12-2023

Design Space Visualization And Exploration For Many Goal Problems Under Uncertainity

Niharika Balaji
Florida Institute of Technology, nbalaji2022@my.fit.edu

DESIGN SPACE VISUALIZATION AND EXPLORATION FOR MANY GOAL PROBLEMS UNDER UNCERTAINTY

by

Niharika Balaji

A thesis submitted to the Department of Mechanical and Civil Engineering of
Florida Institute of Technology
in partial fulfillment of the requirements
for the degree of

Master of Science
in
Mechanical Engineering

Melbourne, Florida
December, 2023

We the undersigned committee hereby approve the attached thesis,
"DESIGN SPACE VISUALIZATION AND EXPLORATION FOR MANY GOAL PROBLEMS UNDER UNCERTAINTY."
by
Niharika Balaji

Anand Balu Nellippallil, Ph.D.
Assistant Professor
Mechanical and Civil Engineering
Major Advisor

Luis Daniel Otero, Ph.D.
Associate Professor
Mathematics and Systems Engineering

Chiradeep Sen, Ph.D.
Associate Professor
Mechanical and Civil Engineering

Ashok Pandit, Ph.D., P.E.
Professor and Department Head
Mechanical and Civil Engineering

Abstract

Title: DESIGN SPACE VISUALIZATION AND EXPLORATION FOR MANY GOAL PROBLEMS UNDER UNCERTAINTY

Author: Niharika Balaji

Advisor: Anand Balu Nellippallil, Ph.D.

Designing a complex engineered system is challenging due to many conflicting goals, uncertainties, and multiple interactions. Traditional optimization approaches often yield single-point solutions, which may not be suitable for early design stages due to their susceptibility to changes in conditions and uncertainties. To address this challenge, a satisficing approach is employed. This approach enables designers to effectively navigate the design space and identify satisficing solutions that balance conflicting goals in the face of uncertainties and changes in conditions. From a systems design perspective, we view design as an iterative process that involves making informed decisions based on available information and supported by simulations. The Decision-Based Design (DBD) paradigm is the foundation for design methodology in this thesis, empowering designers to navigate the complex design landscape by making informed decisions grounded in available information. In this thesis, the DBD technique called compromise Decision Support Problem Technique (cDSP) is employed to address the issue of many (more than three) goals and uncertainty in the system.

Model-based complex system design involves one crucial step: exploring and visualizing the solution space. This procedure provides insightful information about the system's behavior, enabling designers to make informed decisions. Accurately predicting future states is

difficult because of the intrinsic constraints of models and the inherent uncertainty in search methods and solvers. By exploring the solution space and displaying its complexities, designers find satisficing solutions relatively insensitive to uncertainties.

In this thesis, a Decision-based Design framework is proposed. The novelty of this framework is that it integrates the compromise Decision Support Problem (cDSP) technique accounting for many conflicting goals, incorporated with robust design metrics to address the issue of uncertainty with a machine-learning-based visualization technique called interpretable Self-Organizing Maps (iSOM) to visualize and explore the solution space for many goals effectively. The efficacy of this framework is validated, considering vehicular crashworthiness problems as an example.

Once the DBD framework has identified feasible solutions, selecting a standard satisficing solution proves crucial for understanding the system's behavior and performance. This thesis presents a systematic approach for identifying common satisficing solutions from the visualized plots generated by the DBD framework. The effectiveness of this approach is demonstrated through the design of a composite beam as an illustrative example.

Table of Contents

List of Figures

List of Tables

Acknowledgement

I want to express my deepest gratitude to my thesis advisor, Dr. Anand Balu Nellipallil, for his unwavering support, guidance, and encouragement throughout the research process. His expertise, patience, and steadfast belief in my abilities have been instrumental in shaping this thesis. I am particularly grateful for his willingness to share his time and knowledge, even during challenging moments.

I thank Professors Janet K. Allen and Farrokh Mistree of the Systems Realization Laboratory at the University of Oklahoma for their guidance and support. I thank all the members of the System Realization Laboratory at Florida Institute of Technology for their support and intellectual stimulation, especially Mathew Baby, for guiding me and supporting me throughout my Master's. Our discussions and shared experiences have greatly enriched my graduate experience. I also thank Dilshad Digonta and Akshay Guptan for their support.

I thank Gehendra Sharma of the Center for Advanced Vehicular Systems at Mississippi State University for helping me formulate the composite design example problem for my thesis. I thank Professor Palaniappan Ramu and Rashmi Rama Sushil from the Indian Institute of Technology Madras for their help and collaboration with the Interpretable Self-Organizing Maps (iSOM) construct.

I would also like to thank my thesis committee members, Dr. Chiradeep Sen and Dr. Luis Daniel Otero, for their insightful comments and feedback on my research. Their expertise and critical evaluation have significantly improved the quality of this work.

Finally, I would like to express my heartfelt gratitude to my family and friends for their unwavering love and support. I am grateful to my mother, Asha Venkatesh, my father, Balaji Malagal Dasashetty, as well as my grandparents, Venkatesh and Vijayalakshmi and my uncle, Shiva Kumar V who has served as a great mentor and supporter to me and I thank my sister Nidhi Balaji for her love and care. Their understanding and encouragement have been a constant strength throughout my academic journey.

Chapter 1 : Realization of Complex Engineered Systems - Navigating Solution Space through Visualization and Exploration

This chapter lays the foundation for the thesis, starting with Section 1.1, discussing motivation for complex systems. In Section 1.2, the need for realization of complex systems is discussed. Section 1.3 explains the challenges in the realization of a complex system. Section 1.4 and Section 1.5 discuss the background of Decision-based Design (DBD) and solution space exploration, respectively. A detailed discussion of research gaps and questions is in Section 1.6. Finally, Chapter 1 is concluded by discussing the validation square and outline of the thesis in Section 1.7.

1.1 Motivation: Realization of Complex Systems

In contemporary times, engineering problems are predominantly complex in nature. Complex systems are those that depend on several system components or features, with each component or feature having a separate set of priorities and goals. The fact that there are many (more than three) goals that conflict with one another, multiple ways in which various components interact, and the existence of uncertainties make managing these systems a challenging task. Interconnectivity, evolving behavior of the system, robustness, and many conflicting goals are all characteristics of the complex systems. The ability of one component or feature in a system to have an impact on other components or features is known as interconnectivity and, the interactions inside the system cause the system's dynamic behavior which also impacts the performance of the system and properties of individual components, these interactions are non-linear in nature i.e., any small change in a component or feature may result in huge behavioral changes of the system. In a complex system, variations or uncertainties are always present, arising from various factors. Additionally, these systems often involve many conflicting goals, posing a challenge for designers to identify feasible solutions. For effective solutions to be developed, it is crucial to comprehend and acknowledge the need for the realization of complex systems and their challenges.

1.2 Need for Realization of Complex Systems

In designing engineered complex systems, we come up with mathematical modeling that is used to model the physical systems. To make the models manageable designers must simplify and approximate the model. This simplification may include altering certain details, assuming the system to be under ideal conditions, or using linear approximations for non-linear methods. This simplification becomes necessary as we cannot capture every detail of a complex system. In some cases, understanding of the complete physical system plays a vital role in developing the mathematical solvers used to simulate and analyze the models. Typically, models are imperfect but provide means of understanding, predicting, and making decisions about physical systems. The model must capture the key characteristics of the system, and it should be able to make predictions or offer insights that aid in decision-making while still being reasonably accurate. A British mathematician and professor of statistics George Box states that "*Essentially, all models are wrong, but some are useful*" (Box and Draper 1987). In recent times there has been an increase in the use of model-based complex system design due to growing accessibility and diversity, customization, efficiency, complexity of modern systems, adaptability, rapid prototyping and iterative methods, and cost efficiency. To further explore model-based complex system design, it is first required to clarify what a system is, Blanchard and co-authors define a complex system as a " Combination of components that act together to perform a function not possible with any individual parts" (Blanchard and Stiglitz 1992). According to Shupe and co-authors complex system can be defined as " Grouping of associated entities characterized by a mental construct" (Shupe, Muster and co-authors). Bloebaum and co-authors define the complex system as "Tightly coupled interacting phenomena yield a collective behavior that cannot be derived by simple summation of the behavior of the parts" (Bloebaum and McGowan 2012). Some of the characteristics of complex system are:

- Complex systems have different components interacting with each other.
- Complex system consists of uncertainties.
- They have many conflicting goals.

Once the complex system is understood the next step is to understand the model-based design. A model-based design is a virtual representation of physical systems that help in development of new systems, implementation, and analysis, prototyping and validating a system. Some of the principles of the model-based design are:

- Use of mathematical constructs to model physical complex systems.
- Use of computer models for testing and implementing the models.
- Evaluation and analyzation of the models.

The modelling of physical complex system or real world is shown in Figure 1.2. In a model-based design the information available from a physical system is limited, every aspect of the physical system cannot be captured. Therefore, the mathematical models developed are the approximations of actual systems. In addition to knowledge limitation mathematical models also must deal with the solver limitations which results in the manifestation of uncertainty in the system. To solve this problem, the decision-making process needs to incorporate a lot of analysis, review, and interpretation. This is necessary in order for the end user to comprehend the outcome and use it to accomplish their desired goals and develop a structured methods (Miñón, Paternò and co-authors 2016).

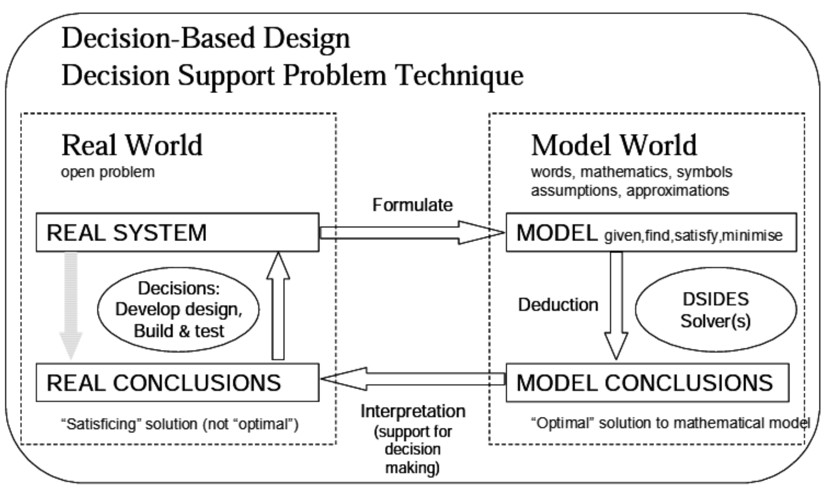


Figure 1.1: Representation of modelling of Physical systems (Xie, Peacock and co-authors 2002)

Advantages of the model-based design are mentioned below:

- It is helpful in the early stages of the design by reducing the prototyping cost and identifying errors. It also provides designers with information that helps to explore and analyze the system completely.
- Facilitating a common design environment that provides data documentation, analysis, and visualization, model verification and interaction between the variables in the system.
- The design can be reused to upgrade and modify to expand the capabilities.

In conclusion, realization of complex engineered systems is crucial for understanding the world around us and for obtaining new insights, making predictions, developing, and optimizing systems, and managing complexity. It is essential in decision-making, risk assessment, risk management, and a variety of applications in many different fields of study. Next, in Section 1.3 the challenges faced in realization of complex systems are discussed.

1.3 Challenges in Complex Systems

The key challenges and characteristics of a complex system are provided by Mexiner and co-authors (Meixner, Paternò and co-authors 2011), Ruiz and co-authors (Ruiz, Serral and co-authors 2019), Some of the challenges faced by the designers in a complex system are mentioned below :

- To understand the behavior of a complex system we require intricate simulations and mathematical models, it becomes challenging to develop efficient models to account the behavior of the complex system.
- Identifying the type of uncertainties in the complex system and coming up with a robust solution that satisfies the design requirements.
- Understanding the interactions within the system, which presents a challenge to designers, is important to comprehend the emergent behavior of the complex system.
- It becomes challenging for the designers to identify feasible solutions that achieve an appropriate balance between many conflicting goals.

- Exploring the design space and visualizing feasible solutions in complex systems becomes challenging due to the multitude of dimensions involved.

Considering the complexity of the engineering problems it becomes challenging for the designer to come up with effective solutions. **Hence, the principal goal of this thesis is to establish a Decision-Based Design (DBD) framework for complex engineered systems with many conflicting goals, addressing the uncertainty in the system and use an effective machine learning based visualization technique to explore and visualize the solution space**. The key characteristics of complex system are it consists of many conflicting goals (more than three), there are multiple interactions among variables and uncertainties, or variations are present, and these characteristics are shown in Figure 1.2. In the next Section 1.4 the foundation about the Decision Based Design (DBD) is explained in detail.

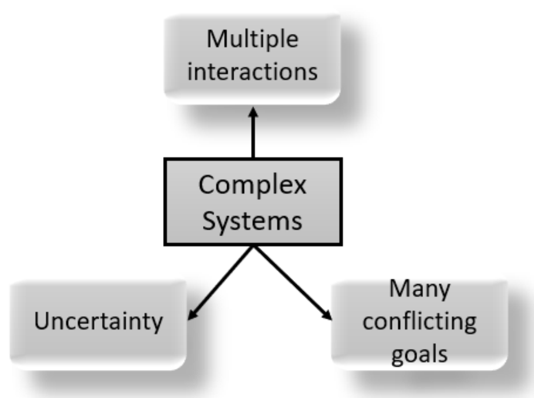


Figure 1.2: Complex system features

1.4 Background for Decision Based Design (DBD)

The work of Herbert Simon (Simon 1969) serves as a foundation for Decision Based Design. A human designer's primary responsibility is to make decisions, hence the DBD design paradigm was created to help establish design methodologies to support human designers. In decision making process the information is converted into knowledge and some of the characteristics of the design decisions are discussed below (Mistree, Smith and co-authors 1990).

- Design decisions are governed by multiple measures of merit and performance.
- Decisions in design involve information coming from different sources and disciplines.
- Decisions in design are invariably multidimensional and multileveled in nature.
- All the information needed to make decisions may not be available.
- Some of the information required to decide may be hard, that is based on scientific principles and some of the information may be soft, that is, based on the designer's judgement and experience.
- The problems for which design decisions are being made are invariably loosely defined and open and are characterized by the lack of singular, unique solution. The decisions are less than optimal, which represent satisficing solutions.

Bras B A (Mistree, Smith and co-authors 1990) developed a Decision based design equation shown in equation 1.1 and Figure 1.2, where the I represents the information , T represents the transformation matrix and K represents knowledge

$$K = T(I) \qquad \textbf{Equation 1.1}$$

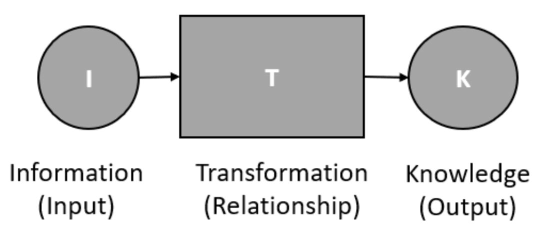


Figure 1.3: Design equation (Brass 1993)

Based on the design equation a meta equation is developed as an approximation shown in equation 1.2 and the equation converts the design parameters into functional requirements, the T function in equation 1.2 satisfies multiple decision support problems. Therefore, the Decision support problems represent the practical application of the design equations within the Decision based design framework. Figure 1.2

In the Decision support problem technique, one fundamental aspect is the recognition that the primary responsibility of an engineer during the design of an artifact is to engage in decision making (Muster and Mistree 1988). One way to apply decision-based design is through the Decision Support Problem (DSP) Technique, which is described in more detail in the next section.

1.4.1 Decision Support Problem (DSP) Technique

DSP technique is one of the ways that DBD is carried out. Muster and Mistree (Muster and Mistree 1988) affirm that the DSP technique helps human designers in employing human reasoning to make logical decisions. The designers are primarily concerned with two tasks: analyzing symbols and arriving at decisions and for this they need an approach or method to come up with satisficing solutions rather than the optimal solutions because the systems are complex and have uncertainties in them. The DSP technique helps designers summarize and formulate complex problems so that they can be solved satisfactorily while remaining near the real system without removing its sources of uncertainty. DSP technique consists of two phases: (i) Meta design and (ii) Design phase. In meta design phase the decision support problem is structured and separated into basic DSPs, the main goal of this phase is to develop the design process to be implemented. The actual DSP problem is solved in the design phase and analysis of the solution is carried out. Design-related decisions can be modeled using decision-support problems, and the resulting domain-specific mathematical models are known as templates or decision-support problem templates. All the decisions obtained in the DSP technique are classified as selection or compromise and these decisions are considered primary and derived decisions respectively (Mistree and Bras).

Selection DSP - Allows the designer to choose from a variety of possible factors to be considered. The focus in the selection process is accepting specific alternatives while rejecting others based on various measures of merit, which are referred to as attributes, which represent the functional requirements.

Compromise DSP – Allows the designer to identify the right combination of design variables to obtain the best satisficing system design considering constraints and many goals.

The compromise DSP is discussed in greater detail in Chapter 2, and this is the foundational DSP construct used in this thesis. In the next section foundational philosophy used in this thesis is explained in detail.

Derived DSP – Allows the designers to use combination of both primary DSPs to model the complex system decision like selection/selection, compromise/compromise, selection/compromise (Karandikar and Mistree 1993, Simpson, Rosen and co-authors 1996, Sharma, Allen and co-authors 2019)

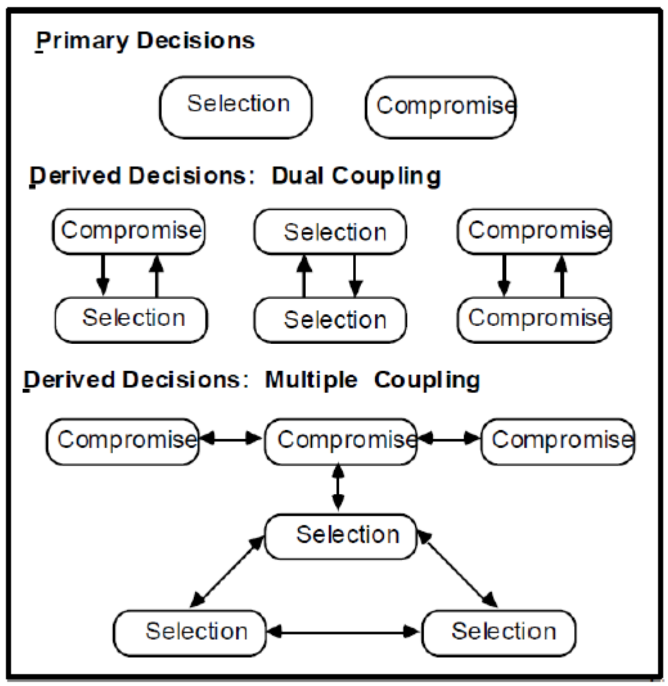


Figure 1.4: Primary and derived decisions (Mistree and Allen 1997)

1.5 Satisficing Philosophy versus Optimizing

In modeling of decisions there are two types of reasoning i) Optimizing philosophy and ii) Satisficing philosophy. Our research foundation is anchored with rationality proposed by Herbert A, Simon (Simon 1969) given all models are inaccurate, incomplete, and not of equal fidelity. We consider a "**Satisficing solution**" for exploring the solution space for design problem. A satisficing solution is the one that satisfy and suffice the requirements of the designers for a conflicting goals problem. For a complex problem the characteristics and information are mentioned below (McDowell, Panchal and co-authors 2009):

- Design problems could be loosely defined and open.
- Design information may be quantifiable, and some may be qualitative.
- There will be multifunctional requirements in design, and they are governed by multiple measures of merits and performances.
- All information required for design may not be available and thus the designer may have to work with incomplete, inaccurate and infidel models and information.

From the characteristics of a complex system the solutions obtained are less optimal and sought satisficing solutions. A complex problem can be formulated in two ways: i) Solving the complex problem in approximate approach and ii) Solving an approximation of complex problem in an exact way.

In the first method, designer obtains optimal solutions based on simple models, the optimal solutions found are approximation and not optimal in reality. Whereas in the second method an algorithm that provide solution close to reality is used rather than simple models. The solutions obtained by using approximate models are closer to reality and satisficing.

Sequential linear programming (SLIP) is used in solving cDSP formulations has it consists of single algorithm for solving a set of DSP's in engineering design (Mistree, Hughes and co-authors 1993). The sequential linear programming (SLIP) and multilevel version (SLIPML) is altered resulting an algorithm called adaptive linear programming (ALP). This algorithm is for multilevel, multigoal features which is accounted in decision support on the design of engineering systems (DSIDES) (Mistree and Kamal 1985, Reddy, Smith and co-authors 1996). The key characteristics of Adaptive linear programming are (Mistree, Hughes and co-authors 1993):

- During linearization second order terms are used.

- The constraints and goals are normalized and are transformed into well-behaved convex functions in the region of interest.
- A constraint suppression and accumulation technique that is intelligent.

Understanding the fundamental philosophy of this thesis allows us to go on to the following section, which discusses background on solution space exploration.

1.6 Background: Solution Space Exploration and Visualization

In a model-based complex system design the key step is to explore and visualize the solution space that provides the information and insight about the system to the designer's that facilitate them in decision making. As the physical world cannot be represented completely and accurately it is difficult to predict the futuristic state, even the different search algorithm used are inaccurate and solvers used also exhibit the uncertainties in the results. By exploring and visualizing the solution space, a designer is able to find robust solutions, which are relatively insensitive to inaccuracies in the system (Triantaphyllou and Sánchez 1997). To go deeper into the solution space exploration instance, the concept of multi-objective formulation in model-based design is discussed. A multi goal formulation involves conflicting goals that are assessed based on different performance criteria, designers have the option to employ, the satisficing approach to address these conflicts (Simon 1969). On the contrary, when employing a single goal or multi goal formulation, disputes over the solution are minimized as it relies exclusively on the pre-determined criteria for making the choice. Consequently, the inclusion of multiple objectives elevates the complexity involved in the decision-making process. There exists different type of visualization techniques, but the question is: *Is there a visualization technique that can be utilized to visualize and explore the solution space for complex problem with many goals and how to choose the appropriate visualization technique?* It is also important to understand that hear of decision-making is the human designer leverages these process and methodologies to determine the optimal variable settings, design parameters and understand the system in more detail. In general sense, the evolution of design process and methodologies provides tools for focusing attention and improving human judgment to enable well-informed, knowledge driven decision making. Computers and methodologies possess the capacity to enhance designer's capabilities and they are commonly used to develop complex systems, for example designing aircraft. The significance of designers is increasingly acknowledged, where design methodologies play a vital role that enhances the design process and promotes concurrent work. Solution space needs to be visualized and explored to understand the interactions between the variables, the effect of each goal on one another and identify the satisficing solutions. Visualization techniques streamline the decision-making process for designers by providing straightforward means of interpreting solutions for conflicting goals problems. There is a need for developing techniques that can be utilized to visualize and explore the solution space that approximates complex systems, which is typically incomplete, inaccurate, and not of equal fidelity. Many visualization methods primarily used are limited to two-dimensional visualization leveraging the graphical optimization to facilitate intuitive thinking for designer's (Nagar, Pannerselvam and co-authors 2022). However, it becomes challenging when a designer must visualize complex problems that involve a high number of dimensions. Hence there is a need to come up with new approaches where solution space can be visualized and explored for more than three dimensions and help designers make decisions on various factors such as design variables, relation between these variables and Region of Interest (RoI). Chapter 2 provides a more detailed discussion about existing visualization techniques and their limitations. Moreover, it delves into the visualization technique employed in this thesis, highlighting how it successfully surmounts the limitation encountered in existing methods. In the net section research questions and research gaps are discussed.

1.7 Identifying Gaps and Research Questions

After discussing design choices for engineered systems, it is necessary to develop a framework that may help designers deal with the problem of uncertainty for robust performance of complex systems. Besides robust performance of the complex systems we need effective solution space visualization to interpret the data easily. There are some challenges associated with developing framework for complex system mentioned below,

- Defining and understanding the type of uncertainty in a complex system is a fundamental challenge.
- Choosing appropriate methods to quantify the uncertainties.
- Complex systems consist of multiple interactions, and it becomes challenging to analyze the effect of change in variables affecting the overall performance of the system.
- Identifying appropriate data visualization techniques that facilitate the designers to interpret the solutions easily.

In the context of these challenges, the focus in this thesis is to establish research questions and address the research gaps required for complex systems under uncertain conditions.

Research Gaps

Gap 1: Address many conflicting goals and understand their interrelations under uncertainty.

