of thousands of threads could be simultaneously processed on the GPU. The number of threads that are recommended to be launched on the GPU is in the millions. This is because of the following reasons:

- On the CPU all threads need to be in a running state
- Context-switching/multiplexing is a common process enforced in both CPU and GPU to hide memory latency. As part of this process, the processing core
temporarily switches execution of a thread if non-processing tasks such as loading from memory is currently underway.

- A CPU is designed for task parallelism, i.e. performing different types of operations whereas a GPU is designed for data parallelism i.e. performing the same task multiple times.
- A thread in the GPU is therefore a mere instance of a pointer to a function that needs to be executed along with some unique constants.
- The main goal is to hide memory latency i.e. delay in time introduced due to loading data from memory to the core for processing.
- This task is achieved on the CPU by allocating more CACHEs of local memory/thread and a greater number of schedulers that smartly enforce context switching.
- On the GPU this task is achieved by using a greater number of threads per core. “This means that for the GPU to work efficiently a very large number of threads need to be scheduled than the actual number of threads that can be concurrently processed on each core”. This would allow the scheduler to schedule new threads whenever possible.
- Moreover, a significant overhead in data transfer results in movement of information between the main memory (RAM) and the memory for the GPU.
- In summary, the above points highlight pros and cons of using the GPU to process data and employing it for problems that justify and significantly outperform the cons involved.
- Monte Carlo techniques are therefore ideal candidates for GPU computing since they rely heavily on task parallelism to find the solution of a governing equation in millions of points in the solution space independently of each other.

To conclude this brief introduction to GPU computing the following figure 6-2 summarizes the workflow for a typical parallel computing algorithm. When a GPU
computing subroutine conventionally known as a *Kernel* is instantiated, data from the main memory (RAM) is transferred to the on-chip GPU memory. In tandem information pertaining to allocating resources such as cache, registers per processing thread is directly communicated to the GPU by the CPU via per kernel execution syntax which are customizable. The structure in which several thread bundles known as blocks are organized can also be customized based on the problem dealt with.

As mentioned earlier, thread organization is vital to efficiently achieve optimal concurrency. This fact is even more apparent if the problem calls for the launch of millions of threads on the GPU. Unlike thread execution on the CPU, groups of 32 threads known as *Warps* are processed together by each core on the GPU referred to as a Streaming Multi-Processor. In order to simplify resource allocation to these warps, groups of warps known as blocks are aggregated together. On the highest-level block groups are arranged as grids. These blocks and grids could either be two or three dimensional in nature based on the problem requirement. Table 3 previously summarized shows how many blocks can be concurrently processed per Streaming
Multiprocessor. This number varies from GPU to GPU. For the Tesla K20 it is 64 blocks. Figure 6-3 below shows this thread hierarchical organization and allocated memory resources associated with each organizational level. This illustration shows a 2D grid of 2D blocks. All threads within the same block share certain resources which threads from other blocks cannot access.

![Figure 6-3: GPU process structure and memory hierarchy][25]

### 6.3 Implementation Hardware Benchmark Study:

The following table 2 and 3 summarizes benchmark studies performed on the TESLA K20Xm GPGPU and the Intel Xenon CPU processor currently being used for this research. The bandwidth or the amount of data that can be transferred per second is clearly mentioned for comparison. The total number of threads that could be concurrently run is also summarized. It can be observed that data transfer rate between CPU and GPU is roughly about 5 times slower than data transfer rate between the CPU and RAM. The global memory available for use in the GPU is also about 20 times lesser than that available in the RAM.
Table 2: Utilized CPU Specifications