Gap 2: To effectively visualize and explore robust solutions for problems involving many conflicting goals.

Gap 3: Identifying satisficing solutions for many (more than three) conflicting goals.

The main objective of this thesis is to lay the research foundations necessary for the complex engineered system design for many goals problem in the presence of uncertainties in design variables and visualization and exploration of solution space. Such systems require the design of information from multiple areas and the incorporation of design, production, and material expertise and experience. The following research topic for this thesis is thus prompted by the necessity of having systematic techniques in expressing such data and how they interact with each other and visualizing the data effectively, which results in the following research questions:

Research Question (RQ1): ***What are the mathematical and computational foundations necessary for the formulation, visualization, and exploration of problems involving many (more than three) conflicting goals?***

In decision making modelling plays an important role, designers frequently choose to develop models and simulate various complex systems instead of investing in costly prototyping and experimental work. Therefore, the approach used to model the decisions in complex system should enhance the decision-making process by providing a cost-effective and efficient means of gathering insights and improving human judgement.

A multi-objective problem is solved with an optimization framework that minimizes the variation in complexity allocation and maximizing degree of modularity for train bogie (Sinha and Suh 2018). A pareto analysis method is used in food sector to understand the effect of critical factors in this system (Fotopoulos, Kafetzopoulos and co-authors 2011). Adaptive systems like learning systems consisting of conflicting objectives, to solve this problem concept of pareto optimality combined with machine learning technique (Jin, Gruna and co-authors 2009). A decision choice model (DCM) integrated with mixed integer linear programming (MILP) is used to solve a profit maximization problem of a parking service operator and the proposed methodology can be altered to supply related decisions to the users. Regression trees and gradient boosted regression trees are used in aviation domain to understand the turbulence forecasting at different levels of altitude. A framework is proposed which diagnose a car problem and a multifunction with the help of Bayesian approach and this is also validated with experimental results (Budiharto 2013). Topology optimization is used in designing of efficient car bodies for different design stages (Leiva 2011). In the context of improving car crashworthiness, a multi-objective task is being tackled by using radial basis functions and genetic algorithms to create composite absorbers (Lanzi, Castelletti and co-authors 2004). Parrish and co-authors present a technique that combines two different fidelity models, the incremental step solver and the one-step solver to act as a correction function in an algorithm for artificial bee colonies. they used a sheet metal forming example to demonstrate this method (Parrish, Rais-Rohani and co-authors 2012). To get around the limitations of conventional techniques, vehicle occupant restraint systems are designed using a hybrid approach that combines the non-dominated sorting genetic algorithm (NSGA II) with the Kriging model (Gu, Sun and co-authors 2013). In order to construct thin-walled energy absorption tubes for crashworthiness applications, Acar and co-authors provides a comparative examination of polynomial response surfaces, radial basis, and Kriging models (Acar, Guler and co-authors 2011).

The approaches discussed in the above paragraph are different optimization techniques used to solve multi-objective complex problems. The optimization techniques entail computationally intensive iterative procedures but are intended to help designers locate single point solutions. As a result, these methods might not be appropriate for the preliminary phases of design exploration. In these early phases, the focus of designers is to quickly find a wide variety of solutions that meet their needs. Creating a wide range of satisficing solutions is more important in the early stages of design exploration than obtaining a single, targeted solution. Higher levels of uncertainty and flexibility characterize this phase, as designers try to comprehend the trade-offs and find potential routes by exploring different design alternatives. For this early, more experimental stage of the design process, traditional optimization-based techniques which require many iterations and substantial computer simulations are frequently time consuming and resource-intensive. Therefore, to address this research gap, the concept of satisficing solutions(Simon 1996) or the solutions that are good enough is used considering a compromise Decision Support Problem Technique in thesis.

Robustness is a basic goal in complex systems that has become more and more important in the changing world. Complex systems have multiple interconnected components and interactions, often exhibiting emergent behaviors that are not predictable through the analysis of individual parts and they have many conflicting goals. In such complex systems, achieving robustness implies the capacity to withstand disturbances, adapt to changing conditions and continue functioning effectively under a variation.

A comparison between multi-objective optimization with single-objective optimization from a Pareto perspective is presented by Hou and co-authors (Hou, Li and co-authors 2009). Using the Kriging technique, a multi-objective genetic algorithm (GA) is utilized to create crash-worthy cars using foam-filled bitubal structures (Zhang, Sun and co-authors 2012). For topological optimization of the front rail structure of a vehicle, a gradient approach is employed (Soto 2004). A simulation-based design framework for vehicle crashworthiness is presented, taking into account an effective global optimization method (Hamza and Shalaby 2014). A comparative analysis of four meta-modeling strategies is conducted by Jin and coauthors (Jin, Chen and co-authors 2001) and their application to diverse objective design challenges is discussed. In their comparison of the deterministic and reliability-based approaches to vehicle crashworthiness optimization under multiple impact crashes, Gu and co-authors (Gu, Dai and co-authors 2017) take sampling strategies and reliability analysis into account. A reliability-based optimization technique is described by Youn and co-authors (Youn, Choi and co-authors 2004) to take into consideration the degree of uncertainty in the process of designing cars for side crashes. In order to address a parametric uncertainty in the design of foam-filled thin-walled structures, Sun and co-authors (Soto 2004) employed a resilient design approach. In reference to other approaches, a review of the many kinds of uncertainty and the usefulness of robust design approaches is provided by Aspenberg and co-authors (Aspenberg, Jergeus and co-authors 2013). To account for uncertainty in design variables and enhance the effectiveness of probabilistic design for a vehicle crashworthiness example during a side collision, a sequential optimization and reliability evaluation technique is applied (Du and Chen 2004).

Some of the approaches discussed are deterministic approaches used in design and optimization of complex systems across various domains. These approaches are helpful in finding the best possible solution specific to input parameters, objectives, and constraints. However, the limitation of these approaches is their incapability to account for uncertainties that can affect the design process of a complex system. To enhance the reliability and efficiency of complex systems, it is essential to incorporate probabilistic and robust approaches. These methods prepare complex systems to perform reliably and meet their objectives under a wide range of real, uncertain conditions.

The robust optimization approaches discussed above address the issue of uncertainty in complex systems they work by optimizing system performance under various conditions. These approaches are useful when dealing with two or three specific goals, allowing designers to strike a balance between them to ensure robustness. In real world applications, complex systems often have a multitude of goals that go beyond the scope of two or three. These goals can be various aspects like performance, cost, safety, efficiency and more, satisfying all the goals is a formidable challenge. To address this challenge the designers, need to identify trade-off solutions that can balance many conflicting goals. In essence, they must find a compromise solution that satisfies as many of these goals as possible while acknowledging the prioritization on one another. To address this research gap, a framework is proposed in this thesis.

The research question (RQ1) is supported by following hypothesis (H1):

- ***Problems involving many conflicting goals under uncertainty could be effectively formulated, visualized, and explored from a decision-based design perspective using compromise decision support problem construct, robust design metrics and an effective machine learning-based solution space visualization and exploration technique.***

The mathematical models do not represent the physical system exactly, but the important features of the system are captured and give the brief information about the system to the designer for decision making, the compromise Decision Support Problem is used in this thesis to model the decisions of a complex system, which is discussed in more detail in Chapter 2 and the example problem are discussed in Chapter 4 and 5.

The exploration of robustness in complex systems delves into the intricacies of developing resilience and adaptability into complex systems that are increasingly unpredictable. In this thesis the issue of uncertainty in the design variables of a complex system is addressed by proposing a framework and using a mathematical construct called Design Capability Index (DCI) discussed in Chapter 3 and examples are discussed in Chapter 4 and 5.

Research Question (RQ2): ***How can designers effectively interpret and select satisficing solutions for many (more than three) goals?***

Visualization of solution space is vital in decision making process for complex engineered systems with many conflicting goals. It facilitates the designer's insights and prioritize the goals and helps them to choose the satisficing solutions that algin's with design requirements. Effective visualization techniques enhance the decision-making process by making the tradeoffs and interactions between the goals more understandable.

Some of the visualization techniques used in various domains are discussed below. A ternary plot is a triangular graph consisting of three axes, each axes representing one component or feature of a system and points on the plot represent the relative proportions or percentages of these components. A ternary plot is used to in visualization of earthquake determination problem, to understand the thrust, normal and strike-slip motion (Frohlich 1992). For understanding the concentration gradient, inequality constraints in designing of Yangtze river delta port a ternary plot is used (Feng, Grifoll and co-authors 2020). To understand the complex network of hardware and software in vehicle a data loggers is used and to interpret the data resulting from the tests parallel axis plots are used (Theissler, Ulmer and co-authors 2010). Parallel axis plots are data visualization technique to visualize high dimensional data into two-dimensional data (Zhou, Yuan and co-authors 2008). To understand the weather prediction various climate simulations are carried out and to interpret this easily nested axis plots are used (Wang, Liu and co-authors 2016).

There are many visualization techniques used to visualize and explore solution spaces. Each of these techniques has its own set of limitations and it's important to understand these limitations while appreciating the significance of efficient visualization in problem-solving and decision-making. One common limitation is to deal with high-dimensional data, as the number of dimensions increases, it becomes challenging to visualize the data effectively. Conventional two dimension or three-dimension visualizations that are not suitable for spaces with many dimensions. As the dimensionality increases interpreting the information becomes challenging, especially when complex system involves multiple interaction between the variables. Hence there is a need for efficient visualization technique that provides intuitive to understand complex data and solution spaces. They enable the discovery of patterns and trends that might not be apparent through other techniques. Effective visualization techniques allow designers to aid decision making by presenting information in a format that is easier for humans to comprehend. And helps decision makers to grasp complex problems, compare options and make informed choices. In decision-based design domain, visualizing solution spaces helps in identifying the satisficing solutions by evaluating the trade-off and choosing the suitable solutions.

To address this research gap in exploring and visualizing the solution space, a machine learning based visualization technique is used. This technique is discussed in more detail in Chapter 2.

The research question (RQ2) is supported by following hypothesis (H2):

- ***It is hypothesized that this could be addressed by proposing a systematic approach to evaluate and identify common satisficing design scenarios for many goals through solutions space visualization, and exploration.***

Visualization techniques are very helpful for understanding complex solution spaces. It's important to be aware of their limitations. Choosing the correct visualization method for a given problem and understanding how to interpret the visual representations are critical. Efficient visualization is essential for making informed decisions, solving complex problems, and communicating effectively. Chapter 6 discusses the machine learning based visualization techniques used to visualize the solutions for the two-example problem considered.

1.8 Verification and Validation of the Thesis Chapters

In engineering research formal and quantitative methods are used for validation which involves logical thinking and deduction which is a process of concluding from standard principles. The conventional formal validation methods are not suitable for all design methods that involve subjective elements. Whereas the validation square strategy validates the design method by considering the knowledge validity depending on context and dependent on the intended purpose rather than considering standard principles.

Pederson and co-authors (Seepersad, Pedersen and co-authors 2006)introduced the validation square framework which is used in this thesis for implementing the verification and validation strategy. This framework examines whether a design method is successful and efficient in its work, and whether the final solutions operate well in real-world situations (efficiency and operational performance). The effectiveness and efficiency of the design method with respect to operational performance are two important factors that determine the design method's overall usefulness inside the framework. The validation square construct to validate design methods is shown in Figure 1.4

The process of increasing trust in the usefulness of a purpose is represented by the Validation Square in Figure 1.4. Philosophically speaking, verification deals with the justification of knowledge assertions, whereas validation pertains to internal consistency. But from the standpoint of modeling, validation is the defense of knowledge assertions, while verification is the internal consistency.

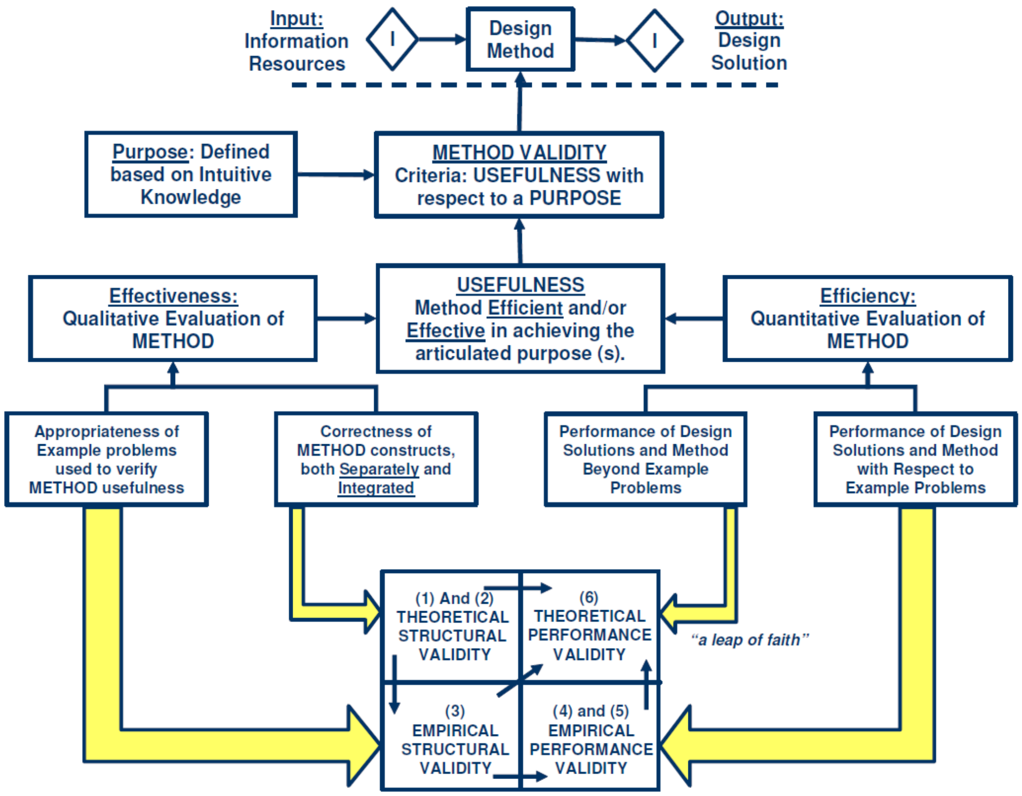


Figure 1.5: The validation square (Seepersad, Pedersen and co-authors 2006)

Validation Square consists of four quadrants as explained below:

Theoretical Structure Validity **(TSV):** It entails evaluating the internal consistency of the design methods, or the logical soundness of each individual construct. The fact that TSV is useful and can be applied to empirical structural validity (ESV) serves as validation for it. It requires the following steps:

- The first step involves determining the design method requirements that is considering both the desired outcomes and the steps or process involved.

- Thoroughly analysing the literature review which involves comprehensive review of existing research, knowledge and information related.

- Next step involves ensuring internal consistency, this involves analysing the coherency in design and implantation.

Empirical Structural Validity **(ESV):** It involves verifying the performance or effectiveness of a particular process or system used in the research. In this context, it specifically involves using practical examples to validate and then confirm the functionality of the given framework. It requires the following steps:

- First step is to ensure the sustainability of the example problems chosen for the design method. This involves confirming that the selected example problems are appropriate and relevant for evaluating the design method.

- Next step validates the test results supporting the design method's utility, this involves evaluating the efficacy and applicability of the framework or design method proposed.

Empirical Performance Validity (EPV): It involves evaluating the practical efficacy of the framework or design method in real-world complex system applications, it is rooted in its potential for being employed in Theoretical Performance Validity (TPV), typically referring to a theoretical condition. This involves following step:

• This step involves validating the practical utility and efficiency of the method in obtaining desired solutions. This demonstrates the design methods or framework capability in addressing real-world challenges.

Theoretical Performance Validity **(TPV):** Verifying the broad applicability of the design method is the main goal of this step. Although it includes a hypothetical aspect, TSV, ESV, and EPV are its fundamental building blocks. Evidence from the three quadrants (TSV, ESV, and EPV) supports the verification for Theoretical Performance Validity (TPV), indicating a thorough approach to validation. Regarding the validity of TPV, the main point is that the technique can be expanded upon or used in situations other than the ones that are discussed in the thesis. The goal of this procedure is to build confidence and trust in applying the design method outside of the thesis's particular instances. This involves following steps:

- Validation supported by the information in TSV, ESV, and EPV.

- Validating the design method's ability to yield valuable outcomes beyond the scope of the example problems and demonstrating the versatility of the design method or framework for different design problems.

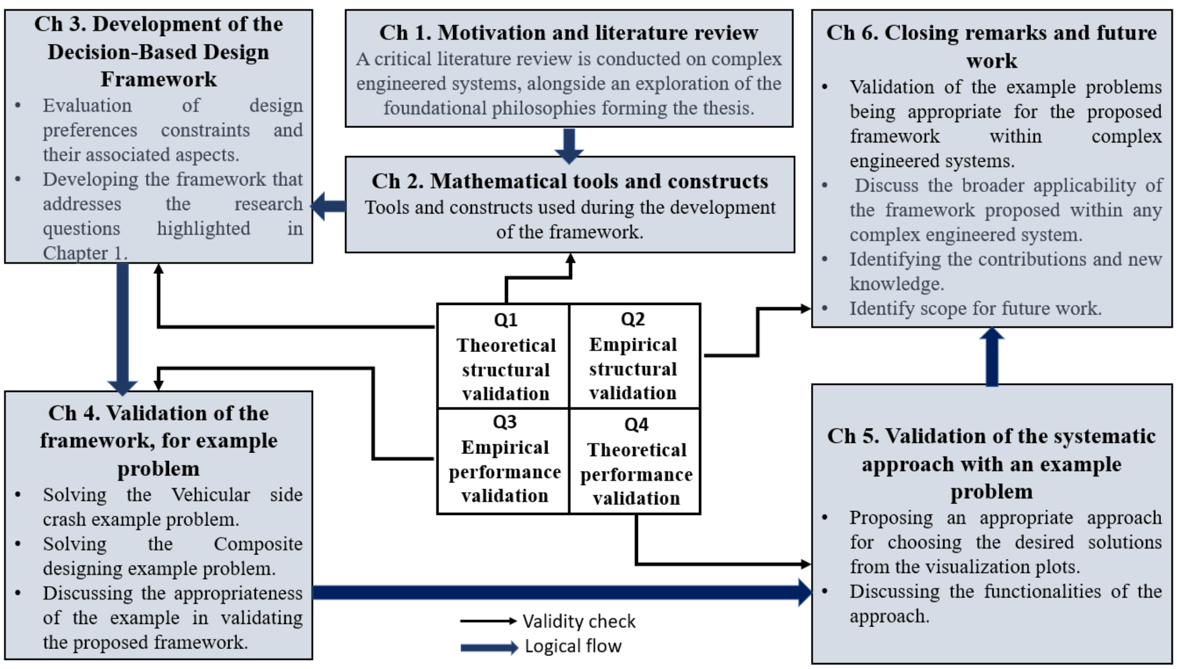


Figure 1.6: Verification and Validation square for this Thesis

Further details outlining each quadrant and its respective association with the related chapters are explained below. The table below shows verification and validation applied to the thesis chapters.

Table 1.1: Verification and Validation strategy applied to the Thesis chapters

Quadrants in validation square	Chapters applied
1.	**Theoretical structural Validation** Chapters 1, 2 and 3 delve into assessing the internal consistency to establish the logical soundness of the design method. In chapter 1 the background of design methodologies, complex system and solution space is discussed. Finally, the scope of the work, including the research questions posed, hypothesis proposed, and of the present work is detailed. In Chapter 2 intense literature review is carried out and mathematical tools used are discussed. In particular, the discussion is on types of robust design, compromise Decision Support Problem (cDSP) construct and Design Capability Index (DCI) are discussed. Chapter 3 contains the detailed discussion about decision-based design framework proposed in this thesis.
2.	**Empirical structural Validation** The analysis of the suitability of the chosen test problem to demonstrate and validate the design method is discussed in Chapter 4 & 5. In this Chapters, the results obtained from example problems are presented and discussed. The results pertaining to each mathematical formulations in Chapters 4 and 5 are presented. In Chapter 4, vehicular

	side crash example problem is formulated and in Chapter 5 designing of composite material example problem is formulated. In detail, the discussion about the validity and usefulness of the method is outlined.
3.	**Empirical Performance Validation** Chapters 4 & 5 discuss the assessment of the suitability of the comprehensive test problems chosen to demonstrate and validate the design method. In Chapter 4, design decision making in vehicular side crash is introduced as a design problem. This followed with the DSP based mathematical formulations and DCI mathematical construct for solving problem. In Chapter 5, decision problem in the design of composite structures is presented to validate the systematic approach. This step in assessing the real - world practical effectiveness of the framework or design method in complex engineered system. In detail, the discussion about the validity and usefulness of the method is outlined.
4.	**Theoretical Performance Validation** The approach involves speculation but is firmly grounded in the principles established by TSV, ESV and EPV, the verification for TPV is derived from all three quadrants (TSV, ESV and EPV). The

	validation of TPV is based on the notion that the method can be expanded, thus establishing the utility of the presented method in examples not explicitly addressed in the thesis. Chapter 6 focuses on building confidence in the generality of the framework. In this chapter, the results concerning the test problems are presented and their utility is discussed. Subsequently, the focus shifts to the broader applicability of the framework. The initial section comprises a summary of this thesis. Following, there is a revisit to the research question of the accomplishments and contributions made in this thesis.

1.9 Organization of the Thesis

The purpose of Chapter 1 is to provide background information on complex engineered systems and to inspire the development of such systems. To set the stage for the rest of this thesis, the purpose of this discussion is to highlight the significance of decision making in model-based designed systems. In Section 1.6, the background information and relevant literature study on solution space exploration are covered. As seen in the thesis organization, a number of mathematical constructs and techniques are used for developing decision-based design framework are covered in the following chapter. These tools and constructs are utilized due to their relevance to one or more research questions posted in 1.7. The compromise DSP is discussed in Section 2.1, DSIDES in section 2.2, Utility of cDSP in complex systems is discussed in 2.3, robust design, and its types in 2.4, different types of visualization technique and their limitations is discussed in 2.5.

The tools and constructs introduced in Chapter 2 are then employed to develop the framework that addresses the research gaps in Chapter 3. There are different parts involved in this method: in Section 3.1 and 3.2 the existing framework and their limitations are discussed. In Section 3.3 the Decision-Based Design framework is proposed. The foundational mathematical construct with robust design metrics is discussed in Section 3.4. Solution space visualization and exploration is discussed in Section 3.5. The machine learning based visualization technique used in this thesis is discussed in section 3.6.

The method proposed in Chapter 3 is then tested through design example. In Chapter 4, vehicular crashworthiness example is considered to validate the proposed framework and the limitation of the framework is discussed. In Chapter 5, the limitation in framework is addressed by developing a systematic approach and tested considering and composite design problem. In Chapter 6, the functionalities of the framework and summary of the thesis are discussed. In section 6.3 the research questions are revisited, and hypothesis is discussed. Finally, the chapter is concluded with future work.

Figure 1.7: Layout of Thesis Chapters

Introduction and problem identification	**Chapter 1:** Frame of reference, design methods for complex engineering problems, research questions and hypothesis	*Relevance* **Chapter 1:** Introduction to the complex problem features, DST technique. Identify research gaps and questions.	*Hypothesis* Establish problem background
Mathematical constructs and methods	**Chapter 2:** Literature review, mathematical constructs used, and cDSP technique.	**Chapter 2:** Critical literature review, mathematical models used in thesis work	Detail discussion of tools used
Framework and testing example problems	**Chapter 3:** Robust design framework **Chapter 4:** Vehicular side crash problem (addressing RQ 1) **Chapter 5:** Designing of composite beam problem (addressing RQ 2)	**Chapters 3,4 and 5:** Proposing a robust design framework for addressing the issues discussed in Chapter 1. Validating the efficacy of the framework with test examples	Establish a framework and validate its efficacy
Closure	**Chapter 6:** Summary of thesis, closing remarks and way forward	**Chapter 6:** Summarizing the results and discussion of closing remarks	Discussion of future work and closure.

Chapter 2: Mathematical Tools and Constructs for Developing Decision-Based Design Framework

In Chapter 1 background on complex systems, design as decision making process and solutions space were discussed, the mathematical constructs and tools used in thesis work are discussed in the current Chapter 2. In this chapter we discuss the tools used to model the design decisions and mathematical constructs used to address the issue of uncertainty present in complex systems. This chapter begins with Section 2.1 explaining about compromise Decision Support Problem (cDSP) used for modeling the complex engineering systems. In Section 2.2 DSIDES (Decision Support in the Design of Engineering Systems) are discussed. In Section 2.3 Robust design for complex system and their sources are discussed. In Section 2.4 there is a discussion of and its use in complex systems.

2.1 Compromise Decision Support Problem (cDSP)

The compromise Decision support problem (cDSP) is a decision model which is a hybrid combination of mathematical and goal programming (Mistree, Hughes and co-authors 1993). cDSP provides a set of satisfied design variable values that obey design goals and constraints. It helps designers in decision making by analyzing the constraints and tradeoffs between conflicting goals and coming up with design preferences associated with conflicting goals (Vadde 1995). compromise DSP offers following functionalities:

- Can handle single objective problems or multi-objective problems.
- Design goals can be formulated as preemptive or Archimedean.
- Feasible solutions can be generated more frequently.
- Results can be generated rapidly for several weight scenarios.

The compromise DSP has been used in many industries like aircraft designing (Lewis and Mistree 1995), designing of thermal energy systems (BASCARAN, MISTREE and co-authors 1987, Fuchs, Karandikar and co-authors 1990), designing of mechanisms (Mudali 1987), designing of damage tolerant structural systems (Shupe and Mistree 1987), designing of ships (Mistree, Smith and co-authors 1990), and designing of composite material (Fuchs, Karandikar and co-authors 1990). In the next section we discuss formulating the cDSP.

2.1.1 Compromise DSP Formulation

The compromise Decision Support Problem is a decision model that facilitates designers to model the decision [12]. Many quantified goals are modeled to find the feasible solution space that helps designers to make decisions [13], hence cDSP is an effective decision model supporting human judgement. The four key words used in cDSP are Given, Find, Satisfy and Minimize. The word formulation of the compromise Decision Support Problem is shown in Table 2.1

Table 2.1: Compromise DSP word formulation

Given The system parameters and goals for design Assumptions are used to model the domain of interest. **Find** System variables (describe the attributes of system) Deviation variables (describe the extent to which the goals can be achieved) **Satisfy** System constraints System goals Upper and lower bounds **Minimize** Objective function Z (It is a measure of the deviation of the system performance to actual set of goals and their associated weight priorities.

Comparison between single objective and cDSP formulation considering a two-dimensional problem is shown in Figure 2.1. From the figure we can observe that both conventional single objective formulation and cDSP formulation represent similar feasible design space. This feasible design space is bounded by system bounds and system constraints of the system. We obtain a single objective function z (see Figure 2.1a), and this objective function can either be minimized or maximized. Whereas in cDSP formulation we obtain a set of objectives defining an aspiration space (see Figure 2.2b). We can obtain solutions that satisfy the goals and constraints of the given complex problems only to some extent which is represented by the aspiration space. The tradeoff solutions obtained between the desired solutions space (aspiration solution space) and achieved solution space (design space) are modeled using cDSP by minimizing the deviation function. The mathematical formulation of cDSP is shown in Table 2.2 where A_i(X) represents the actual goal that can be attained for the ith goal for a targeted value of G_i. there exist two deviation variables for each goal, the deviation variable d_i^- defines a goal that is underachieved from its target or desired value and d_i^+deviation variable defines a goal that is overachieved from its target or actual value. Deviation variables are always positive, and the value of these variables are determined by the extent the achievement function A_i(X) achieves the targeted value G_i and they are dependent on the system variables X and k represents number of priority levels.

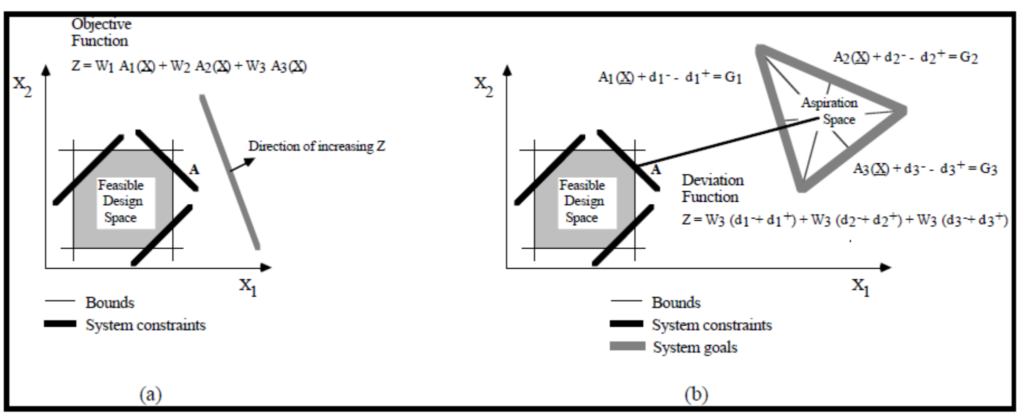


Figure 2.1: A single objective optimization and cDSP formulation (Mistree, Smith and co-authors 1990)

Table 2.2: Mathematical formulation of a compromise DSP (Lin, Krishnapur and co-authors 1999)

Given

An alternative to be improved, domain dependent assumptions.

The system parameters:

n	number of system variables
q	inequality constraints
$p + q$	number of system constraints
m	number of system goals
$g_i(\mathbf{X})$	system constrain functions
$f_k(d_i)$	function of deviation variables to be minimized at priority level k for the preemptive formulation

Find

System variables

$X_i \quad i = 1, 2, \ldots, n$ (They describe the physical attributes of an artifact.)

Deviation variables

$d_i^-, d_i^+ \quad i = 1, 2 \ldots, m$ (They indicate the extent to which the goals are achieved)

Satisfy

System constraints: These must be satisfied for the solution to be feasible (linear, non-linear)

$g_i(\mathbf{X}) = 0;\ i = 1 \ldots . p$

$g_i(\mathbf{X}) \geq 0;\ i = p+1 \ldots . p+q$

System goals: These need to achieve a specified target value as far as possible (linear, non-linear)

$A_i(\mathrm{X}) + d_i^- - d_i^+ = G_i;\ \ i = 1 \ldots m$

Bounds: Lower and upper limits on the system variables.

$X_i^{\,min} \leq X_i \leq X_i^{\,max}\ ;\ i = 1 \ldots n$

$d_i^-, d_i^+ \geq 0,\ d_i^- * d_i^+ = 0;\ i = 1 \ldots m$

Minimize

Case a: Preemptive formulation (lexicographic minimum)

$$\mathbf{Z} = [f_1(d_i^-, d_i^+), \ldots \ldots, f_k(d_i^-, d_i^+)]$$

Case b: Archimedean

$$\mathbf{Z} = \sum_{i=1}^{m} W_i(d_i^- + d_i^+);\ \sum_{i=1}^{m} W_i = 1$$

To maximize the achievement, function the below equation can be used.

$$[\, \boldsymbol{A_i}(\mathrm{X}) \,/\, \boldsymbol{G_i}] + \boldsymbol{d_i^+} - \boldsymbol{d_i^-} = \mathbf{1} \qquad \text{Equation 2.1}$$

To minimize the achievement, function the below equation can be used.

$$[\, \boldsymbol{G_i}/\boldsymbol{A_i}(\mathrm{X})] + \boldsymbol{d_i^+} - \boldsymbol{d_i^-} = \mathbf{1} \qquad \text{Equation 2.2}$$

The motive of compromise DSP is to minimize the deviation function using deviation function. Deviation functions represent the region between the feasible solutions space and the aspiration space and the range pf these variables depend on the goals. Two types of deviation function exist in compromise DSP namely preemptive and Archimedean formulation. In Preemptive formulation goals are satisfied orderly according to designer's requirement and there is no compulsion to assign the weights to goals whereas in Archimedean formulation weight for each goal must be assigned and these weights can be determined by using pair-wise methods or relative weighting method.

Assigning weight to goals depends on the designer and problem requirements. Therefore, the weights W_i assigned to goals are sequential affecting the solution space based on the designer's requirement and these weight are normalized to a sum of one (Mistree, Hughes and co-authors 1993). The preemptive formulation and Archimedean formulation are shown in Table 2.2 and lexicographically minimized. After a detailed discussion about word and mathematical formulation the next section discusses DSP technique.

2.1.2 Decision Support Problem Technique

In decision making approach there are two categories selection and synthesis, corresponding to selection and compromise in Decision Support Problem Technique (Sen and Yang 2012). Multiple attribute decision making (MADM) is referred to selection, which involves selection from ranged set of alternatives from a catalogue according to the attributes priorities whereas compromise method is refereed to Multiple Objective Decision-Making (MODM) which involves alternatives based on the goal priorities.

Any complex systems can be modeled with a network of DSP (compromise and selection) The core constructs of the technique are the axioms described in References and the ability to work with the complexity of these decision networks. Typically, models can be modeled with no more than three DSP's that can be coupled together (Mistree, Smith and co-authors 1993).

Compromise decision support has been implemented in many application like designing of ships, thermal energy systems, composite materials, damage tolerant structural and mechanical systems, designing aircraft and concurrent design of multi-scale, multi-functional material and products (Mistree, Muster and co-authors 1990). Key applications specification development (Lewis, Smith and co-authors 1999), robust (Chen, Allen and co-authors 1996, Chen, Allen and co-authors 1997, Allen, Seepersad and co-authors 2006) product families (Simpson, Chen and co-authors 1999, Simpson, Maier and co-authors 2001, Simpson, Seepersad and co-authors 2001) the integrated realization of materials and products (Choi, Mcdowell and co-authors 2008, Choi, McDowell and co-authors 2008, McDowell, Panchal and co-authors 2009) and a variety of mechanical (Chen, Meher-Homji and co-authors 1994, HERNANDEZ and Mistree 2000, Sinha, Bera and co-authors 2013).

After formulating compromise DSP, DSIDES with operations research tool adaptive linear programming algorithm is used to model conclusions (Mistree, Hughes and co-authors 1993). This iterative process often requires substantial justification, particularly in instances where goals are conflicting. Therefore, it is crucial to articulate and comprehend the realms of design and aspiration, enabling the exploration of these domains. Upon considering the compromise Decision Support Problem (DSP), similarities can be observed with demands and wishes framework introduced by Pahl and co-authors (Beitz, Pahl and co-authors 1996). Here demands can be met by adhering to DSP constraints and bounds, similarly wishes can be met by satisfying goals. Solution space is the combination of both feasible and aspiration space, feasible region consists of constraints and bounds, and aspiration region consists of goals. The key words used during formulation of compromise DSP problem are *Given, Find, Satisfy* and *Minimize.*

The compromise DSP offers several advantages as a decision framework. It provides contextual and structural support for making decisions and remains independent of specific domains and remains independent of specific fields. When employing DSIDES to solve DSP's, it enables the exploration of both design and solution space by considering design requirements and the preferences of the designer. This can be achieved through formulating problems in Archimedean or preemptive manner thereby enhancing the decision-making process. They can be formulated rapidly even with minimal information, making them adaptable for use at any stage within specific timeline. They also prioritize offering diverse viewpoints that contribute to decisions which capture the design intent. Additionally, conducting post-solution sensitivity analysis becomes essential to provide de3signers with insights when faced with uncertainties.

2.2 The Decision Support in the Design Engineering System (DSIDES)

The principles of decision support technique are implemented in DSIDES. The conceptual DSP and compromise DSP are utilized in many domains like designing of aircraft, designing of ships, damage tolerant structural and mechanical systems (Mistree, Hughes and co-authors 1993). For the problems involving Boolean and continuous variables and multi-goals DSIDES can be used with both selection and compromise formulation.

DSIDES is a tailored computational environment for solving the compromise DSP, DSIDES require a user specific input file in the form of cDSP template which consists of data file and user supplied FORTRAN file (Reddy, Smith and co-authors 1996). The size of the problem, variable names, goals, constraints, bounds, and convergence criteria are defined in the input data file. Data file is created with a number of mandatory blocks like SYSVAR this provides a description of system variables name, type, bounds, and guess value and XPLORE optional blocks provides best initial points to explore the design space based on the pattern search. An example of the data file is provided in Appendix and mandatory and optional blocks used in creating a data file are shown in Figure 2.2.

The FORTRAN file consists of routines that are user specified such as USRMON for the user specific monitoring of the solution space. And these users specified subroutines are used to analyze the nonlinear constraints and goals. Constraint evaluation routines and design evaluation routines are used for input data similarly, for output data format routines are used. In some cases, we can use design analysis routines like REFPROP routine to capture interface like thermal properties associated with synthesis cycle.

Mandatory Blocks

PTITLE	1	Problem title
NUMSYS	2	Number of System Variables
SYSVAR	3	Description of System Variables - name, type, bounds and guess value
NUMCAG	4	Number of Constraints and Goals
LINCON	5	Linear Constraints - names and data *(if specified in NUMCAG)*
LINGOL	6	Linear Goals - names and data *(if specified in NUMCAG)*
DEVFUN	7	Deviation Function - number of levels and weights of deviation variabl
STOPCR	8	Stopping Criteria (run and principal print flags, NITER, EPSZ, EPSX)

Optional Blocks

NLINCO	9	Names of Nonlinear Constraints (default names: NLCO##)
NLINGO	10	Names of Nonlinear Goals (default names: NLGO##)
INITFS	11	Automatic Generation of Initial Feasible Solution
ALPOUT	12	Flags for Output Level, Post Processor and Time Statistics
USRMOD	13	Flags for User Modules (USRINP, USROUT, USRMON, USRLIN)
USRDAT	14	User Data Block for Access From USRINP
OPTIMP	15	Optimization Parameters (VIOLIM, REMO, STEP)
ADPCTL	16	Nonlinear Inequality Constraint Adaption Flag (LADAP)
USERAN	17	Information for USRANA (maximum cycles - NANCY, NSYCY)
FIXVAR	18	Fixing of Variables
SUPCON	19	Suppression of Nonlinear Constraints
PVALFX	20	Particular Values for Stationarity of System Variables
PVEPSZ	21	Particular Values for Stationarity of Deviation Function Levels
PVSTEP	22	Particular Values for STEP
PVCVIL	23	Particular Values for VIOLIM
PVREMO	24	Particular Values for REMO
ADREMO	25	Adaptive Reduced Move Parameters
XPLORE	26	Explore the design space for best initial points
ENDPRB	27	End of Problem Definition

Figure 2.2: Mandatory and optional blocks used in DSIDES data file.

- `USRINP` (for user specific input)
- `USRSET` (for evaluating nonlinear constraints and nonlinear goals)
- `USRLIN` (for updating linear constraint and linear goal coefficients)
- `USRMON` (for user specific monitoring of the solution process)
- `USRANA` (for relevant analysis cycle calculations)
- `USROUT` (for user specific output)

Figure 2.3: User specified routines used in FORTRAN file of DSIDES.

Adaptive Linear Programming algorithm is used in solving compromise DSP which is incorporated in DSIDES providing vertex solutions (Mistree, Smith and co-authors 1993) and other approach is zero order search which is referred to as XPLORE in DSIDES. Utilizing the referenced algorithm (Aird and Rice 1977), it is employed to assess various designs within the prescribed bounds of system variables. The top n designs are retained, serving as potential initial points for a more extensive search process. Initial feasible Solution module is used as a second method for pattern search algorithm. These methods help in implementing Adaptive Linear Programming algorithm more effectively and better understanding of solution space. The compromise DSP can be solved with different optimization techniques depending on the type of problem. However, in this thesis we use Adaptive Linear Programming (ALP) algorithm in DSIDES which comes under the classification of solving the approximate problem in an exact manner. ALP algorithm solves the problem in linearized manner and the three main characteristics of the ALP algorithm are mentioned below (Mistree, Hughes and co-authors 1993):

- The use of second-order terms is linearized.

- The normalization of the constraints, goals and their transformation into generally well-behaved convex functions are in the region of interest.
- A "intelligent" constraint suppression and accumulation scheme.

The modification of second order algorithm is ALP algorithm, here the values of the variables and derivatives of constraints and goals are required. The derivates are calculated numerically by central difference formula. The implementation of the Adaptive Linear Algorithm using computer is shown in Figure 2.4.

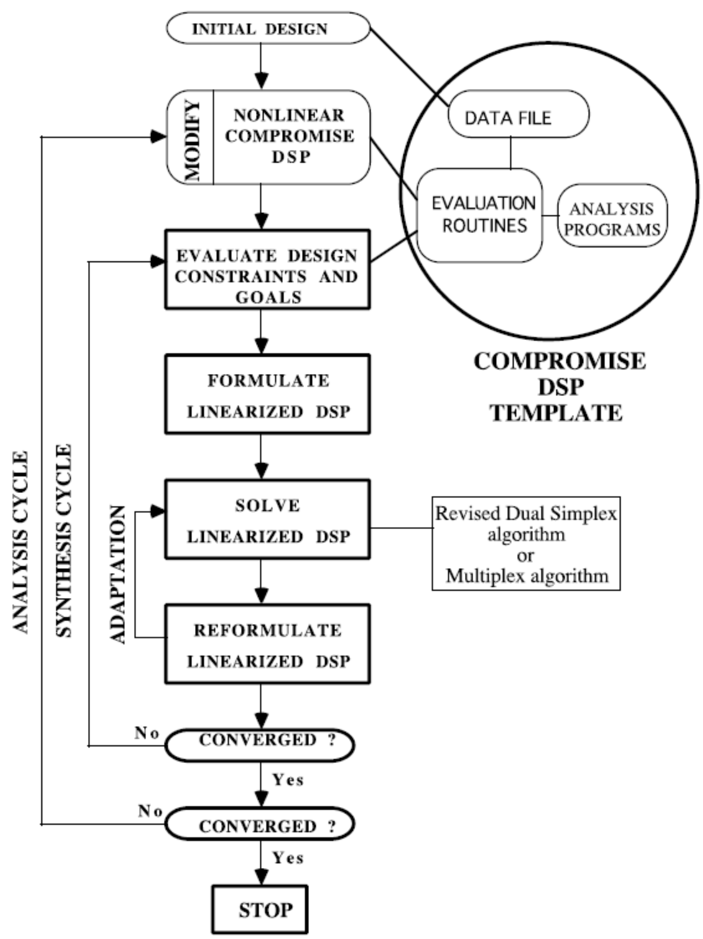


Figure 2.4: Flowchart of implementation of ALP algorithm for solving cDSP (Mistree, Smith and co-authors 1993)

The FORTRAN routines within DSIDES are utilized for assessing nonlinear constraints and objectives, inputting necessary non-linear constraints and objectives, inputting necessary data for constraint evaluation and design analysis, and generating results in the desired user-friendly format. The algorithm consists of two main cycles, the analysis cycle and synthesis cycle. Within these cycles, access to a design-analysis program library is facilitated enabling the utilization of both cycles and exclusively within the synthesis cycle. In Figure 2.4 the implementation of ALP algorithm for solving cDSP is shown, when formulating and evaluating the nonlinear cDSP's through user-specified routines a linear approximation is employed, this approximation aids in solving the linear programming problem. The solution of the linear programming problem is calculated using multiplex algorithm and obtaining solutions, a post-solution analysis is performed (Kyprioti, Zhang and co-authors 2020).

In solving cDSP, the Adaptive Linear Programming is very effective, but it has the limitation of solving only Boolean and continuous variables, suppose if a problem consists of integer or discrete variables then it becomes difficult to ALP algorithm to solve these types of problems. When system constraints are highly non-linear ALP algorithm suppress the constraints, in such scenarios it's designer's choice to whether to analyze these suppressed constraints. Another limitation is, sometimes the designer might not have complete information about constraints in actual design itself, in such cases the data file might not work as it requires complete information of the design problem (Mistree, Hughes and co-authors 1993).

In this section, the components of DSIDES and type of search method used are discussed, and in the next section the usefulness of compromise DSP in complex engineered systems are discussed.

2.3 The Utility of Compromise Decision Support Problem in Complex Systems

The compromise DSP are used to model multiple tradeoffs and decisions (Mistree, Patel and co-authors 1994), the cDSP allows for the exploration and assessment of how a complex system designing evolves across various design scenarios. This can be accomplished by employing the compromise DSP in a diverse manner. Various deviation functions within the compromise DSP offers tools to investigate design decisions over the course of the design timeline as the understanding of the design progresses. At the initial stages of design preemptive formulation is more beneficial as it facilitates in exploring the tradeoffs between goals at different levels of priorities, when the information or knowledge about the complex system is increased Archimedean formulation can be used to explore design priorities. Alternatively, you can explore the constraints at any point during the design process to learn more about their robustness and practicality, and then adjust the design as needed. A fast method of examining design space and gaining understanding of the tradeoffs in design is to use the DSIDES module XPLORE. Given that the analytical models are imprecise, inconsistent, and lacking in certain areas, this kind of investigation utilizing compromise DSP broadens our understanding of design. Additionally, XPLORE offers a comprehensive overview of the entire design space along with details on the intriguing and satisfying areas that warrant more investigation. Two approaches exist in modelling complex systems, first one is using the exact system functions to predict and explore the complex system behavior and second approach is accurate analysis of the system. When heuristics is used to obtain the solutions that are good enough or satisficing, the solutions are no longer optimal or exact. " In a perfect and stable world, with perfect knowledge, designers could establish optimum designs for all their individual product and process requirements" (Chen, Allen and co-authors 1996).

Distinguishing between optimizing and satisficing, they both hold distinct perspectives on what constitutes an effective design across the entire design timeline. Optimization emphasizes seeking the best available solution at each stage of the design process, while the satisficing approach advocates maintaining a degree of openness at each stage to accommodate potential solutions that might arise. These concerns stem from the limitations and accuracies present in the models, leading to uncertainties, particularly in the initial stages of design.

Complex systems are complete and accurate in optimization approach and the optimal solution obtained for a complex system becomes less relevant for the overall design when variations occur during process across the timeline, causing the design to deviate from the originally identified optimal solution. To overcome the limitation of optimization approach, we consider the satisficing approach where the solutions remain satisficing even when the boundary conditions change. The optimization and satisficing approach are shown in Figures 2.5 and 2.6 respectively.

The significance of the compromise DSP in modeling decisions for complex systems lies within the realm of model-based system design. This approach proves particularly beneficial when dealing with incomplete, inaccurate and models not of equal fidelity, especially prevalent in the initial design process, where the information is limited. The compromise DSP offers the capability to identify solutions that are good enough or satisficing, which can later be refined and enhanced as more data and analysis are gathered throughout the design process. The ability to iteratively improve solutions based on the evolving information and analysis makes the compromise DSP a valuable tool for navigating the uncertainties inherent in complex designs.

The compromise DSP is used in the two example problems that are discussed in Chapter 4 and 5. In the next section the concept of robust design and types of uncertainties are discussed in detail.

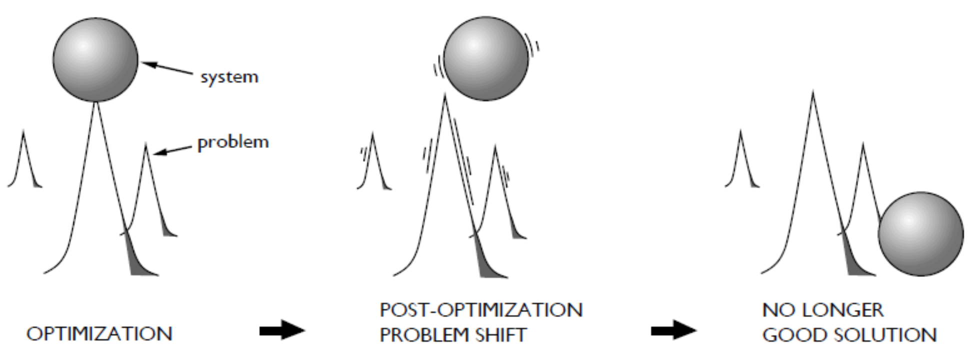


Figure 2.5: Optimization approach

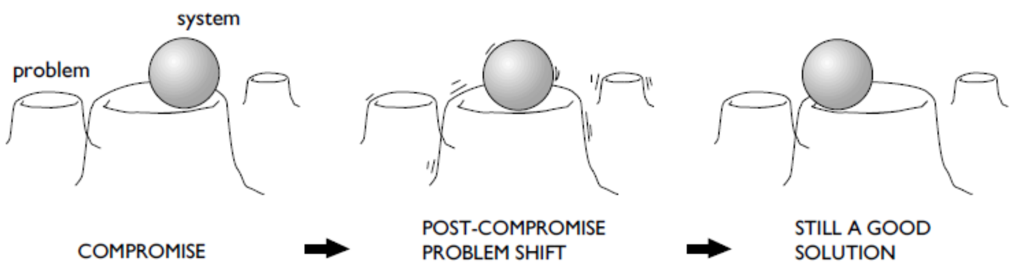


Figure 2.6: Satisficing approach

2.4 Robust Design of Complex Engineered Systems

In the context of this thesis, robust design is defined as the one with solutions that are relatively insensitive to uncertainties. In the designing of complex engineered systems, the challenge for designers lies in accounting for the uncertainties present in the system. It is important to manage these uncertainties effectively to develop a robust solution. The computational models used in design are the approximations and not exact representations of the real world. Because of this inherent abstraction, it becomes crucial to develop design solutions that are less sensitive to uncertainties. In this section we discuss different sources of uncertainty that exist in complex engineered systems and different types of robust design methods established to mitigate the effect of these uncertainties on the design process.