<table>
<thead>
<tr>
<th>Property</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Device name</td>
<td>Intel Xenon CPU E5-2690 v2</td>
</tr>
<tr>
<td>No of CPU physical cores</td>
<td>$2 \times 8 = 16$ (2 Octa Core CPUs)</td>
</tr>
<tr>
<td>No of CPU logical cores</td>
<td>$2 \times 8 = 16$ (Intel hyperthreading)</td>
</tr>
<tr>
<td><strong>Total no of cores</strong></td>
<td>32</td>
</tr>
<tr>
<td>No of Threads/CPU core</td>
<td>16</td>
</tr>
<tr>
<td>Max memory size</td>
<td>384 GB</td>
</tr>
<tr>
<td>Maximum no of memory channels</td>
<td>4</td>
</tr>
<tr>
<td>CPU maxed turbo frequency</td>
<td>3.80 GHz</td>
</tr>
<tr>
<td><strong>Available RAM Memory</strong></td>
<td>116 GB</td>
</tr>
<tr>
<td>RAM working frequency</td>
<td>933.2 MHz</td>
</tr>
<tr>
<td><strong>CPU-RAM bandwidth</strong></td>
<td>$933.2E06 \times 4 \times 64 = 29.86 \text{ GB/s}$</td>
</tr>
</tbody>
</table>

Table 3: Utilized GPU device Specifications.

<table>
<thead>
<tr>
<th>Property</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPU device name</td>
<td>Tesla K20Xm +</td>
</tr>
<tr>
<td>Architecture</td>
<td>Kepler</td>
</tr>
<tr>
<td><strong>Global device memory</strong></td>
<td>5.7 GB</td>
</tr>
<tr>
<td>Memory clock rate</td>
<td>2600 MHz</td>
</tr>
<tr>
<td>Memory bus width</td>
<td>384 bit</td>
</tr>
<tr>
<td><strong>Shared memory bandwidth</strong></td>
<td>249.6 GB/s</td>
</tr>
<tr>
<td><strong>Host (CPU) to Device (GPU) Bandwidth</strong></td>
<td>6.1 GB/s</td>
</tr>
<tr>
<td><strong>Device to Host Bandwidth</strong></td>
<td>6.5 GB/s</td>
</tr>
<tr>
<td><strong>Device to Device Bandwidth</strong></td>
<td>173.7 GB/s</td>
</tr>
<tr>
<td>No of Streaming Multiprocessors</td>
<td>14</td>
</tr>
<tr>
<td>CUDA cores/SMs</td>
<td>192</td>
</tr>
<tr>
<td>Warp size</td>
<td>32</td>
</tr>
<tr>
<td>Maximum no of threads/multiprocessor</td>
<td>2048</td>
</tr>
<tr>
<td>Maximum no of threads/ block</td>
<td>1024</td>
</tr>
<tr>
<td>Maximum thread block dimension</td>
<td>$1024 \times 1024 \times 64$</td>
</tr>
<tr>
<td>Maximum grid dimension</td>
<td>$2147483647 \times 65535 \times 65535$</td>
</tr>
</tbody>
</table>
6.4 Recently Developed Dynamic Parallelism for Monte Carlo Problems:

The current research aims to use the recent technique introduced in GPU computing enabling task/work trigger by the device (GPU). Previously, any task needed to be mandatorily triggered by the CPU which introduced significant data transfer bottlenecks. Following this innovation, a GPU thread can launch its own parallel child kernel based on problem parameters evaluated during its execution. Consequently, resource optimization could be improved significantly. For example, a concept known as adaptive mesh refinement in Finite Element Techniques could be quickly carried out on the GPU. This technique calls for discretization/dividing a domain into discrete elements for solving governing differential equations. With dynamic parallelism, based on certain criteria the number of discretized elements could be increased in regions of interest. Figure 6-4 below shows a sample of an adaptive meshing performed around an airfoil domain. Notice that regions closer to the airfoil, especially the trailing edge where fluid flow instabilities occur and call for finer meshing is performed on the GPU [37] [38]

Figure 6-4: Adaptive mesh refinement performed on the GPU
6.5 Application of GPU parallelization to current research:

For the current research, this technique could be harnessed to alter the number of random walks taken per solution data point based on heuristically available analytical solution in nearby data point location. This strategy is summarized as follows:

- Analytical solution at Monte Carlo data points inside looped coil with similar geometric dimensions as real helical coil is performed.
- Interpolated analytical solution at Monte Carlo data points in real helical coil obtained.
- A weight between 0-1 is assigned to each Monte Carlo point based on variance of solution with neighboring points within a sphere of influence.
- The main kernel is launched to compute solutions concurrently in all requested data points in real helical coil domain.
- Based on the pre-determined weight, child kernels are launched independently within each threads processing solution associated with a single data point
- The number of child kernels launched depends upon the weight.
- This method is predicated on the hypothesis that solution at a data point located close to points that do not vary much in theoretical solution converges quicker thereby allocating lesser resources in terms of walk number for this point.
- Points in “hot spots” where more eddy currents are induced due to proximity effects are allocated more resources as per this technique.
- Once all child threads associated with a walk number of a data point are completed, they are synced and averaged to find the converged solution.
- It is obvious from this technique that certain regions of the helical coil such as the region interconnecting two turns are inherently given higher weightage and therefore allocated more resources since the analytical solution in these regions
are not available. This means that the technique is capable of inherently handling such deficiencies.

The location of a particle in each step of the random walk with respect to the coil (inside or outside coil) is an important parameter to determine several important variables involved in the Monte Carlo formulation seen in the previous chapter. A conventional cylindrical point location algorithm as discussed in [27] would translate to scanning through each composite cylindrical segment comprising the helical coil and described by the parametric curve of the coil. This scan therefore corresponds to a temporal bottleneck of the order $O(n)$ where $n$ corresponds to total number of cylinders in the coil. Employing dynamic parallelism, the following procedure is carried out instead:

- Initial cylindrical segment location of Monte Carlo data point is recorded.
- A hop size which is a small fraction of the skin depth is carried out.
- Check if the travelling particle has exited coil domain, remained in same segment or transmitted to adjacent coil segments.
- If particle has exited coil domain, next hop size is equal to maximally sized sphere touching domain boundary.
- Now, the particle could have entered cylinder segments further away from the initial segment. The aim is to avoid the order $O(n)$ coil segment check. The distance between the current location and previous location inside the coil is first found and notes here as the variable $d$. Having access to the total coil length variable $L$, and the length of each cylindrical segment comprising the total coil $dl$, $N-k$ no of coil segments from either end are scanned instead. Where $k$ corresponds to the fraction of total domain radius and the variable $d$. In this way, in order to determine the location the total number of scans vary with each hop and therefore dynamic parallelism is employed.
6.6 Program Execution Time- Commercial FEM Software Vs CPU Vs GPU:

The following table briefly summarizes the performance comparison for different types of implementations of the research problem at question. It could be clearly observed that the performance gain in switching from CPU to GPU is incomparable and in the order of about 120 times execution time speedup. In evolving code usage within the CPU, python’s concurrent list module has been extensively employed for parallel execution of the monte Carlo code within the CPU without using the GPU. The performance of these cases are shown in rows 2 through 4 of table 4. This is possible due to recent advances made in hyperthreading for Intel CPUs that enable extension of a synchronous execution beyond the conventional hardware limited cores of the CPU.

<table>
<thead>
<tr>
<th>CPU Usage %</th>
<th>GPU Block Count &amp; Threads/Block</th>
<th>Total Code Runtime s</th>
<th>COMSOL Runtime s</th>
<th>Speed Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>14819</td>
<td>2996</td>
<td>0.2</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>1488</td>
<td>2996</td>
<td>2.0</td>
</tr>
<tr>
<td>50</td>
<td>-</td>
<td>965</td>
<td>2996</td>
<td>3.1</td>
</tr>
<tr>
<td>90</td>
<td>-</td>
<td>841</td>
<td>2996</td>
<td>3.5</td>
</tr>
<tr>
<td>-</td>
<td>7</td>
<td>1024</td>
<td>25</td>
<td>2996</td>
</tr>
</tbody>
</table>

It is important to point out the exact number of threads per GPU block and the total number of blocks used as part of the kernel execution given in table 4. These number have been arrived at based on literature survey of performance associated with the current Tesla GPU employed which peaks in its performance and data occupancy at these values which results in the most optimal performance overcoming any bottlenecks associated with data transfer.
6.7 Advantages of Employing Monte Carlo simulations on GPU:

This section is dedicated to the broad justification associated with pursuing the previously implemented Monte Carlo probabilistic technique over conventional finite element-based methods available in commercial numerical solvers. Concurrency achieved in the solution process enables rapid solution times. The following table summarizes the test duration for the Eddy Current Monte Carlo scheme. It is important to note that as the frequency increases the duration of run increases unlike conventional solvers. This is because the underlying formulation of the random walk inside the coil domain is predicated on taking hops of size restricted to a small fraction of the skin depth inside the copper coil. As previously discussed, this skin depth is a strong function of frequency. In the frequency range currently being analyzed i.e. between 50-2000 Hz and given the small coil radius of 1.3 mm this is not a major concern since skin effect starts to play a noticeable role only after 1800 Hz at this coil radius. The following simulations are performed for an AC drive current of 20 A. Although the current geometry used is a simple dual helical coil system for purpose of validation in this research the true potential of this method is in its application to complicated multiscale winding geometries with anisotropic materials. One such example is a stranded litz wire embedded in a ceramic insulation matrix or copper in neodymium winding configuration used in superconductors. These kinds of geometries cannot be practically simulated in commercial software.
6.8 Conclusion:

In summary the use of the CUDA programming model suggested previously improves the performance associated with the Monte Carlo model by about 120 times in comparison to conventional finite element schemes used in commercial softwares such as COMSOL. At this juncture it is important to point out that the Monte Carlo simulations have been performed in an isolated Tesla GPU. The concurrent performance of the parallel code has been extensively improved by carefully considering and removing time consuming process associated with having too many conditional or cumulative summations within each parallel kernel execution for optimal run of the code. In theory the use of multi-GPU programming is further predicted to improve the performance of the code used. Although the author was privy to the availability of multiple GPUs it was concluded that multi-GPU execution was to be extended as part of future effort since it required significant alteration on the hardware end of the system architecture requiring the use of specialized SLI bus architecture in replacement of the tradition PCIe architecture for GPU connection with the motherboard. If such a model is to be developed in the future, portions of the coil current density simulation could be split between different GPUs and performance could be significantly improved as a result.
Chapter 7: Final Remarks and Future Work

Commercially available software such as COMSOL have a robust and well-developed library of material properties. In consequence, the temperature dependence of coil resistivity (important at high signal frequency when joule heating losses are high) can be enabled for more accurate results. Considering temperature dependence on resistivity, the COMSOL predicted coil power loss increases by 4.6%, and maximum coil current density decreases by -1.7% at all frequencies.

Some of the **advantages** of using the proposed **Monte Carlo method are** listed below:

- Monte Carlo techniques facilitate the analysis of complex coil geometry.
- The Monte Carlo computation was completed in **841s** utilizing 80% of all **CPU** resources. The same simulation in **COMSOL** takes **2996s** using **110 GB of RAM**.
- The same simulation, performed on the GPU, was completed in **25s** i.e. 35 times faster than Monte Carlo in CPU and 120 times faster than COMSOL. This illustrates the unequivocal performance advantage obtained by employing GPU computing.
- This study was performed for a simple 17 turn, 1 layered primary + pickup coil configuration with 1.5 million Monte Carlo data points. The performance advantage using GPU based computation grows exponentially with the number of data points and coil complexity.
- The Monte Carlo method estimates DC power losses and eddy current power losses independently, by separately summing successive higher order current-field effects, enabling a clear understanding of the individual contributions of DC and AC losses.
- Based on the significant advantage in computational performance, Monte Carlo based techniques could be used to simulate millions of machine configurations as part of design optimization studies, which would be impossible using available finite element tools.
• By altering the data point distribution type within a coil geometry (uniform to exponential) highly accurate results that capture skin effects could be obtained. As next development step, evaluation of the induced eddy current density in an iron core inside an electromagnetic coil system will be developed. Magnetization and hysteresis effects could be further added to the calculation to enhance the accuracy of the estimation of power losses in electrical machines.
References