2.4.1 Classification of Uncertainties in the Complex System

The term uncertainty was first coined by the Greek scholar in 4th century BC which is within the scope of epistemology. The word epistemology originates from the Greek word's "episteme", signifying knowledge and "logy" which encompasses various meanings that includes "theory" (Thunnissen 2003). The study of uncertainty has attracted research from diverse fields such as social services, economics, engineering, medicine and more. Uncertainty has been categorized from management science. Within the domain of management science, particularly in the probabilistic risk analysis community, uncertainty is defined as "that which disappears when we become certain" (Bedford and Cooke 2001). These classifications and their definitions are shown in Figure 2.7 and Table.

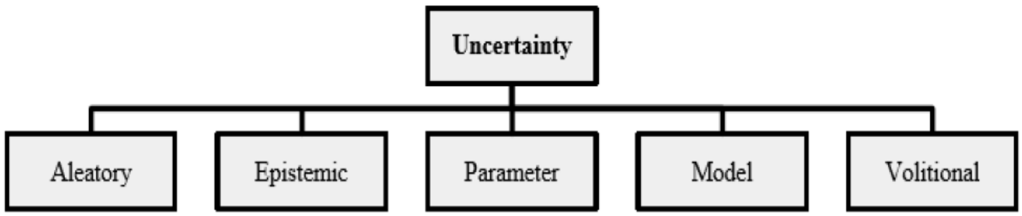


Figure 2.7: Uncertainty classification for Management science (Bedford and Cooke 2001)

Table 2.3: Definitions for uncertainty for Management science (Bedford and Cooke 2001)

Uncertainty	**Definition**
Aleatory	Arises through natural variability in a system
Epistemic	Arises through lack of information in a system
Parameter	Uncertainty about the "true" value of a parameter in a mathematical model
Model	Uncertainty about the truth of the model
Volitional	Uncertainty that an individual has in whether what will do he agreed to do.

Other ways to categorize the sources of uncertainty is mentioned below (Kennedy and O'Hagan 2001) :

- *Parameter uncertainty*: This type of uncertainty arises from the model parameters that are the inputs for computer based mathematical models. These parameters are unknown and cannot be precisely controlled in physical experiments nor can their values be accurately considered through statistical methods.
- *Parametric uncertainty*: This uncertainty arises due to variations present in the input variables of the system.
- *Structural uncertainty*: This uncertainty arises due to a lack of complete information or knowledge about the fundamental physics involved in a particular problem. It also

depends on how accurately a mathematical model represents the true system, considering the models are typically only approximations to reality.

- *Algorithmic uncertainty*: This uncertainty arises from numerical errors and approximations during the implementation of computer model. Given that many models are highly complex, they often are too complex to solve precisely. This leads to uncertainty arising from the inherent limitations in numerical methods and approximations employed in computational models, introducing potential errors due to the complexity and computational constraints involved in solving these models accurately.
- *Experimental uncertainty:* this uncertainty arises from variability in experimental measurements. It is an inevitable aspect of experimental work and becomes apparent when the same measurement is repeated many times under identical conditions for all the variables.
- *Interpolation uncertainty:* This type of uncertainty arises due to a scarcity of available data obtained from computer simulations or experimental measurements. When predicting data for inputs that lack simulation data information, interpolation becomes necessary. The uncertainty emerges from the process of estimating or forecasting responses based on this interpolated information which may introduce uncertainties due to the assumptions and methodologies used in this process.

It is vital to understand various types of uncertainties as it forms the foundation for developing methods aimed at quantifying and addressing them. These methods help in managing uncertainties by reducing their impact. Within uncertainty quantification there are two major types of problem that exist. The first type is the forward approach, which involves the propagation of various sources of uncertainty through the model to anticipate the overall uncertainty in the system's response. This approach focusses on understanding how uncertainties in the system's response. This approach focuses on understanding how uncertainties input parameters influence the uncertainty in the final system output or response. And the second type is the inverse assessment of model uncertainty and parameter uncertainty. In this scenario the model parameters are calibrated simultaneously using test data. This method helps refine the model by adjusting its parameters based on observed data, aiming to reduce the discrepancy between model predictions and actual observations.

Some of the foundational concepts of uncertainty classification and robust design methods are discussed in this thesis are anchored from the concepts of Chen on co-authors (Chen, Allen and co-authors 1996, McDowell, Panchal and co-authors 2009, Sharma, Allen and co-authors 2021). Uncertainty quantification by Iskupalli and co-authors (Isukapalli, Spendiff and co-authors 2010) . And the type of uncertainties based according to choi and co-authors are mentioned below (Choi, Austin and co-authors 2005):

- Natural uncertainty (NU): This uncertainty arises from inherent randomness or incomplete information. This type of uncertainty is fundamental and cannot be eliminated and can only be assessed through statistical models.
- Model structure uncertainty (MSU): This type of uncertainty arises from model uncertainty during formulation due to approximations and simplifications considered in the system. This form uncertainty can be reduced by enhancing the model's formulation.
- Model parameter uncertainty (MPU): This uncertainty arises due to insufficient data or information due to inaccurate data. This form of uncertainty is reduced by sufficient data or accurate measurements.
- Propagated uncertainty (PU): This uncertainty is a combination of the above two types of uncertainty in system. Consequently, the ultimate performance estimation of the sequence of models may exhibit a considerable level of uncertainty.

We need robust design methods to address the numerous sorts of uncertainty that are common in complex engineered systems. One strategy would be to lessen the uncertainty itself, and another would be to control or lessen the effects that result from these uncertainties. This thesis addresses uncertainties in complex engineered systems for many (more than three) goals. In general, there are four types of robust design methods (Panchal, Choi and co-authors 2005) and these are discussed in below section.

2.4.2 Type I Robust Design Method in Complex Engineered Systems – Taguchi Method

In robust design, the goal is to make products and processes better by making them less affected by differences without getting rid of these differences altogether (Taguchi 1986). Robust design principles, inspired by Genichi Taguchi's philosophy, involve three main categories of information that effect the system:

- **Control factors (design variables):** These are the variables that a designer can change or adjust to achieve a specific product or system.
- **Noise factors:** These are the external variables that impact the process or product but are beyond the designer's control.
- **Responses:** These are the measurements used to evaluate how well the product or process is performing.

The categories of information are shown in Figure 2.8.

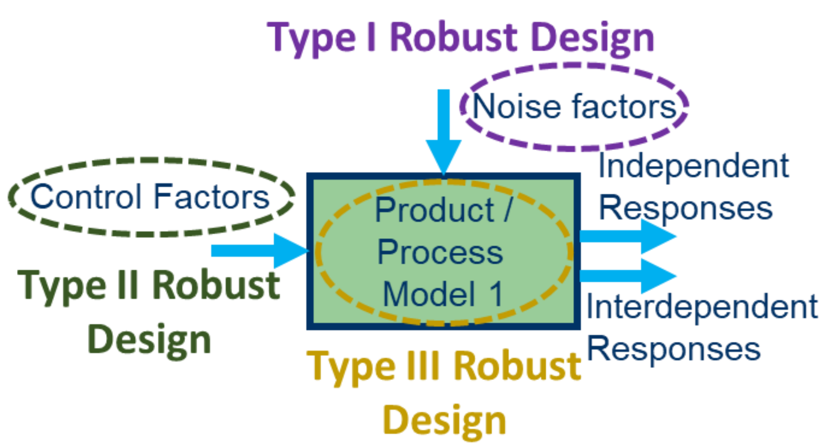


Figure 2.8: A P-diagram showing the categories of information in robust design. (Nellippallil 2018)

Type I robust design, introduced by Taguchi, aims to find values for design variables that meet performance requirements even when there are variations in noise factors. Taguchi's approach, widely influential in Japanese industries, focusing on reducing variability's impact without eliminating its sources. It uses experimental designs like orthogonal arrays, a quality loss function, and the signal to noise ratio. The quality loss function assesses the societal loss caused by the product from the moment it's shipped, in Taguchi's method, higher product quality means minimizing the societal loss (Perona 1998). The equation for Taguchi's loss function is given as:

$$\mathbf{L = k(y - T)^2} \qquad \text{Equation 2.3}$$

In the above equation, L represents loss function, k represents cost efficient, y represents the value of quality characteristic and T represents the target value. The quality loss function is shown in the below Figure 2.9.

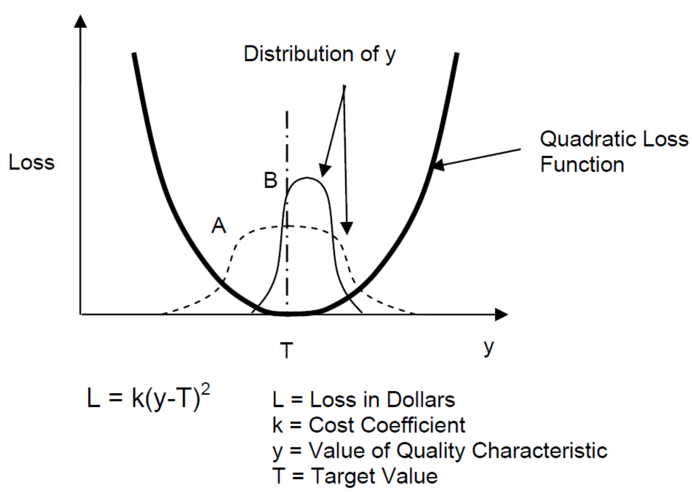


Figure 2.9: Taguchi's quality loss function (Choi 2005)

The two probability distributions A and B which represent product outputs are shown in Figure 2.9. B has the smaller average loss of quality compared to A, this prediction is based on how average value of y deviates from the target (T) and the mean squared deviation of y around its own mean according to Phadke (Phadke 1986). Taguchi emphasizes three crucial stages for engineering design: System design, parameter design and tolerance design. Among these Taguchi particularly stresses the significance of the parameter design stage in identifying the best parameters to minimize quality loss.

In Taguchi's robust design, the parameter design phase begins by separating parameters into control factors and noise factors. Control factors are adjustable, while noise factors are either beyond control or costly to manage. Using an experimental design called an orthogonal array, control factors are organized in one array, and noise factors in another. Every possible combination of these factors is another. Every possible combination of these factors is tested in the experiment. By varying noise factors constant, average responses are measured. Taguchi's signal to noise ratio assesses how responses react to noise factor changes. Designers can then select the best level for each control factor based on mean response and signal-to-noise ratio. Though Taguchi's principles like statistical methods (orthogonal arrays) are popular in many industries they do have limitations. Some of the limitations are mentioned below:

- Taguchi's methods, such as orthogonal arrays and signal to noise ratio, might be a complex to understand and implement for those unfamiliar with statistical techniques.
- These methods often assume linearity in the relationships between parameters, which might not always hold true in practical systems. Real-world systems can be more complex and non-linear than Taguchi's methods assumptions.
- The classification of the factors into control noise might sometimes oversimplify the complex interrelationships between variables in a system.
- These methods are computationally expensive and require a large number of experiments.
- These methods heavily rely on the selection of specific parameters and their ranges. If these parameters are not chosen appropriately, it can impact the effectiveness of the approach.

Taguchi's robust design theory has been widely applied in industry, producing positive results, despite limitations. One important accomplishment of Taguchi's robust design approach is the creation of designs that are less susceptible to sounds and other environmental variables.

2.4.3 Type II Robust Design Method in Complex Engineered Systems

This type of robust design method deals with complex systems that are relatively insensitive to uncertainties in design variables (that are controllable). Mostly the robust design focuses on the detailed design stage, which assumes that an initial design is already established with specific layout and specifications, which isn't often the case. Some researchers have concentrated on incorporating robustness in the early stages of the design, especially in the conceptual design phase. Decisions made at this early stage influence the final products performance and quality. When exploring ideas, designers need to deal with continuous design spaces. There is a need to develop solutions that are not just insensitive to variations in noise factors but also to control factors. To address this challenge, chen and co-authors introduced Type II robust design to handle variation in both control factors and noise factors. They have developed the Robust Concept Exploration Method (RCEM) to systematically identify robust solutions that are less affected by variations in both control and noise factors during the early stages of design. The RCEM framework is discussed in more detail in Chapter 3.

Chen and co-authors classify problems pertaining to minimizing performance variations and reaching the target mean according to the origin of these variances (Chen, Allen and co-authors 1996).

- Type I - This robust design deals with minimizing variations in performance caused by variations in noise factors.
- Type II – This type of robust design deals with minimizing variations in performance caused by variations in control factors.

Figure 2.10 shows the robust design types developed by Chen and co-authors. The impact of variations in performance due to changes in both the noise and control factors are shown on right hand side of Figure 2.10. In robust design type II, the goal is to find solutions that are relatively insensitive to variations in control factors rather than aiming for optimal solutions. This nearly flat region indicates less sensitivity of system performance to variations in control factors contrasting with the optimal solution that significantly degrades system performance even with slight variations in control factors.

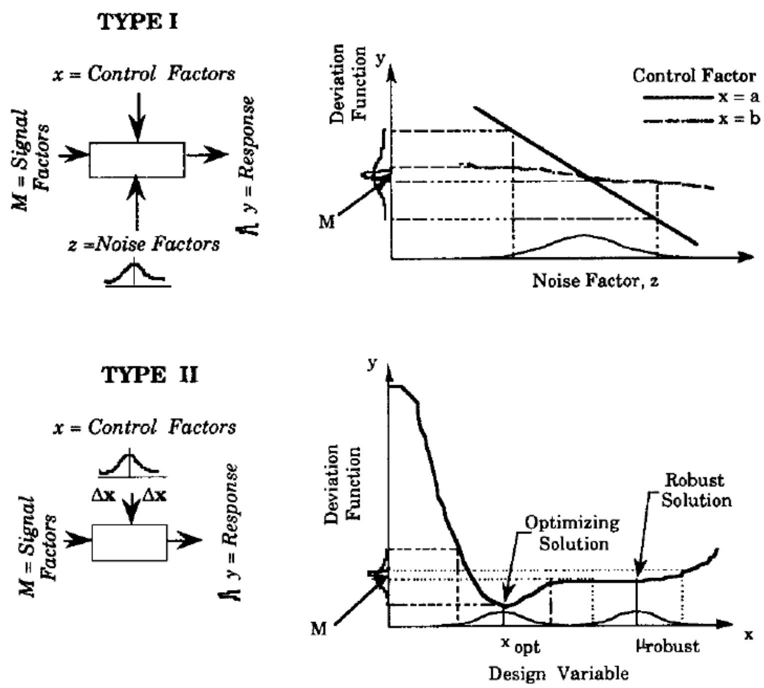


Figure 2.10: Robust design for variations in noise factors (Type I) and control factors (Type II) (Chen, Allen and co-authors 1996)

2.4.4 Type III Robust Design Method in Complex Engineered System

The robust design methods discussed do not address the issue of uncertainty in the models themselves. This uncertainty isn't about control or noise factors but about the models used. It could arise from uncertain parameters, model constraints, metamodels, assumptions within the model, other aspects. To address this issue Type III robust design is used which deals with system the is insensitive to variability in the model themselves. The robust design Type I and II and robust design Types I, II and III along with optimal solution is shown in Figure 2.11.

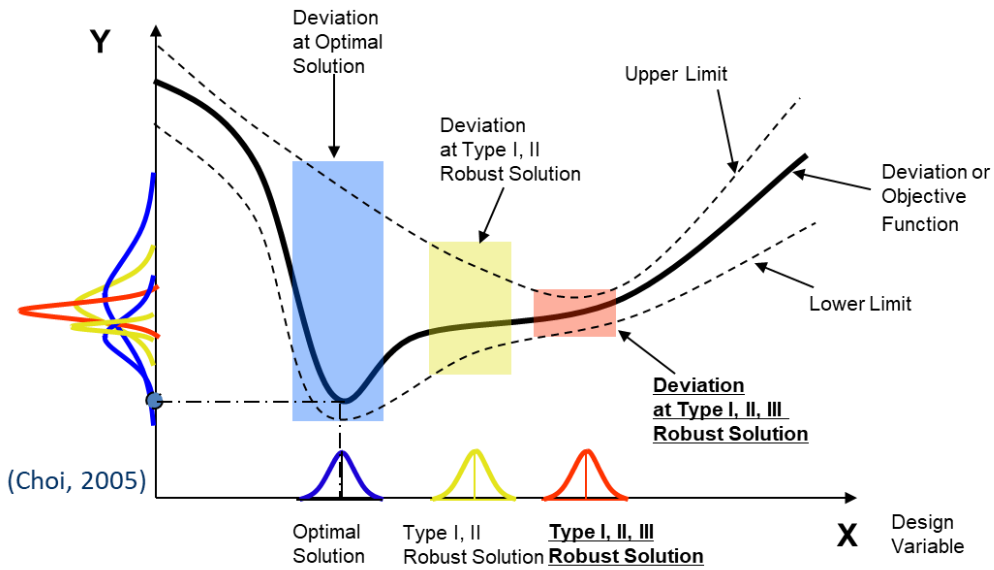


Figure 2.11: Robust design type I, II and III (Choi 2005)

In Figure 2.11 the two curves represent uncertainty limits for the system response and the solid curve represents system response. As compared to Type I and II robust solution and optimal solution with Type I, II and III solutions there is a least deviation in system performance. Therefore, the primary goal for robust design Type III is to identify these adjustable ranges in design variables that not only meet specified performance requirements or ranges but also insensitive to uncertainties within the model.

Type IV robust design delas with the integrated multiscale design of material and product. This robust design type is used to identify design variable values that satisfy a set of performance requirements despite the propagation of uncertainty (PU) through scales (Choi, Mcdowell and co-authors 2008).

The previously mentioned methods constitute types of robust design employed in a system to secure solutions that remain unaffected by uncertainties. For better understanding of the solutions and to get insight into system performance there is a need to visualize them effectively. For this we need explore the existing techniques to use the appropriate technique for solution space exploration and visualization. In the next section various types of visualization techniques that are in use are discussed in detail.

2.5 Visualization Techniques used for Solution Space Exploration.

In complex engineered systems, decision-based design involves making informed decisions at various stages of the design process. Data visualization plays a vital role in this context for several reasons some of them are mentioned below (Thole and Ramu 2020):

- Complex engineered systems generate large amounts of data. Visualizing the data helps designers to understand the behavior of various system components, interactions between them and their emergent behavior under different conditions. For example, understanding the temperature changes or pressure fluctuations in a complex system facilitates designers to grasp the real-world scenarios easily.
- Visualization techniques help designers identify patterns, trends and irregularities within the data. They help in recognizing correlations between variables that might not be apparent when looking at raw data. This helps designers in understanding how changes in one aspect of the system might affect the other components or overall performance.
- In decision-based design there always exist making trade-off decisions between many conflicting goals. Data visualization enables clear representation of these trade-offs. For example, visualization techniques can illustrate the impact of different choices, making it easier for designers to weigh options and make informed decisions.
- Effective visualization helps simplify the communication of complex technical information and grasp complex data and understand the implications of design choices. This helps with collaboration and easy communication among multidisciplinary teams.

- Visualization assists in comparing data. By visualizing simulated or experimental data alongside predicted outcomes, designers can validate and refine their models.
- Visualization helps designers identify the potential risk within the complex system and allow risk mitigation strategies to be implemented in the design.
- Designers can observe the effects of design modifications in real time through data visualization. This facilitates an iterative design process, where changes can be made based on observed performance which leads to incremental improvements in the system.
- Data visualization empowers decision makers by presenting complex information in an easily understandable format. This allows for informed decisions which are robust and efficient.

Data visualization is the graphical representation of information and data. It utilizes visual elements like charts, graphs, and maps to help viewers understand complex data sets by displaying the data in a more accessible and easily understandable format. The primary goal of data visualization is to communicate information clearly and effectively through visual representations. It allows for the exploration, analysis and understanding of data patterns, trends and correlations that might not be apparent from raw data. By presenting information visually, data visualization enables quick identification of key insights, aiding in decision-making processes. Some of the basic visualization techniques involve charts and graphs, maps, infographics, and dashboards.

In the context of design engineering that involves expensive computer models, the use of metamodels serves as an efficient solution to mitigate the high computational cost associated with running these models (Thole and Ramu 2020). Meta models, also known as surrogate models or response surface models are simplified mathematical approximations of complex and expensive computer simulations or models. The meta models used depends on the basis functions, kernels and minimum sample size directly that influences the accuracy of the metamodel (Shan and Wang 2004). The accuracy of the metamodel is affected by several factors, such as the choice of basis function, kernels in the case of kernel-based methods like support vector machines and the sample size used to construct the metamodel. In metamodeling there exists a tradeoff among metamodel accuracy, the number of samples, the size of the design space and the number of dimensions. Designers often struggle with these trade-offs when attempting to build accurate metamodels within constraints such as limited sizes, expansive design spaces and higher dimensions. In engineering design process particularly during the conceptual or early design stages, it's crucial to gain insights into specific regions within the design space that correspond to good designs. The design space refers to the range of possible configurations, values, or combinations of design variables that can be chosen for a given product or system. Identifying these regions of interest (RoI) involves determining sets of design parameters or variables where the system or product performs well or meets design requirements. This can be viewed as a phase of exploration, by gaining the ability to comprehend or explore design spaces, rather than solely seeking the single point solutions, designers can make informed decisions. Finding the peaks and valleys, flat regions that correlate to robust regions, splitting the design space into favorable and unfavorable design regions, and offering insights on the function variation with respect to various variables are all part of the process known as "Design Space Exploration" (DSE) (Koch, Evans and co-authors 2002, López-Rubio 2013, Gan and Gu 2019).

A designer with a lot of data or information, cross-disciplined constraints and challenges will be more concerned about problem formulation, design space exploration and region of interest. For such scenarios advanced visualization enables enhanced comprehension and exploration in high dimensions. Incorporating visual techniques into a Design Space Exploration (DSE) method proves highly advantageous, facilitating well-informed decision-making. Therefore, it becomes imperative to employ an effective visualization technique in the process of Design Space Exploration (DSE). Some of the existing visualization techniques and their limitations are discussed below.

2.5.1 Tile Plot

A tile plot is a visualization technique that depicts hierarchical structures shown in Figure 2.12. It works especially well for displaying proportions and relationships within a dataset. In Tile plots a particular value, referred to as the baseline value, is selected for each dimension in these displays. While all other possible combinations of values across the remaining dimensions are investigated, this baseline value stays fixed or constant. These matrices represent functions or relationships between variables. Each matrix essentially displays the contours or shapes formed by these functions. Each cell within the matrix represents a value resulting from combining specific variables (Forrester, Sobester and co-authors 2008).

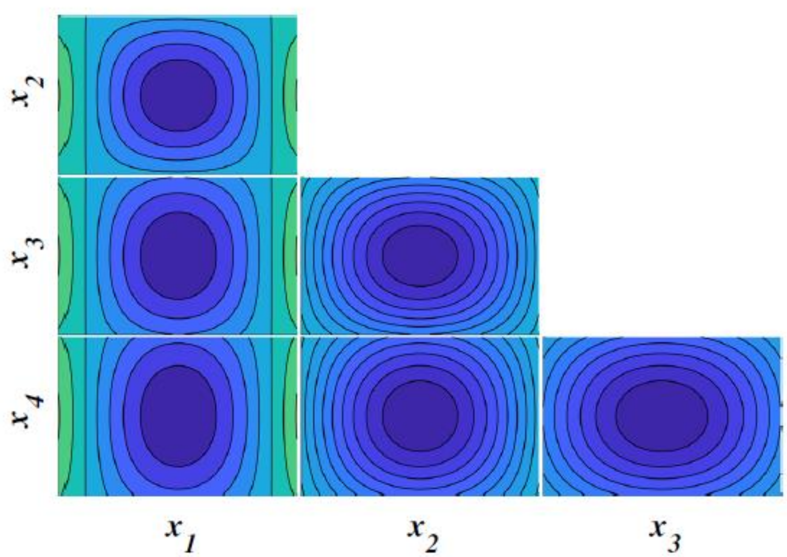


Figure 2.12: Tile plot (Thole and Ramu 2020)

The following describes how a tile plot works.:

- The plot starts with a single, usually rectangular, frame representing the entire dataset.
- This frame is then divided into smaller rectangles or tiles. The size of each tile is relative to the proportion of the data it represents. For example, larger tiles represent a larger share of the dataset, and smaller tiles represent a smaller share.
- Tiles are further divided into smaller tiles based on subcategories or hierarchies, continuing until the smallest level of detail is reached.
- Additionally, color or shading can be used to represent additional information within the tiles, such as a gradient for numerical values or different colors to represent different categories.

For each dimension (or variable) in the plot, a particular value is chosen as the "baseline." This baseline value remains constant while exploring the relationships between other variables. This means that one dimension is held steady, allowing examination of how the other variables interact or change concerning this fixed baseline. While the baseline value within each dimension remains fixed, the plot is constructed by varying the other dimensions across all their possible values. This enables an exploration of how the function contours change concerning different combinations of these other variables, with the baseline value held constant.

Limitation: In tile plot, focus is on a limited set of variables while keeping others constant, the visualization method may overlook or fail to represent the potential interactions between the fixed variables and the ones being actively plotted. Factor interactions, which could play a significant role in the overall relationship between variables, might not be fully captured in these plots due to the chosen method of visualizing only specific dimensions at a time and holding the remaining variables constant.

2.5.2 Parallel Axes Plot

Parallel axes plot is a data visualization technique used to display multivariate data in a two-dimensional space. Figure 2.13 shows a parallel axes plot.

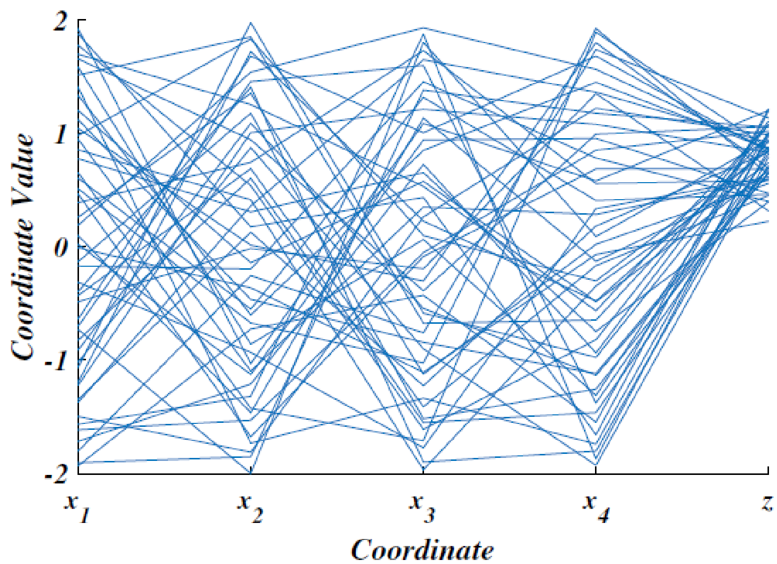


Figure 2.13: Parallel axes plot (Thole and Ramu 2020)

The following describes how a parallel axes plot works:

- The plot consists of parallel vertical axes, where each axis corresponds to a different variable or dimension of the dataset. These axes run parallel to each other.
- Next, every data point in the dataset is shown as a collection of connected line segments or as a polyline. For each given data point, the lines connect the points on each axis that represent the values of the variables.
- The values of the variables are indicated by the lines that meet the axes at various heights as you go horizontally across the plot. A data point's location on one axis is unrelated to its location on the other axes.
- The way the lines overlap along the axes can be used to identify patterns, trends, and correlations between the variables. In multivariate datasets, parallel coordinate maps are very helpful for locating relationships, outliers, and broad patterns.

Limitation: Each variable has its own axis in a plot with parallel axes, and data points are shown as lines joining values along these axes. The probability of lines crossing and intersecting one another grows with the number of dimensions. This may lead to a plot that is visually convoluted and complex. With more axes, the density of lines on the plot grows. As lines overlap, it becomes challenging to distinguish individual lines and perceive the relationships between variables accurately. This visual overlap can obscure patterns and trends in the data.

2.5.3 Nested Axes Plot

A nested axes plot involves using multiple parallel axes to represent different variables. Figure 2.14 shows a nested axes plot.

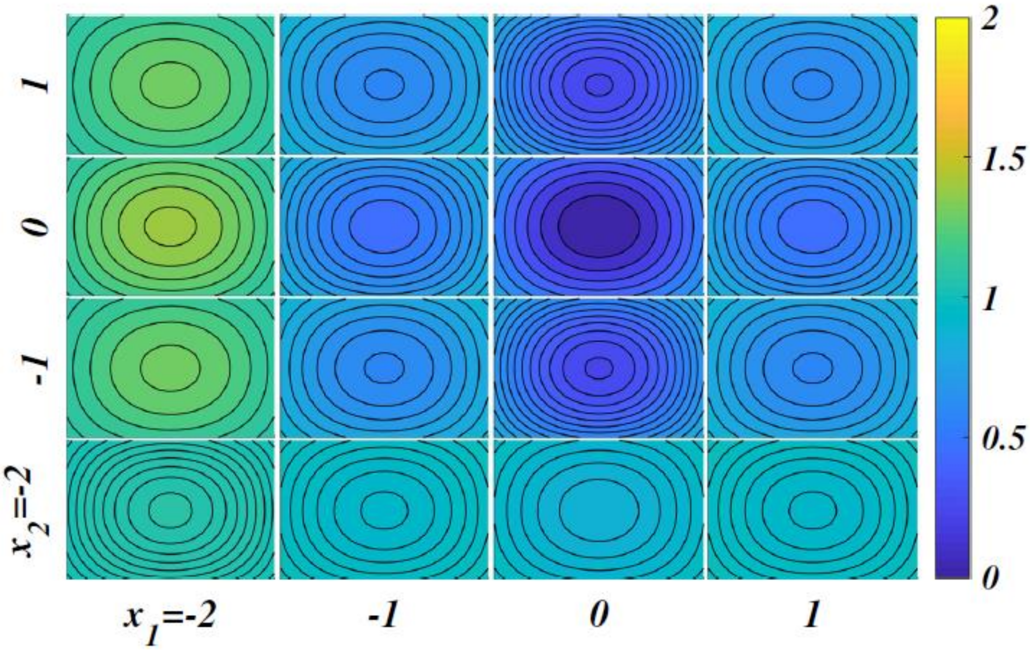


Figure 2.14: Nested axes plot (Thole and Ramu 2020)

The following describes how a nested axes plot works:

- The first step is axis alignment, each axis represents one variable, and they run parallel to each other.

- Next, each data point in the dataset is represented by a polyline, a set of connected line segments. Each line segment connects points on each axis corresponding to the values of the variables for that specific data point.
- The next step is axes nesting, here nesting might involve plotting specific variables against each other while keeping others constant. For example, x1 and x4 might be the primary variables of interest, and the plot could show how they interact while x2 and x3 are kept constant.

Limitation: As the number of variables (axes) increases, the plot can become visually cluttered and complex. Distinguishing individual lines and interpreting patterns may become challenging, particularly when dealing with a high-dimensional dataset. When several lines overlap, it's known as overplotting and makes it challenging to distinguish between individual data points or lines. This problem may mask significant trends or patterns in the data.

2.5.4 Ternary Plot

Ternary plots are graphical representations used to display data with three variables, often in contexts where the sum of the variables is constrained to a constant. Figure 2.15 shows a nested axes plot. The following describes how a ternary plot works:

- Ternary plots are typically represented by an equilateral triangle, where each corner of the triangle represents one of the three variables or components.
- The triangle represents all possible combinations of the three variables, and any point within the triangle represents a specific composition or mixture of the three components.
- One key feature of ternary plots is that the sum of the three variables always adds up to a constant value 1.

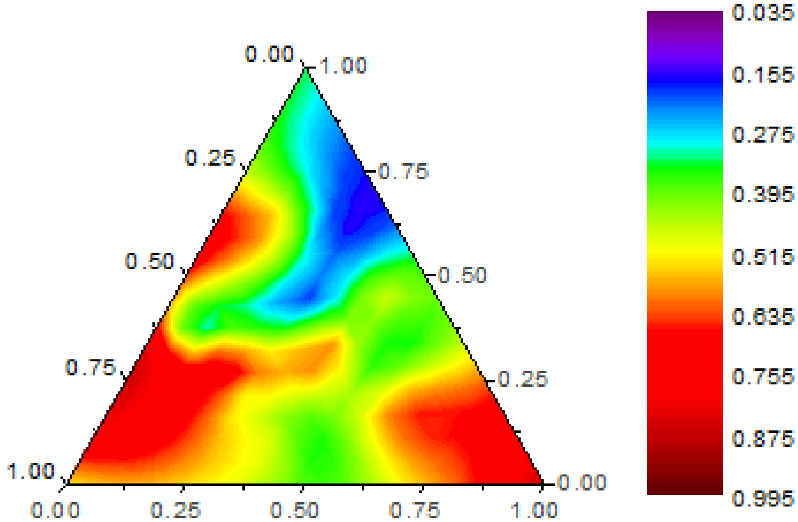


Figure 2.15: Ternary plot

- Each data point in the ternary plot corresponds to a specific composition, and its position is determined by the proportion of each component. The point is located along the lines connecting the corners of the triangle, with the distance from a corner indicating the proportion of the corresponding component.

Limitation: Ternary plots are specifically designed for visualizing data with three components. While ternary plots are excellent for understanding relative proportions and mixtures, they are not well-suited for precise quantitative readings. Extracting exact values from points on the plot can be challenging, and more accurate measurements may require additional numerical analysis.

There are many other visualization techniques used as well rather than the above-mentioned techniques. A 3D-rad technique is used to visualize the shape of multi objective optimization data (Ibrahim, Rahnamayan and co-authors 2016). The techniques of hierarchical axes and generative topographic mapping are discussed by Holden and Keane (Holden and Keane 2004) for visualizing huge design spaces, although they come with an additional computational load.

The number of plots needed for an effective visualization grows quadratically with the number of attributes in the dataset. This highlights the necessity of having a multi-dimensional Design Space Exploration (DSE) visualization method that is easily implementable. A method to find Regions of Interest (RoI) and record factor interactions and correlations easily is needed (Thole and Ramu 2020). The goal is to create a visualization technique that effectively grows in complexity with the dataset, enabling a thorough examination of the design space while addressing the difficulties brought on by an increasing number of features.

To overcome the above-mentioned limitations of visualization techniques a machine learning based visualization technique is used in this thesis, which is explained in detail in Chapter 3. This need leads us to Research Question (RQ 2), justification of this challenge is discussed in Chapter 5.

Chapter 3: Decision Based Design Framework for Visualization and Exploration for Problems with Many Goals

In Chapter 3 the proposed Decision Based Design (DBD) framework is discussed addressing research question 1 (RQ 1). In Section 3.1 and 3.2 we discuss some of the existing frameworks in use and their limitations. In Section 3.3 we discuss Decision based design framework used in this thesis. In Section 3.4, the utility of the framework is discussed and in Section 3.5 we discuss the foundational mathematical construct and robust design metrics. In Section 3.6 we discuss the machine learning based visualization technique called Interpretable Self Organizing Maps (iSOM).

3.1 The Inductive Design Exploration Method (IDEM)

Choi (Choi 2005), introduced the Inductive Design Exploration Method (IDEM) with the specific aim of attaining Type IV Robust Design, particularly for the integrated multiscale design of materials and products. IDEM addresses the propagation of uncertainty through scales (Choi, McDowell and co-authors 2008). IDEM streamlines the exploration of robust solutions and employs a metric called the Hyper-Dimensional Error Margin Index (HD_EMI) to evaluate the mapping across scales, as outlined by Choi (Choi 2005). A higher HD_EMI value signifies that the mapped region is distant from the boundary of the feasible region of interest, indicating reduced sensitivity to changes. Therefore, the HD_EMI value serves as an indicator of the reliability of a chosen design variable in meeting constraints and bounds. IDEM is formulated to furnish a range of robust solutions that account for propagated uncertainty (PU) and operate under model structure uncertainty (MSU). This is achieved by iteratively conveying the feasible solution range in an inductive manner, starting from the specified performance range, and extending into the design space. The IDEM comprises three steps, as illustrated in Figure 3.2 and are mentioned below:

- Concurrently conducting discrete function evaluations at each level of the design process, involving both bottom-up simulations and experiments.
- In the step referred to as Inductive Discrete Constraints Evaluation (IDCE), a top-down exploration of the feasible design space is performed using metamodels. This exploration leverages the Hyper-Dimensional Error Margin Index (HD_EMI) metric to evaluate the mapping from higher space to lower space, identifying robust solution ranges within the feasible space.
- The compromise decision support problem (cDSP) is formulated to determine the optimal solution under model structure uncertainty (MSU). It enables designers to pinpoint the most desirable robust solution from the set of feasible solutions obtained. This is accomplished by conducting a trade-off analysis among the HD_EMI values acquired. The cDSP serves as the fundamental computational framework within IDEM for facilitating design decision-making.

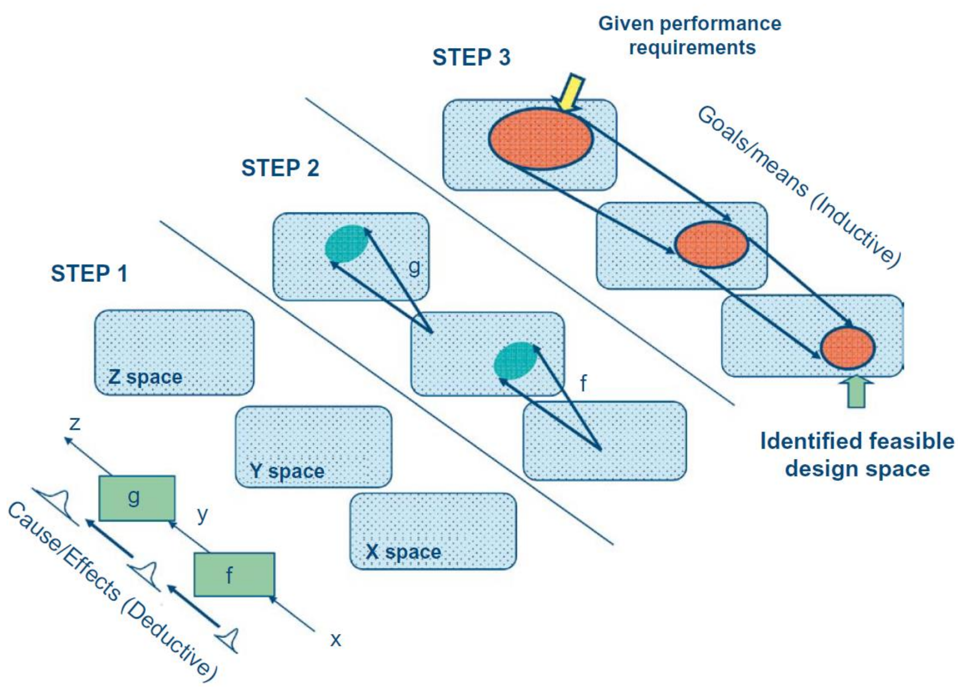


Figure 3.1: Solution search procedure in IDEM (Choi, McDowell and co-authors 2008)

With the use of IDEM we can identify robust solutions ranges with the consideration of uncertainty that is propagated across a process chain. And with the use of IDEM we can reduce the design iterations. Designers can effortlessly modify or update analysis models since there are no computational interfaces between the models. Designers only need to reassess their designs based on the modified model.

Limitation: IDEM employs a three-dimensional visualization space utilizing the HD-EMI metric for exploration. In this space, only a maximum of three design variables can be examined simultaneously, with the remaining variables assuming predefined values. This limitation constrains the scope of the simulation study and its outcomes. IDEM faces a constraint concerning the quantity of design variables applicable to a given design problem under examination. IDEM does not permit designers to introduce new goals or requirements at various levels during the design process. This restriction arises because the method relies on mapping to feasible spaces of 'Y' and 'X' for a given 'Z' space.

3.2 Robust Concept Exploration Method (RCEM-EMI)

The Robust Concept Exploration framework with Error Margin Indices (RCEM-EMI), designed to address Type I, II, and III robust designs was introduced by Choi and co-authors (Choi, Austin and co-authors 2005). Error margin indices are mathematical constructs that indicate the location of the mean response and the spread of the response considering the variability associated with design variables and system models. EMIs represent the margin against failure due to uncertainty in both model and design variables. The Error Margin Indices (EMIs) contribute to the development of Type I, II, and III robust designs. Subsequently, these are integrated as goals in the cDSP formulation, aiming to design the system while accounting for uncertainties in both model structure and model parameters. The RCEM-EMI procedure, as outlined by Choi, Austin, and their coauthors in 2005, comprises the following steps: (a) clarification of the design task, (b) DOE and simulation, (c) integrated metamodel and prediction interval estimation, and (d) design space search using the cDSP for the RCEM-EMI.

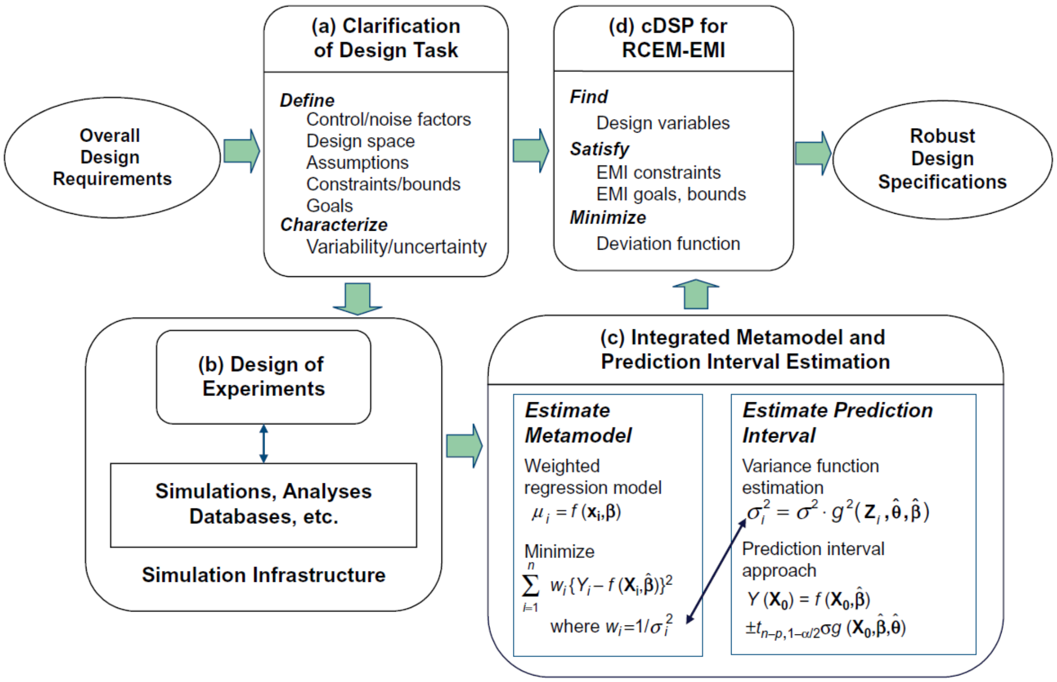


Figure 3.2: The RCEM-EMI framework (Choi, Austin and co-authors 2005)

Within the framework of RCEM-EMI, the Error Margin Indices (EMI) serve as robust design metrics, providing insights into the reliability level of a decision that meets system constraints and bounds. The comprehensive process of RCEM-EMI is illustrated in Figure 3.1.

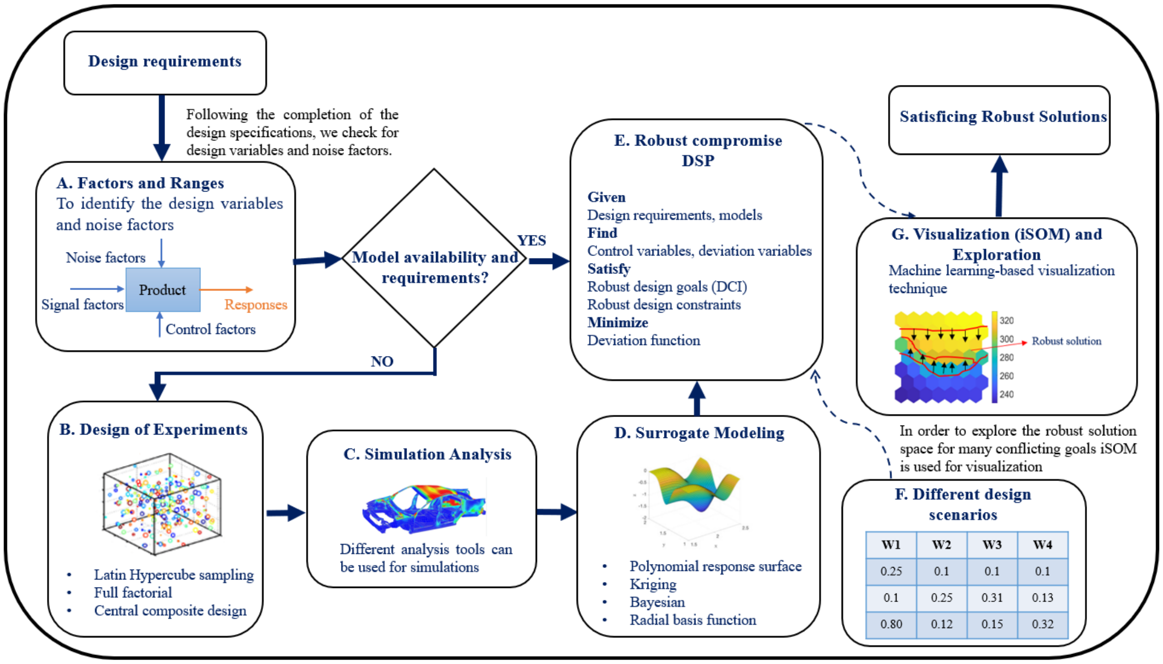
One significant benefit of RCEM-EMI, distinguishing it from alternative methods, lies in its capacity to yield precise outcomes in the realm of design exploration. This capability stems from RCEM-EMI's ability to account for uncertainties related to noise factors, control factors, and the model itself. By doing so, RCEM-EMI aids designers in making informed decisions within the context of a system's stochastic variability and/or uncertainties in model parameters.

Limitation: The RCEM-EMI framework is incapable to effectively handle multiple goals or performances that necessitate distinct types of robust design. The current form of RCEM-EMI, as described, lacks the capability to manage the propagation of all forms of uncertainty throughout process chains. Additionally, it demands a substantial number of experiments for uncertainty analysis even in a single evaluation during the design exploration phase, making it computationally expensive. In essence, these limitations underscore challenges in addressing diverse design objectives and managing uncertainty propagation efficiently, while also highlighting the computational demands associated with the methodology.

The methods outlined above reveal a limitation by accommodating only three goals. However, in complex systems, the number of goals often exceeds three, necessitating a framework that can effectively handle this complexity. Therefore, in this thesis, we introduce a decision-based design framework, which will be detailed in the following section, specifically addressing the challenges posed by problems involving many (more than three goals) conflicting goals under uncertainty. This need leads us to Research Question (RQ 1), justification of this challenge is addressed by proposing Decision based design framework discussed in next section.

3.3 Decision Based Design Framework for Visualizing and Exploring for Problems with Many Goals

In this section, we delve into the discussion of the Decision based design exploration framework, as depicted in Figure 3.3. The proposed framework draws inspiration from the Robust Concept Exploration Method (RCEM) introduced by Chen and coauthors (Choi 2005). This framework accounts for many (more than three) goals and uncertainty in the complex problems. The framework's description unfolds through the Steps A to G outlined below. The frameworks start with identifying the design requirements as shown in Figure 3.3. This step is an important task since design goals and strategy for design exploration are determined in this process. First the design goals need to be identified from the design requirements. This entails determining which performance criteria or responses should be selected. For example, considering the design of electric vehicle battery system, here one goal could be to enhance the system’s energy efficiency. Once goals and requirement limits are formulated, then the next step is identifying control and noise factors that affect the system performance.



W1	W2	W3	W4
0.25	0.1	0.1	0.1
0.1	0.25	0.31	0.13
0.80	0.12	0.15	0.32

Figure 3.3: Decision based design framework

Step A (Identifying factors and ranges): This step involves identifying the design requirements which is in Step A as shown in Figure 3.3. During this step, the designer identifies problem-specific information related to design variables, their bounds, and constraints based on the given design requirements. The figure shown in step is called the P-diagram proposed by Phadke (Phadke 1989), it represents the quality characteristics of a system, product or process. As shown in the figure, Phadke classifies parameters that can influence the quality characteristic or response of a product into the three types, which are signal factor (M), noise factor (x), and control factor (z). Signal factors are the parameters set by a user or an operator of a product to express the intended value for the response of the product (Phadke 1989). For example, speed, torque or acceleration are signal or input factors.

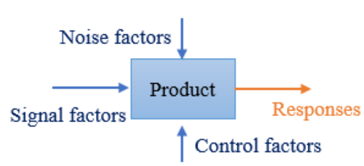


Figure 3.4: The block diagram of a product/process: P diagram (Phadke 1989)

The signal factors are selected by the design engineer based on the engineering knowledge of the product being developed. Noise factors are some parameters that cannot be controlled by the designer or are very expensive to control. Only the statistical characteristics (such as mean and variance) of a noise factor can be known or specified, but the actual value is uncertain. The noise factors make the response deviate from the target specified by the signal factor, and lead to quality loss. Example environmental factors or system interaction. Control factors are parameters that can be specified freely by a designer. Designers are responsible for selecting (designing) the control factors. For example, hardness, density, or thickness can be considered as design variables. Following this, a check is carried out to identify if mathematical models that relate the design variable to the goals are available explicitly. The designer moves on to Step E if models are available. If models are unavailable, the designer moves on to Step B.

Step B (Design of Experiments): In step DOE is carried out as shown in Step B of Figure 3.3. Considering the control and noise factors identified in the preceding section, it becomes essential to carry out design of experiments. Design of Experiments (DOE) is a systematic and efficient approach used in experimentation to understand how different factors or variables influence the outcome of a process or system. There are different types of DOE techniques that can be used based on the requirement. Central composite design technique can be employed when the relationship between response surface of a system and the independent variables (factors) needs to be understood. Nevertheless, designers have the flexibility to utilize alternative Design of Experiments (DOE) techniques, such as two or three-level factorial design, Latin Hypercube sampling (LHS) (McKay 1995), Box-Behnken (Borror, Montgomery and co-authors 2002), and others. This allows them to obtain the most precise response surface model with the minimal number of experiments or simulations. After carrying out DOE the next step would be simulations.

Step C (Simulations): In a decision-based design approach, simulations following Design of Experiments (DOE) play a crucial role in informing decision-making. Simulations allow designers to assess the performance of different design points identified through the DOE. By simulating the outcomes under various scenarios, designers can make informed decisions. Common simulation techniques include Monte Carlo simulations, numerical simulations using mathematical models, and computer-aided simulations to mimic real-world conditions. Simulations provide a powerful tool for analyzing and gaining insights into complex systems and processes.

Step D (Surrogate modeling): In this step, the process involves surrogate modeling, where the developed models serve as approximate representations of the actual underlying function models. These surrogates are crucial tools for the designer, facilitating the mapping of the design space to the response or performance space. The data generated from simulations in Step C forms the basis for constructing these surrogate models. The primary purpose of surrogate modeling is to create efficient and computationally fewer demanding approximations of complex and resource-intensive simulations. Design space comprises all possible combinations of input variables, while response or performance space represents the corresponding system behavior or outcomes. Surrogate models serve as a mathematical tool to map points in the design space to their corresponding locations in the response space. This mapping enables designers to predict the system's behavior for unexplored design configurations without the need for additional resource-intensive simulations. Once the function models are available from next step is to formulate the problem using cDSP construct.

Step E (Compromise decision support problem): In this step, the designer makes use of the surrogate models developed in Step D and the models identified in Step A and formulates the decision support problem with many goals. The compromise Decision Support Problem (cDSP) can be incorporated with DCI (Design Capability Index) and EMI (Error Margin Index) constructs into the formulation to account for uncertainties in design variables and model respectively. The cDSP allows designers to model problems with many conflicting goals. The cDSP is a hybrid of mathematical and goal programming. The problem specific information is captured in the cDSP using the four keywords (see Figure 3.3- Step E) - Given, Find, Satisfy, and Minimize. Using the cDSP the designer seeks to minimize the weighted sum of deviations of the goal values achieved from their targets. The designer can generate multiple design solutions by assigning different weights to the different goal deviations in the cDSP. This is the foundational mathematical construct used in this thesis and explained in detail in Section 3.4.

Step F (Exercising cDSP for different weight scenarios): Following the formulation of the compromise decision support problem (cDSP) in Step E, the designer proceeds to explore various design scenarios within this framework. Design scenarios are constructed by varying the weights associated with the deviation of goal values from their targets. Each scenario represents a specific set of preferences or priorities regarding the importance of achieving different goals within the design. The weights assigned to the goals indicate the designer's preferences for the different conflicting goals. By adjusting these weights, the designer expresses the relative importance of each goal in the overall decision-making process. This allows for a nuanced exploration of trade-offs and priorities. The outcomes of the cDSP for each design scenario are carefully examined and reported by the designer. These results provide insights into how different weightings impact the robustness and satisfaction of conflicting goals. Can be used to iterate through multiple design scenarios, adjusting weights and refining preferences based on the obtained results. This iterative process allows for a thorough exploration of the design space, fostering informed decision-making.

Step G (Exploration and Visualization of Design Space): The exploration and visualization of the design space are essential aspects of the design process, serving as pivotal tools for informed decision-making. Visualization facilitates the interpretation of complex relationships and patterns within the design space, providing designers with valuable insights into trade-offs and sensitivities. By comprehensively exploring and visually representing the design space, designers can identify robust solutions, uncover areas for improvement, and navigate the intricate interdependencies among design variables. This thesis uses a machine learning-based visualization technique called Interpretable Self-Organizing Maps (iSOM) to explore and visualize complex problems with many (more than three) goals. iSOM is discussed in detail in Section 3.5.

3.4 Compromise Decision Support Problem (Foundational Mathematical Construct) with Robust Design Metrics

This thesis employs the compromise decision support problem (cDSP) to address the many goal problems under uncertainty. The DSP is a hybrid formulation incorporating concepts from traditional mathematical programming and goal programming. cDSP is a mathematical formulation to identify compromised design solutions with many conflicting goals. The similarities between mathematical programming and the cDSP lie in how they address system constraints that need to be met for practicality. The way they model the deviation or goal function varies. Like goal programming, the cDSP models the deviation function using deviation variables rather than system or decision variables. Multiple objectives are stated as system goals, including deviation variables. Nevertheless, the cDSP is different from goal programming because it is designed to manage typical engineering design scenarios where physical constraints show up as bounds on the system variables and system constraints (mainly inequalities).

Given

- n, number of system variables
- p, number of equality constraints
- q, number of inequality constraints
- m, number of system goals
- $g_i(\mathbf{x})$, constraint functions
- G_i, , system goals
- $A_i(\mathbf{x})$, performance functions

Find

- $\mathbf{x}$ (system variables)
- d_i^-, d_i^+ (deviation variables)

Satisfy

System constraints:

$g_i(\mathbf{x})=0$

$g_i(\mathbf{x})\leq 0$

System goals:

$A_i(\mathbf{x})/G_i + d_i^- - d_i^+ = 1 \quad i=1,\ldots,m$

Bounds:

$x_i^{min} \leq x_i \leq x_i^{max}$

$d_i^-, d_i^+ \geq 0$ and $d_i^- \cdot d_i^+ = 0 \qquad i=1,\ldots,n$

Minimize

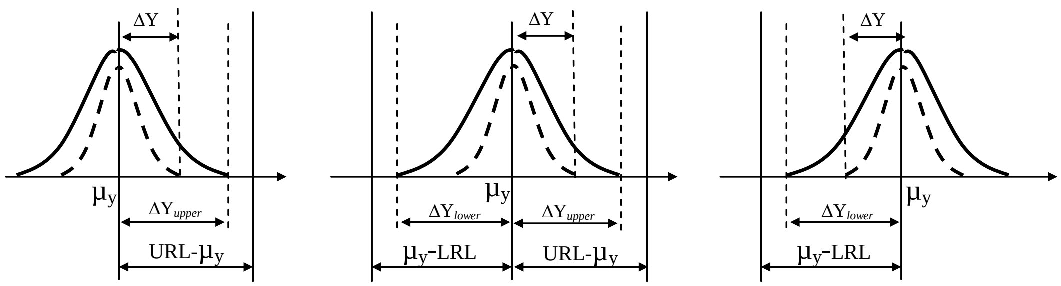
$Z = [f_1(d_i^-,d_i^+), \ldots, f_k(d_i^-,d_i^+)]$ *Preemptive*

$Z = \Sigma W_i (d_i^- + d_i^+)$ $\Sigma W_i=1$ *Archimedean*

Figure 3.5: Mathematical construct of compromise decision support problem

3.4.1 Robust Design Metrics

This section discusses the idea of robustness metrics, namely the Error Margin Index (EMI) and Design Capability Index (DCI), to control and lessen the consequences of uncertainty. The uncertainty bounds resulting from changes in the model and design variable are depicted in the following two images, along with the creation of mathematical constructs to deal with these uncertainties (Choi, Austin and co-authors 2005).



(a) Smaller is better
EMI= $(URL-\mu_y)/\Delta Y_{upper}$
EMI ≥ 1

DCI = $(URL-\mu_y)/\Delta Y$
DCI ≥ 1

(b) Nominal is Better
EMI =Min [EMI_l, EMI_u]
where,
$EMI_l = (\mu_y-LRL)/\Delta Y_{lower}$
$EMI_u = (URL-\mu_y)/\Delta Y_{upper}$
EMI ≥ 1

DCI =Min [DCI_l, DCI_u]
where
$DCI_l = (\mu_y-LRL)/\Delta Y$
$DCI_u = (URL-\mu_y)/\Delta Y$

(c) Larger is better
EMI= $(\mu_y-LRL)/\Delta Y_{lower}$
EMI ≥ 1

DCI= $(\mu_y-LRL)/\Delta Y$
DCI ≥ 1

Figure 3.6: Mathematical constructs of EMIs and DCIs (Choi, Austin and co-authors 2005)

In Figure 3.7, the mathematical formulations for implementing EMIs or DCIs as a goal in DSPs are shown. "Smaller is better" means that we are looking to minimize the targeted function, while "Larger is better" means that we are looking to maximize the targeted process. Further, "Nominal is better" means that we are interested in getting a value as nearer as possible to the target set, that is, we want to avoid underachievement and overachievement.

3.4.2 Design Capability Index (DCI)

DCIs represent the safety margin against systems failure due to uncertainty in design variables. In particular, DCIs mean the degree of reliability by measuring the capability of design decisions (Sharma, Allen and co-authors 2021):

(i) To satisfy the design requirements.

(ii) To tolerate the effect of uncertainty in design variables.

The system model's uncertainty bounds are represented by two adjacent dotted curves in Figure 3.6, while the model's mean response (μ) is shown as a solid red curve. The mean response model predicts that at x, with a variation of + Δx in the design variable, the expected variation in response is ΔY_0. Similarly, as the figure illustrates, the predicted variation in response for the two uncertainty bounds for the identical change in design variable at x is ΔY_1 and ΔY_2, respectively. This will enable us to determine the highest possible predicted deviation in the response for every given combination of x and Δx.

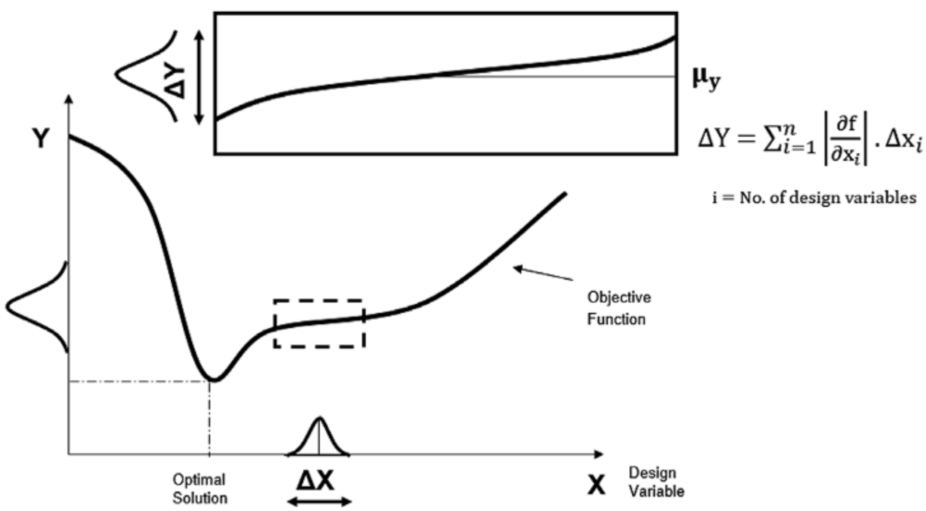


Figure 3.7: Formulation of uncertainty bounds due to variations in a design variable and a model (Choi, Austin and co-authors 2005)

Steps for formulating Goals as DCIs :

Step 1: Using a first order Taylor series expansion, the response variation due to variation in the design variable vector x = {x_1, x_2……, x_n} is estimated. The response variation(ΔY) for small variations in design variables is as

$$\Delta Y = \sum_{i=1}^{n} |\frac{\partial f}{\partial x_i}|. \Delta x_i \qquad \text{Equation 3.1}$$

where, $\Delta \boldsymbol{Y}$ represents function model equation, $\frac{\boldsymbol{\partial f}}{\boldsymbol{\partial x_i}}$ represents the differentiation of the function equation, and $\Delta \boldsymbol{x_i}$ represents the variance allowed in design variable.

Step 2: Using the mean response (μ_y) obtained from the mean response model ($f_0(x)$) and the response variation due to variation in design variables (ΔY), calculate the DCIs. For a 'Larger is Better' case, the DCI is calculated as

$$\text{DCI} = (\mu_y - \text{LRL}) / \Delta\text{Y} \qquad \text{Equation 3.2}$$

where, LRL is the lower requirement limit. A DCI ≥ 1 means that the ranged set of design specifications satisfies a ranged set of design requirements, and the system is robust against uncertainty in design variables. The higher the value of DCI, the higher is the measure of safety against failure due to uncertainty in design variables.

Example Calculation for DCI (Vehicular side crash example problem)

The total energy absorption calculation for a vehicular side crash problem used in Chapter 4 is illustrated below,

Step 1: Establish the functional relationship of total energy absorption in terms of design variables.

$E_{Total} = 0.55646 - 0.10339x_1 + 0.09375x_2 + 0.30379x_3 + 0.86088x_4 + 0.36049x_5 + 0.10462x_1^2 + 0.00639x_1x_2 + 0.0843x_1x_3 + 0.3121x_1x_4 + 0.2085x_1x_5 + 0.07593x_2^2 - 0.0329x_2x_3 - 0.04357x_2x_4 - 0.04462x_2x_5 - 0.07953x_3^2 - 0.08116x_3x_4 + 0.06229335x_3x_5 - 0.39304x_4^2 - 0.36104x_4x_5 - 0.0392x_5^2$

Step 2: Evaluate the partial differentiation of E_{Total} with respect to the design variables

$$\frac{\partial \text{ETotal}}{\partial x_1} = -0.10339 - 0.20924x_1 + 0.00639x_2 + 0.0843x_3 + 0.3121x_4 + 0.2085x_5$$

$$\frac{\partial \text{ETotal}}{\partial x_2} = 0.09375 + 0.00639x_1 + 0.15186x_2 - 0.0329x_3 - 0.04357x_4 - 0.04462x_5$$

$$\frac{\partial \text{ETotal}}{\partial x_3} = 0.30379 + 0.00843x_1 - 0.0329x_2 - 0.159x_3 - 0.08116x_4 + 0.06229x_5$$

$$\frac{\partial \text{ETotal}}{\partial x_4} = 0.8608 + 0.3121x_1 - 0.0435x_2 - 0.08116x_3 - 0.786x_4 - 0.36104x_5$$

$$\frac{\partial \text{ETotal}}{\partial x_5} = 0.3604 + 0.2085x_1 - 0.0446x_2 + 0.0622x_3 - 0.36104x_4 - 0.0784x_5$$

Step 3: Using a first order Taylor series expansion, estimate the response variation due to variation in the design variables. The response variation (ΔY) for small variations in design variables is

$$\text{Y} = \left|\frac{\partial \text{ETotal}}{\partial x_1}\right|.\Delta\partial x_1 + \left|\frac{\partial \text{ETotal}}{\partial x_2}\right|.\Delta\partial x_2 + \left|\frac{\partial \text{ETotal}}{\partial x_3}\right|.\Delta\partial x_3 + \left|\frac{\partial \text{ETotal}}{\partial x_4}\right|.\Delta\partial x_4 + \left|\frac{\partial \text{ETotal}}{\partial x_5}\right|\Delta\partial x_5$$

Step 4: Using the mean response obtained from the mean response model (Equation derived in Step 1) and the response variation due to variation in design variables (ΔY), calculate the DCI. For a 'Larger is Better' case, the DCI is calculated as

$$\text{DCI} = \frac{\text{ETotal} - LRL}{\Delta \text{Y}}$$

where, LRL is the lower requirement limit, which can be set based on the design requirement.

3.4.3 Error Margin Index (EMI)

EMIs represent the amount of safety margin against system failure due to uncertainty in the design model itself. They represent the degree of reliability by measuring the capability of design decisions (Sharma, Allen and co-authors 2021):

(i) To satisfy the design requirements.

(ii) To tolerate the effect of uncertainty in design models.

The solid curve in Figure 3.8 represents the mean response (μ) of the model, while the adjacent dotted curves depict the uncertainty bounds associated with the system model. When there is a variation of +ΔX in the design variable at a specific point X, the mean response model predicts an expected variation in the response denoted as ΔY. Similarly, for the same change in the design variable at X, the expected variations in response within the two uncertainty bounds are ΔY1 and ΔY2. This graphical representation aids in determining the maximum anticipated deviation in response for any given value of X and ΔX, offering insights into the potential variability considering the uncertainties inherent in the system model.

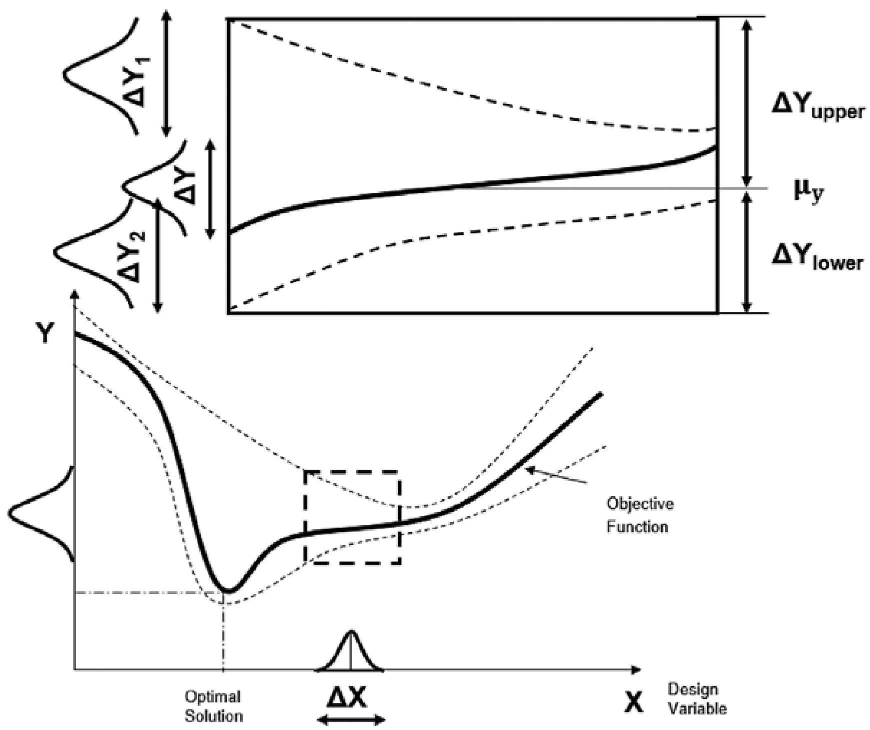


Figure 3.8: Formulation for uncertainty bounds due to variations in a model (Choi, Austin and co-authors 2005)

Steps for formulating Goals as EMIs:

Step 1: If a system model incorporates uncertainty bounds, the calculation of response variation (ΔY) for each of these bounds, considering minor variations in design variables, is carried out as:

$$\Delta Y = \sum_{i=1}^{m} \left| \frac{\partial f}{\partial x_i} \right| \cdot \Delta x_i \qquad \text{Equation 3.3}$$

Step 2: After evaluating various response variations from both the mean response function and uncertainty bound functions due to changes in design variables, the minimum and maximum responses are computed. These calculations factor in the variability in design variables and the uncertainty bounds surrounding the mean response.

$$Y_{\max} = Max\left\{ f_j(\mathbf{x}) + \sum_{i=1}^{m} \left| \frac{\partial f_j}{\partial x_i} \right| \cdot \Delta x_i \right\} \text{ and } Y_{\min} = Min\left\{ f_j(\mathbf{x}) - \sum_{i=1}^{m} \left| \frac{\partial f_j}{\partial x_i} \right| \cdot \Delta x_i \right\} \qquad \text{Equation 3.4}$$

Where *j=0,1,2, ... , n,* $\mathrm{f}_0(\mathbf{x})$ is the mean response model and $\mathrm{f}_1(\mathbf{x})...\mathrm{f}_n(\mathbf{x})$ are uncertainty bound functions.

Step 3: Calculate upper and lower deviations, which are the deviations from the mean response to the maximum and minimum responses, respectively, are represented as

$$\Delta Y_{upper} = Y_{\max} - f_0(\mathbf{x}) \text{ and } \Delta Y_{lower} = f_0(\mathbf{x}) - Y_{\min} \qquad \text{Equation 3.5}$$

Where is the mean response, $Y_{\max}$ is the maximum response, $Y_{\min}$ is the minimum response, ΔY_{upper} is the upper deviation, and is the lower deviation.

Step 4: Using the mean response obtained, the mean response model, and the upper and lower deviations, the EMIs are calculated.

EMI = (μ_y – LRL) / ΔY (Larger is the better case) Equation 3.6

3.5 Visualization and Exploration of Design Space

In design space exploration (DSE), efficient visual representation is crucial for making informed decisions. The common approaches discussed in Chapter 3 involve using projections of axes for visualization. However, these projections have limitations, especially when capturing interactions between different factors and dealing with higher dimensions. In simpler terms, visualizing complex design spaces using axis projections may only partially represent how various factors interact, particularly in situations involving many goals. Other visualization techniques are in use, like Radial coordinate visualization (RadVis,) which maps high-dimensional data into two dimensions (Ibrahim, Rahnamayan and co-authors 2016). Holden and Keane (2004) explore generative topographic mapping and hierarchical axes technique to visualize extensive design spaces. However, these methods introduce extra computational demands. A bubble chart utilizes color and bubble size as additional features to depict different dimensions. Nonetheless, bubble charts are effective only for up to five dimensions. Typically, a grid of scatter plots is employed to analyze the connection between various attributes. As the number of attributes grows, the number of plots increases significantly. Therefore, it's beneficial to create a straightforward method for visualizing design space in Design Space Exploration (DSE) that works regardless of the number of dimensions. This approach should help identify the Region of Interest (RoI) and enable the capture of factor interactions and correlations. To overcome the specified limitation, this thesis employs a visualization technique called iSOM, which is based on machine learning. A detailed discussion of iSOM will follow in the next section. However, before delving into iSOM's functioning, it's essential to comprehend how Self-Organizing Maps operate discussed in next section.

3.5.1 Self-Organizing Maps

A Self-Organizing Map (SOM) is a type of artificial neural network that operates on the principle of unsupervised learning. When given high-dimensional data as input, SOM generates a typically two-dimensional representation of this input data. What sets SOM apart is its ability to preserve the topology of the original high-dimensional space. In this context, topology preservation means that the mapping from high dimension to low dimension maintains the relative distances between different points in the input data. Additionally, SOM exhibits generalization capability, implying that the map can effectively characterize inputs it has never encountered before. This allows SOM to create a condensed, organized representation of complex data while preserving its structural relationships.

SOM Structure consists of a two-dimensional grid as shown in Figure. The features of SOM structure are discussed below,

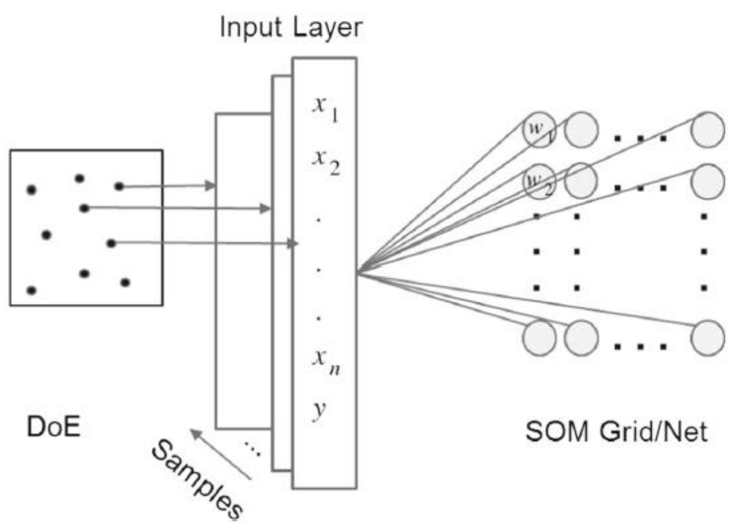


Figure 3.9: SOM structure (Thole and Ramu 2020)

Structure

- Each node corresponds to a set of values called codebook vectors.
- Codebook vectors have a size equal to the number of dimensions in the dataset.
- Nodes can be connected in a rectangular or hexagonal topology.

Input data representation

- If there are n design variables (x1, x2, ..., xn) and a response variable y, each input vector for SOM is represented as v_i = [x_{i1}, x_{i2},, ..., x_{in},, y_i,].

- Each SOM node w_j, has weight vectors [m_{j1},, m_{j2},, ..., m_{jn}, m_{jout}].
- First n weights correspond to design variables, and the last weight corresponds to the response.

Initialization of weight

- Values of weights are initialized using linear initialization.
- Linear initialization involves setting weights in the space spanned by two eigenvectors with the highest eigenvalues.

SOM organizes and understands high-dimensional data in a visually meaningful way and helps identify patterns, relationships, and clusters in the data.

3.5.2 Working of SOM Algorithm

Working of Self-Organizing Maps (SOM) involves a process of training and adaptation to organize and represent high-dimensional data in a lower-dimensional space. Here are the key steps in the working of SOM:

Step 1 (Initialization): First step is to start with a two-dimensional grid of nodes, where each node represents a potential cluster or group in the data and assign random initial values (weights) to the nodes. These weights represent the features of the data.

Step 2: To calculate the Euclidean distance or Best Matching Unit (BMU) between input data and weight of each node and next identifying the closest weights to the input nodes. These nodes are the winning nodes.

Step 3 (Updating weight): Weight of the winning and neighboring nodes are updated according to the winning nodes. The weights of the neighboring nodes are pulled in the direction of the input data point throughout the updating process. The farther you are from the winning node, the less updated.

Step 4 (Iteration): The calculation of the Euclidean distance is repeated for each dataset and each iteration refines the input space representation.

3.5.3 Limitation Of Self Organizing Maps (SOM)

One of the major limitations of SOM is self-folding. Self-folding is a process where each input node in SOM grid is mapped to multiple output grids. In SOM high dimensional data points are mapped to two-dimensional grid points and each of these grid points are associated with weights representing a position in input space. Ideally, each input node should be associated with a specific SOM grid which helps in interpreting the relationships between the input data. But in SOM there exists self-folding resulting where one input node associated with multiple nodes on the SOM grid. Due to this limitation visualizing the plots becomes challenging and the interaction between the variables cannot be interpreted. The Figure 3.10 and 3.11 show the with and without self-folding of the function $z = x^2 + y^2$ respectively, consisting of 20 Latin hyper cube sampling points and corresponding response variables (Thole and Ramu 2020).

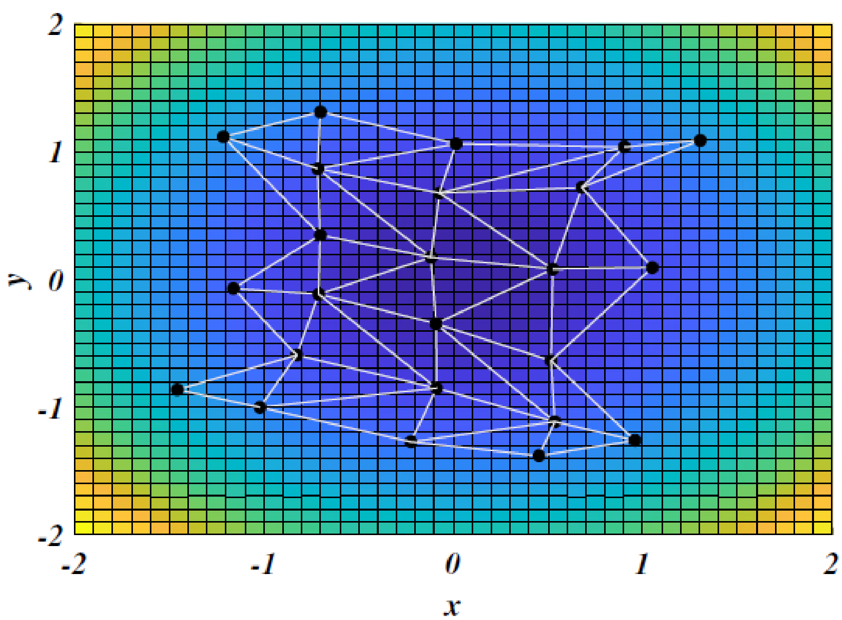


Figure 3.10: SOM plot without self-folding for the given function (Thole and Ramu 2020)

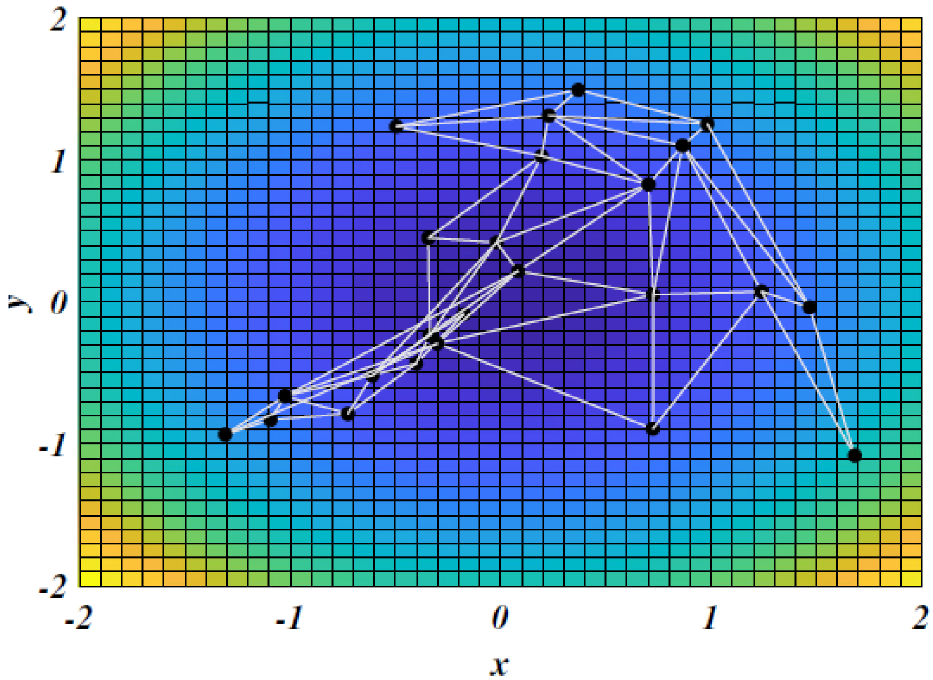
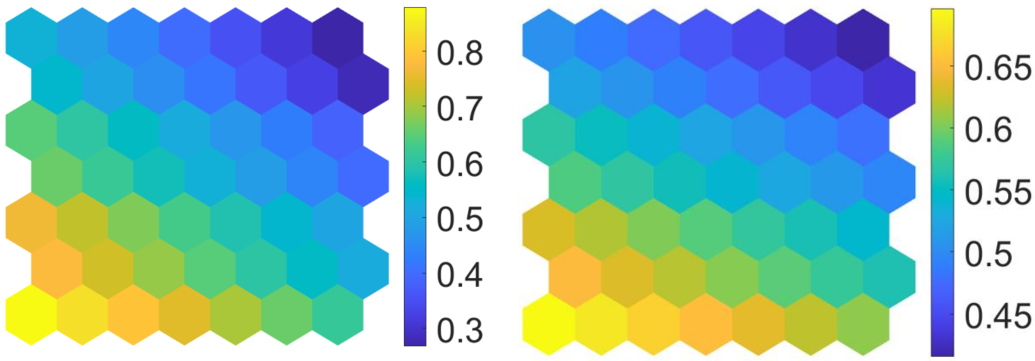


Figure 3.11: SOM plot with self-folding for the given function (Thole and Ramu 2020)

In order to address the challenges posed by self-folding in conventional Self-Organizing Maps (SOM), a specialized machine learning technique known as Interpretable Self-Organizing Maps (iSOM) is employed in this thesis. The operation of iSOM closely mirrors that of traditional SOM, with an alteration in the selection process for the Best Matching Unit (BMU). The details of this adjustment are explained in the next section, highlighting how iSOM helps make visualizations clearer and more understandable by dealing with self-folding.

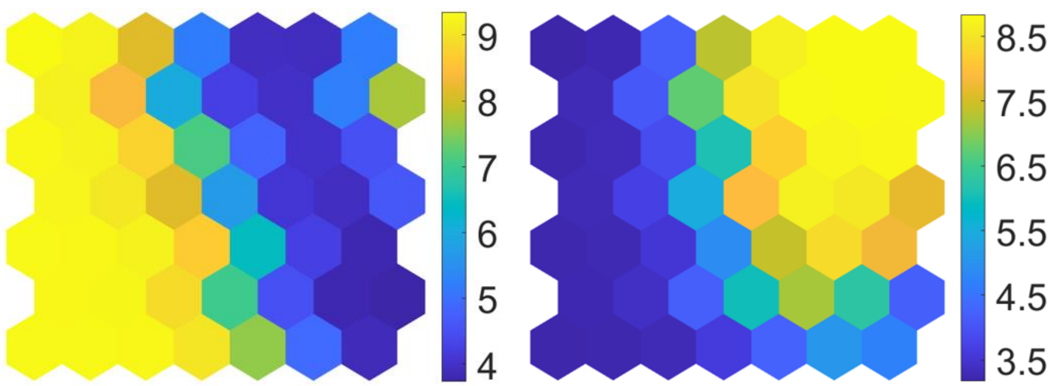
3.6 Interpretable Self-Organizing Maps (iSOM)

iSOM is a machine learning based visualization technique used to address issues related to SOM. Self-intersections and other problems are avoided, and inherently interpretable outputs are obtained by iSOM. The key distinction in iSOM's implementation lies in how it selects data for estimating the Best Matching Unit (BMU). The output data is not considered by iSOM, in contrast to SOM, which solely takes into account the input data while considering the BMU. Additionally, iSOM deletes the input variables during the update stage and only uses the answer value. In iSOM, these modifications avoid folding or self-intersection. Component plane plots that are produced have an ordered structure, which is a benefit that has been noted and makes component planes easier to interpret. The Figures 3.12 and 3.13 show the iSOM input plots and output plots respectively. The iSOM plots provide insights into the trends of the goals and their interconnections. They also allow us to comprehend the impact of input plots on the output plots.



Weight 1(W1) Weight 2(W2)

Figure 3.12: iSOM input (weight) plots



Goal 1(G1) Goal 2(G2)

Figure 3.13: iSOM output (goal) plots

The initial set of plots showcases the input, with each plot (Figure 3.12) representing the weight assigned to goals. For instance, W1 signifies the weightage assigned to the first goal (G1), and W2 indicates the weightage for the second goal (G2). These plots provide insights into how the output influences the input; for example, a high W1 corresponds to high G1 and low G2, while a low W2 leads to low G1 and high G2.

The second set of plots illustrates the output, specifically the goal plots (Figure 3.13). In iSOM plots, it's straightforward to interpret the interconnections among the goals. For instance, a high value of G1 corresponds to a low value of G2, indicating a conflict between these two goals. iSOM proves particularly beneficial in scenarios with numerous goals, as it simplifies the interpretation of relationships.

In order to validate the proposed DBD framework outlined in this chapter, a specific scenario related to vehicular crashworthiness has been chosen for detailed examination. Further insights into this example can be found in the subsequent Chapter 4.

Chapter 4: Vehicular Crashworthiness Example

In Chapter 4 a vehicular side crash test example is presented to evaluate the effectiveness and applicability of the decision-based design framework. In section 4.1 a brief introduction of the problem is discussed and in Section 4.2 the problem is defined in detail. After defining the problem, utility of the framework is discussed in Section 4.3. In Section 4.4 limitation of the proposed framework is discussed. In summary, Chapter 4 follows a structured approach, starting with an introduction to the problem, defining the problem, discussing the utility of the decision-based design framework, and finally, addressing the limitations associated with the proposed framework.

4.1 Introduction to the Vehicular Crashworthiness

In the automotive industry, the ongoing pursuit is to enhance the performance of vehicles and their components, with a primary focus on safety, efficiency, and economy. Improved vehicle design, particularly in terms of safety and fuel efficiency, is a key objective. Despite the advantages of lightweight designs in terms of performance and emissions, there is a constant challenge to strike a balance between achieving efficiency and ensuring safety. In the drive for fuel efficiency and economy, designers often turn to the use of lighter materials. However, this emphasis on lightweight designs can pose a potential conflict with safety considerations. Safety in vehicle design is assessed through the concept of 'crashworthiness,' which refers to a vehicle's ability to protect occupants from injury or fatality during collisions. As regulatory safety standards become more stringent, manufacturers face the imperative to create cars that not only meet efficiency goals but also prioritize occupant safety in the event of a crash. The study of vehicle crashworthiness has thus become pivotal in evaluating the safety aspects of vehicle and component design. Designers working on crashworthiness strive to develop robust, crush-resistant components capable of absorbing and dissipating maximum energy in a controlled manner during a collision. However, the pursuit of maximum energy absorption often leads to designs with increased thickness and subsequently greater mass. Thus, there arises a crucial imperative to facilitate the development of vehicle designs and vehicular components that successfully navigate the conflicting goals of safety and lightweight construction. Automobile crashworthiness is a difficult problem for designers to solve, and the rapid improvements in technology have made this much more complex. Achieving this balance necessitates a deliberate trade-off, wherein designers must preserve the structural integrity of vehicle components while enabling the vehicle's framework to absorb maximal energy through controlled deformation, ensuring occupant safety in the event of an accident.

An established method to ascertain the safety of lightweight vehicular and component designs is through vehicular experimental crash testing. However, this approach, while effective, is characterized by its high cost and time-intensive nature, making it a consideration primarily in the final stages of the design process. To overcome these obstacles, a different approach makes use of computer simulations. Although simulation-based approaches offer relative cost efficiency, they are not without their constraints. These limitations stem from factors such as a) constraints on computational resources, b) the inherent complexity of simulations, and c) design-related challenges. These design challenges encompass the necessity to reconcile many (more than three) conflicting goals, address uncertainties in design variables, and effectively visualize and explore the intricate, high-dimensional design spaces.

It becomes essential to methodically handle the related difficulties while managing these complexities, carefully considering the trade-offs involved in each strategy. Achieving an effective balance of safety and lightweight design through creative techniques emerges as a crucial goal in the constantly changing vehicle engineering scene as technology continues its rapid advancement. When this problem is considered from an optimization approach, a single-point solution is obtained, such as minimizing weight or maximizing fuel efficiency. However, in real-world vehicle design, multiple objectives, such as crashworthiness, performance, and aesthetics, need to be considered simultaneously. These goals often conflict with each other, requiring designers to make trade-offs. conventional deterministic optimization approaches assume that all design parameters are known with certainty. However many design parameters are uncertain due to factors such as material properties, manufacturing tolerances, and environmental conditions. Deterministic optimization methods fail to account for these uncertainties, leading to designs that may not perform as expected in real-world scenarios. Optimization methods are computationally expensive, requiring numerous iterations of simulations and calculations to find a single optimal solution. This makes them unsuitable for early-stage design exploration, where designers need to quickly generate a wide range of potential solutions to explore the design space.

To address these challenges, the decision-based design framework proposed in Chapter 3 is employed for a vehicular side crash problem. This framework stands out by considering many conflicting goals, uncertainties in the design process and effectively visualize and explore solution space. Unlike traditional approaches that focus on finding a single optimal solution through computationally intensive iterations, this framework enables designers to efficiently explore a broad range of potential solutions, making it well-suited for the early stages of design where a variety of solutions need to be quickly identified and evaluated. In the next section the vehicular side crash problem considered is described in detail.

4.2 Problem Definition

Vehicular crash tests are usually done to make sure that a car's design meets the required safety standards. One important is to check how much energy the car absorbs during a crash. When a crash happens, some of the energy is taken in by the car's structure and parts, but the rest gets transferred to the car's components. This transfer of energy can make the car bounce back after a crash and can lead to serious injuries or even death for the people inside. The more energy the car structure and parts can absorb in the early stages of a crash, the less harm is likely to happen to the people inside.

This thesis focuses on problems considering both light and safe during side crashes. When we make a vehicle lighter, it might affect its safety in certain ways. Designing cars that are both lightweight and safe is tricky because these goals often conflict with each other. To achieve this balance, we need to explore ways to reduce weight while ensuring the car can absorb energy effectively during a crash. The structure that absorbs energy during a collision plays a big role, and this thesis specifically looks at a 1996 Dodge Neon car model. The energy-absorbing structures that have been identified consist of five components, which are illustrated in Figure 4.1 and detailed in Table 4.1.

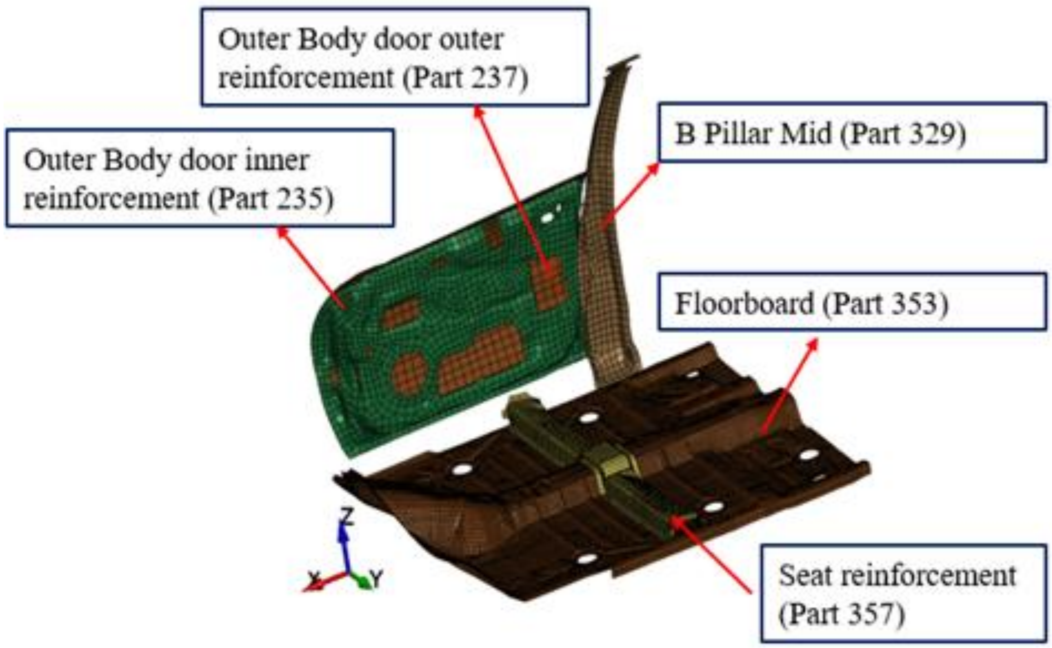


Figure 4.1: Five components considered for vehicle crashworthiness.

Table 4.1: Component's description

Part No.	Part Description
235	OB-DOOR-FT-I-R (Outer body door inner reinforcement)
237	OB-DOOR-FT-O-R (Outer body door outer reinforcement)
329	CH-B-PILLAR-MID-R (B Pillar Mid)
353	CH-CBN-FLOORBRD-FT (Floorboard)
357	CH-CBN-SEAT-REINF-FT (Seat Reinforcement)

The design problem considered aims to maximize the energy absorbed by each of the first three components—Parts 235, 237, and 353 (first three goals) and decrease the total mass of all five components (the fourth goal).

The selection of the three components is determined by two main factors: i) the potential for achieving the highest reduction in mass, and ii) the components capability to absorb more energy. This evaluation is based on data obtained from a simulated side crash trial, where the mass and total energy absorption of car components were measured. Components were chosen if they had either higher mass, absorbed more energy, or fulfilled both criteria. This selection is crucial because having these specific parts offers greater potential to enhance the vehicle design in alignment with the problem's goals. The details of the chosen five components for the vehicular side crash problem are provided in Table 4.1.

The process of creating a lightweight, safe vehicle is difficult for two primary reasons: first, there are numerous design variables and their intricate interactions; second, there are many conflicting things to take into account. There may be thousands of possible designs as a result of these considerations, particularly in the early stages of design. It becomes impractical to rely only on a design engineer's experience or to carry out expensive and time-consuming physical tests. Rather, we use simulations to come up with ideas for designs or fixes. Subsequently, we investigate these design or solution spaces in order to find "satisficing solutions" that reconcile the conflicting goals.

Uncertainties in design variables resulting from manufacturing variations, random noises associated with variations in material properties, and uncertainties in the models used, arising from approximations made in representing the true relationships between design variables and responses, all impact the process of designing safe lightweight vehicles. As a result, it becomes essential to take these uncertainties into account and deal with them while designing safe, lightweight cars using simulation. More than three goals are frequently involved in the design of complex systems, such as the development of lightweight, safe automobiles. As such, it is necessary to enable the comprehensive investigation of design issues involving many (more than three) goals. Therefore, a Decision-Based Design (DBD) framework designed for problems with many goals is employed for this example. With the help of this framework, designers can: i) robust solution space for the complex systems; and ii) visualize and explore the solution space to identify satisficing solutions.

In the next section a detailed discussion about the use of decision-based design framework for vehicular crashworthiness problem is demonstrated.

4.3 Utilization of the Decision Based Design Framework for Vehicular Crashworthiness

The steps to be followed are presented in the decision-based design framework discussed in Chapter 3. Designers begin by specifying the design requirements specific to the problem, and the subsequent steps of the framework are outlined below.

Step A: The four goals considered are to maximize energy absorption (first three goals) during a side impact scenario while minimizing overall mass (fourth goal) by controlling the thickness of the five identified vehicle components and the design variables considered are thickness of the five components. Additionally, the design must be robust to uncertainties in component thickness arising from manufacturing defects, geometric tolerances, and human error. Subsequently, an examination is conducted to determine whether the function models are explicitly available. If not, we move on to Step B.

Step B: In this step Design of Experiments (DOE) is carried out. DOE is carried out to find the design points at which simulation can be done. In this thesis a Latin Hypercube Sampling (LHS) is used to find the design points. Latin Hypercube Sampling is a systematic and efficient method for sampling multidimensional parameter spaces in a way that ensures a representative coverage of the input space while minimizing correlations between variables. Latin Hypercube Sampling (LHS), DoE is used to create a set of points for conducting computer simulations of car crashes. In this thesis, 44 LHS design points are generated for a car side crash scenario (see Appendix). These 44 points represent various combinations of values for the five thickness variables, staying within their allowed upper and lower limits.

Step C: In this FE simulations are carried out. Finite Element (FE) simulations are computational techniques used to analyze and predict the behavior of complex structures and systems. A Finite Element (FE) simulation is carried out to gather data needed for creating a surrogate model. The car model used for simulating side crashes comes from the United States National Crash Analysis Center, and it was adjusted by researchers at the Center for Advanced Vehicular Systems (CAVS) at Mississippi State University (Horstemeyer, Ren and co-authors 2009) . In Figure 4.2, the Dodge Neon FE simulation in a side-crash scenario is shown. Inside the vehicle model, a dashboard, door paneling, steering wheel, driver's seat, and under-the-hood parts are included. This modified FE car model has a total of 221,049 elements and 433,287 nodes. The simulation involves a moving deformable barrier (MDB), which is a model developed by Fang and co-authors(Fang, Rais-Rohani and co-authors 2005), acting as the impacting vehicle. In this thesis, the focus is on side crash scenarios for the car model. The LS-Dyna software is used to simulate vehicular side crashes.

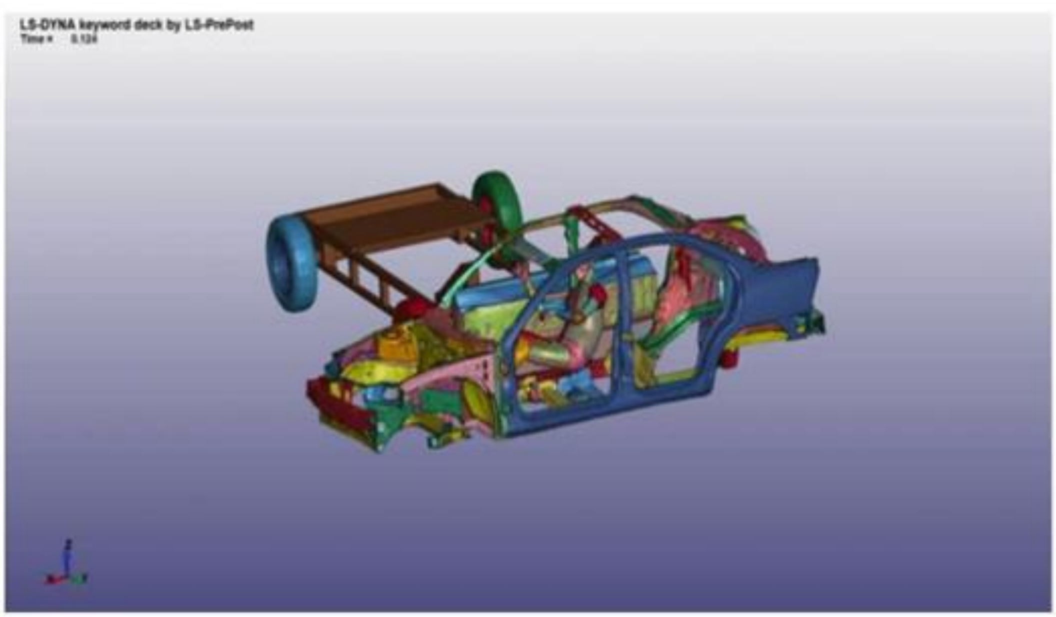


Figure 4.2: LS dyna FE car crash simulation

Step D: In this step surrogate models are developed. Surrogate models are the approximate models of actual models. In this thesis Response Surface Methodology (RSM) is used to create second-order polynomial models that predict the outcomes for the four goals based on the design variables. The Finite Element (FE) simulations, as described in Step C, are conducted for the Latin Hypercube Sampling (LHS), Design of Experiments (DOE) points identified in Step B. Polynomial response surface models of various orders (first, second, and third) for all four goals are generated. After evaluating the goodness of fit using metrics like the coefficient of determination (R2) and Cross-Validation Mean Absolute Error (CV-MAE), the second-order model is identified as the most accurate among the different orders. The second-order polynomial response model is expressed as shown in Equation 4.1.

$$\hat{y} = \beta_0 + \beta_1 t_1 + \beta_2 t_2 + \beta_3 t_3 + \beta_4 t_4 + \beta_5 t_5 + \beta_{11} t_1^2 + \beta_{12} t_1 t_2 + \beta_{13} t_1 t_3 + \beta_{14} t_1 t_4 + \beta_{15} t_1 t_5 + \beta_{22} t_2^2 + \beta_{23} t_2 t_3 + \beta_{24} t_2 t_4 + \beta_{25} t^2 t_5 + \beta_{34} t_3 t_4 + \beta_{35} t_3 t_5 + \beta_{44} t_4^2 + \beta_{45} t_4 t_5 + \beta_{55} t_5^2 \qquad \text{Equation 4.1}$$

Table 2 shows the R^2 values corresponding to the second order fit. Figure 4.3, 4.4, 4.5, 4.6 illustrates the mean response models for all goals while keeping three variables constant. The surrogate models developed is mentioned in Appendix.

Table 4.2: R^2 values

Components	R^2 Value
Part 235	0.9200
Part 237	0.9955
Part 329	0.9985
Part 353	0.8609
Part 357	0.9821
Total Mass	0.9813

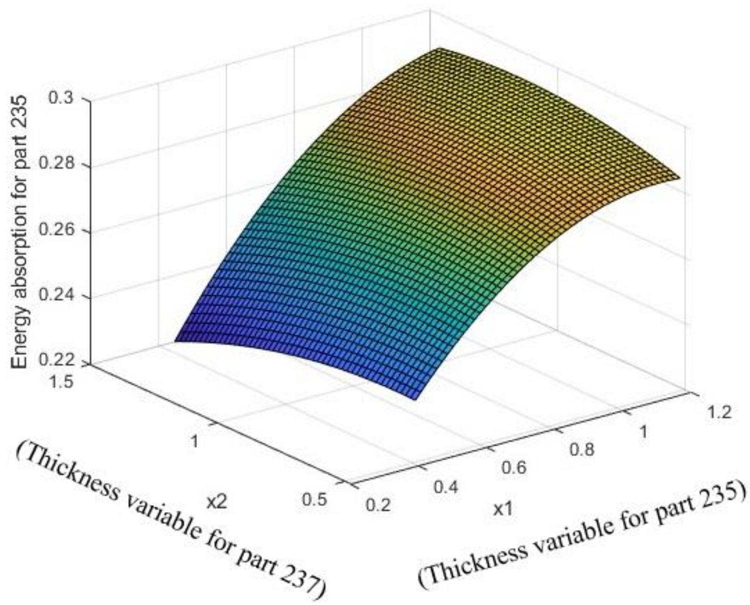


Figure 4.3: Energy absorption for part 235

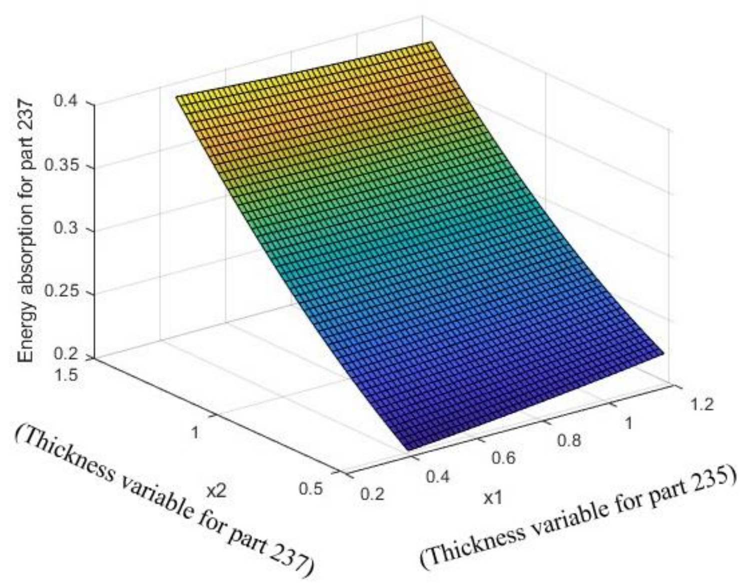


Figure 4.4: Energy absorption for part 237

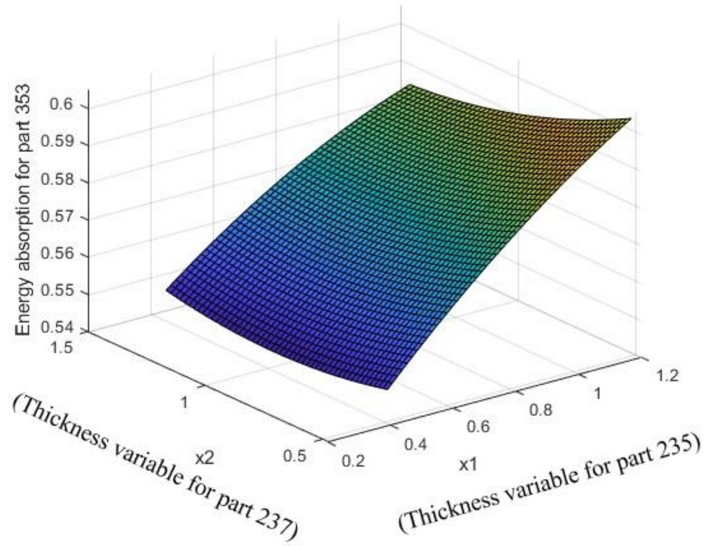


Figure 4.5: Energy absorption for part 353

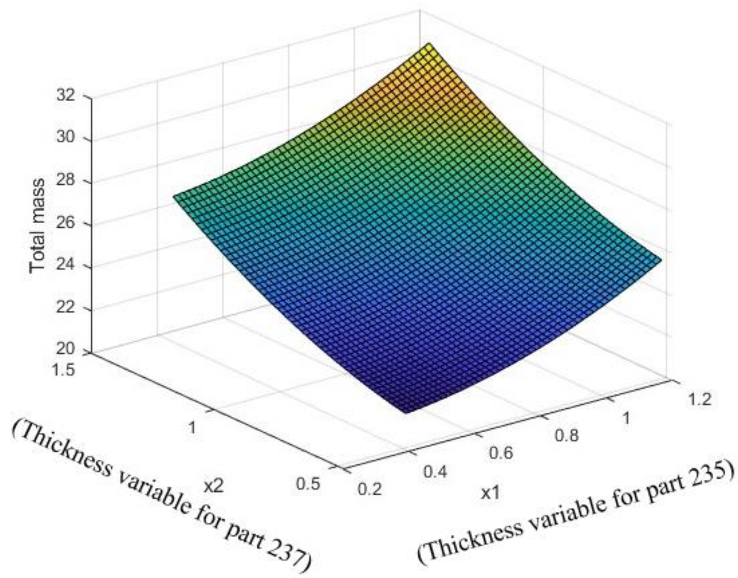


Figure 4.6: Total mass of five components

Step E: After developing surrogate models in Step D, the next Step is formulating the problem in cDSP construct. Compromise decision support problem is a combination of both mathematical programming and goal programming. This differs from conventional mathematical programming, which models numerous goals as a weighted function of only the system variables. cDSP helps in minimizing the deviation function, which is mathematically the weighted sum of deviation on of the goal values achieved from their target values. The four important keywords: Given, Find, Satisfy and Minimize capture the problem specific information. Formulating and solving the given problem for robust design as cDSP construct. A robust design formulation consists of two metrics: i) Design Capability Index (DCI), or ii) Error Margin Index (EMI). DCI is incorporated while considering uncertainty in design variables and EMI is considered when there is uncertainty in the model itself. In this paper we use the cDSP in conjunction with DCI construct discussed in chapter 3. DCI is a metrics developed for evaluating how well a variety of design criteria may be satisfied by a variety of design specifications considering uncertainty in design variables themselves. DCI is employed as goal formulation in the cDSP for the car design problem, the word and mathematical formulation is shown in Table 4.3 and 4.4 respectively.

Table 4.3: Data file for vehicular crashworthiness problem

```
NUMSYS  : Number of system variables: real, integer, boolean
   5    0   0

SYSVAR   : System variable information
T1    1  0.389  1.167  0.778   : Thickness of part 235
T2    2  0.473  1.419  0.946   : Thickness of part 237
T3    3  0.353  1.059  0.706   : Thickness of part 329
T4    4  0.352  1.059  0.705   : Thickness of part 353
T5    5  0.341  1.023  0.682   : Thickness of part 357

NUMCAG  : Number of constraints and goals
 0  8  0  0  4  :  nlinco, nnlinq, nnlequ, nlingo, nnlgoa

ACHFUN  : Achievment function
   1    : level
   1 4  : level 1, 4 terms
  (-1,1)  (-2,0)  (-3,0) (-4,0)

STOPCR  : Stopping criteria
1  0  100  0.005  0.005  : perfm cal, prt intereslts, Mcyles,sta dev, sta var

NLINCO  : Names of nonlinear constraints
E1const 1 : Energy absorption Constraint for part 235
E2const 2 : Energy absorption Constraint for part 237
E3const 3 : Energy absorption Constraint for part 329
```

```
Mconst  4 : Total mass Constraint
DCICT1  5 : DCI value for part 235
DCICT2  6 : DCI value for part 237
DCICT3  7 : DCI value for part 329
DCICM  8 : DCI value total mass

NLINGO  : Names of the nonlinear goals
DCIT1 1 : Maximum Energy absorption for part 235
DCIT2 2 : Maximum Energy absorption for part 237
DCIT3 3 : Maximum Energy absorption for part 329
DCIM 4 : Minimize the total mass

ALPOUT   : Output Control
    1   1   1   1   0   0   0   0  1  1

USRMOD  : User module flags
   1   0   0   0

OPTIMP  : Optimization parameters
   -0.05   0.5  0.005 : VIOLIM, REMO, STEP

ADPCTL : 1
```

Table 4.4: Fortran file for vehicular crashworthiness problem

```
REAL T1, T2, T3, T4, T5
REAL DCIT1, DCIT2
REAL DCIT3, DCIM,F1,F2,F3,F6,F1a,F1b,F1c,F1d,F1e,F1f
REAL F2a,F2b,F2c,F2d,F2e,F3a,F3b,F3c,F3d,F3e,F2f
REAL A1, A2, A3, A4, A5, B1, B2, B3, B4, B5,F3f
REAL C1,C2,C3,C4,C5,G1,G2,G3,G4,G5,G6
REAL Y1,Y2,Y3,Y6

1.0 Set the values of the local design variables (optional)

T1 = DESVAR(1)
T2 = DESVAR(2)
T3 = DESVAR(3)
T4 = DESVAR(4)
T5 = DESVAR(5)

2.0 Perform analysis relevant to non-linear constraints and goals
E1const = 0.23*10**6
E2const =0.22*10**6
E3const = 0.21*10**6
Mconst = 26

Individual energy absorbed for part235
F1a = 0.19739 + 0.14887*T1 + 0.02670*T2 - 0.05124*T3
F1b = -0.08292*T4 + 0.01850*T5 - 0.08935*T1**2 + 0.02951*T1*T2
```

```
F1c = - 0.00579*T1*T3 + 0.07720*T1*T4 - 0.02256*T1*T5
F1d = - 0.02076*T2**2 - 0.0874*T2*T3 - 0.00806*T2*T4
F1e =  0.00479*T2*T5 + 0.03565*T3**2 + 0.04727*T3*T4
F1f = -0.01389*T3*T5 + 0.06025*T4**2 - 0.0511*T4*T5+0.04532*T5**2
F1 = (F1a + F1b + F1c + F1d + F1e + F1f)*10**6
A1=(0.148-0.178*T1+0.0295*T2-0.0057*T3+0.0772*T4-0.02253*T5)*10**6
A2=(0.0267-0.0415*T2-0.087*T3-0.008*T4+0.00479*T5+0.0295*T1)*10**6
A3=(-0.0512-0.0057*T1-0.087*T2+0.0713*T3+0.0472*T4-0.013*T5)*10**6
A4=(-0.0829+0.0772*T1-0.0080*T2+0.0472*T3+0.120*T4-0.051*T5)*10**6
A5=(0.0185-0.0225*T1+0.0047*T2-0.0138*T3-0.051*T4+0.0906*T5)*10**6
Y1=(abs(A1)+abs(A2)+abs(A3)+abs(A4)+abs(A5))*0.01
DCIT1 = (F1 - 0.23*10**6) / Y1

Individual energy absorbed for part237
F2a = 0.1366+0.00607*T1+0.09939*T2+0.03214*T3
F2b = 0.002319*T4-0.02941*T5+0.008624*T1**2-0.04545*T1*T2
F2c = 0.0214*T1*T3-0.00618*T1*T4+0.0287*T1*T5+0.0663*T2**2
F2d =-0.0321*T2*T3+0.031315*T2*T4-0.00037*T2*T5-0.01601*T3**2
F2e=(0.0013*T3*T4)-(0.0001*T3*T5)-(0.0133*T4**2)
F2f = (0.017*T4*T5)+(0.003*T5**2)
F2 = (F2a + F2b + F2c + F2d + F2e+F2f)*10**6
B1=(0.006+0.0172*T1-0.0454*T2+0.0214*T3-0.0061*T4+0.0287*T5)*10**6
B2=(0.099-0.0454*T1+0.1327*T2-0.0321*T3+0.0313*T4-0.0003*T5)*10**6
B3=(0.0321+0.0214*T1-0.032*T2-0.0320*T3+0.0013*T4-0.0001*T5)*10**6
B4=(0.0023-0.00618*T1+0.031*T2+0.0013*T3-0.0267*T4+0.017*T5)*10**6
B5=(-0.029+0.0287*T1-0.0003*T2-0.0001*T3+0.017*T4+0.0006*T5)*10**6
Y2 = (abs(B1) + abs(B2) + abs(B3) + abs(B4) + abs(B5))* 0.01
DCIT2 = (F2 - 0.22*10**6) / Y2

Individual energy absorbed for part329
F3a= 0.015484 - 0.02342*T1 + 0.0294195*T2 + 0.23875*T3
F3b=-0.0417*T4 + 0.011807*T5 + 0.005085*T1**2 - 0.013*T1*T2
F3c=-0.0312*T1*T3 + 0.0513*T1*T4 + 0.01696*T1*T5 +0.001905*T2**2
F3d=(-0.03205*T2*T3)+(0.011237*T2*T4)+(0.000675*T2*T5)
F3e=(0.08*T3*T4)+(0.041*T3*T5)+(0.0006*T4**2)
F3f= -(0.06*T4*T5)-(0.005*T5**2)-(0.03904*T3**2)
F3= (F3a + F3b + F3c + F3d + F3e+F3f)*10**6
C1=(-0.023+0.0101*T1-0.013*T2-0.0312*T3+0.051*T4+0.0169*T5)*10**6
C2=(0.024-0.013*T1+0.00038*T2-0.032*T3+0.0112*T4+0.00067*T5)*10**6
C3=(0.2387-0.0312*T1-0.0320*T2-0.0780*T3+0.0808*T4+0.041*T5)*10**6
C4=(-0.041+0.051*T1+0.011*T2+0.08*T3+0.00012*T4-0.060*T5)*10**6
C5=(0.0118+0.01696*T1+0.00067*T2+0.041*T3-0.060*T4-0.010*T5)*10**6
Y3=(abs(C1) + abs(C2) + abs(C3) + abs(C4) + abs(C5))*0.01
DCIT3=(F3 - 0.21*10**6) / Y3

Total mass
F6a = 6.90989 - 3.708636*T1 - 1.850079*T2+6.84746*T3 + 13.566*T4
F6b=1.251*T5 + 4.57118*T1**2 + 0.007921*T1*T2+5.538308*T1*T3
F6c=-1.3646*T1*T4-1.6955*T1*T5+3.1740*T2**2+1.0138*T2*T3
F6d=2.582*T2*T4 - 1.7904*T2*T5 - 6.903*T3**2-7.156*T3*T4
F6e= 6.10069*T3*T5+2.253031*T4**2+7.952*T4*T5-4.5043*T5**2
F6=F6a+F6b+F6c+F6d+F6e
G1=-3.708636+9.14236*T1+0.007921*T2+5.38308*T3-1.3646*T4-1.6955*T5
G2=-1.85007+0.007921*T1+6.348*T2+1.0138*T3+2.5582*T4-1.7904*T5
```

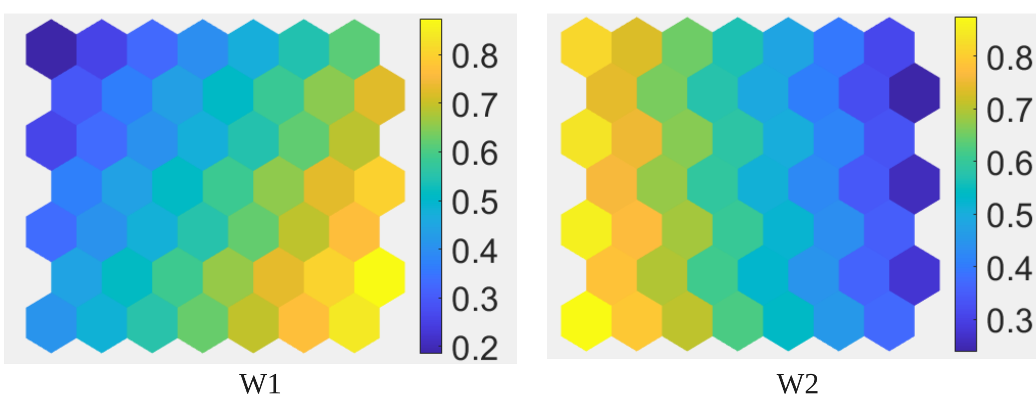
```
G3=6.8474+5.38308*T1+1.0138*T2-12.186*T3-7.156*T4+6.10069*T5
G4=13.566-1.364*T1+2.582*T2-7.156*T3+5.066*T4+7.952*T5
G5=1.251-1.6955*T1-1.7904*T2+6.1006*T3+7.952*T4-9.008*T5
Y6=(abs(G1)+ abs(G2)+ abs(G3)+ abs(G4)+abs(G5))*0.01
DCIM =(26-F6)/Y6

3.0 Evaluate non-linear constraints

FOR PART 235
CONSTR(1) = F1-E1const
FOR PART 237
CONSTR(2) = F2-E2const
FOR PART 329
CONSTR(3) = F3-E3const
FOR TOTAL MASS
CONSTR(4) = Mconst - F6
DCI CONSTRAINTS
CONSTR(5)  = DCIT1 - 1
CONSTR(6)  = DCIT2 - 1
CONSTR(7)  = DCIT3 - 1
CONSTR(8)  = DCIM - 1

END IF

 4.0 Evaluate non-linear goals
Maximize individual energy absorption
Part 235
GOALS(1) = (DCIT1)/(10) - 1
Part 237
GOALS(2) = (DCIT2)/(10) - 1
Part 329
GOALS(3) = (DCIT3)/(10) - 1
Minimize Total MASS
GOALS(4) = (DCIM)/(10) – 1

END IF
```

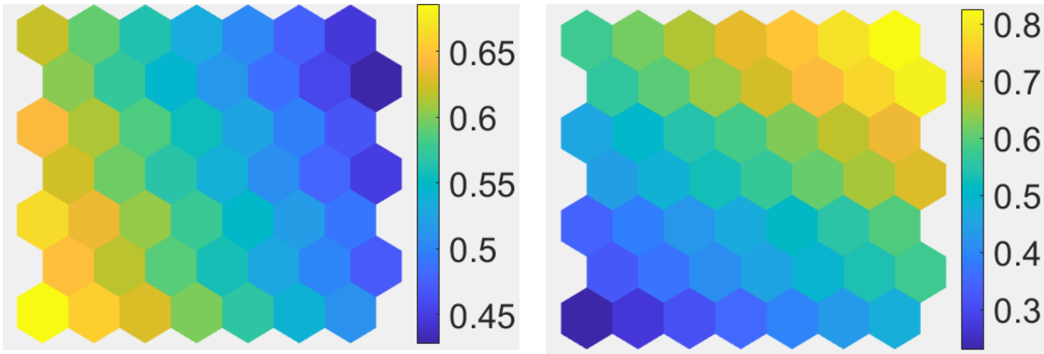
Step F: The cDSP formulation presented in previous Step is exercised for different design scenarios to generate different solutions. 44 design scenarios are considered to explore the design space and to obtain robust solutions for the given vehicular crashworthiness problem. The different weight scenarios represent different preferences for the 4 goals. Selected design scenarios and corresponding weights assigned to goals are shown in Table 4.5. When designers focus on maximizing just one goal, Scenarios 1 through 4 are used, see Table 4. For instance, when we want to maximize goal 3 then scenario 3 is considered where the full weightage is given to the third goal. Scenario 5 represents a situation where all the goals are given equal priority.

Table 4.5: Weight scenarios for the four goals

Scenarios	Weight 1	Weight 2	Weight 3	Weight 4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	0.25	0.25	0.25	0.25
6	0.46	0.23	0.23	0.08
7	0.24	0.17	0.28	0.31
-	-	-	-	-
15	0.3	0.26	0.19	0.25
16	0.26	0.02	0.07	0.65
-	-	-	-	-
43	0.12	0.14	0.39	0.35
44	0.03	0.48	0.35	0.14

Step G: Next step is to visualize and explore the solution space. In this thesis a machine learning based visualization technique is used to explore and visualize the solution space discussed in Chapter 3. The two-dimensional component planes of the iSOM for the weights (inputs) and goals (outputs) of the vehicular crashworthiness problem illustrate the relationship between the design variables (components thicknesses) and the corresponding responses (energy absorption and total mass). The input and output plots are shown in Figure 4.7 and 4.8 respectively.

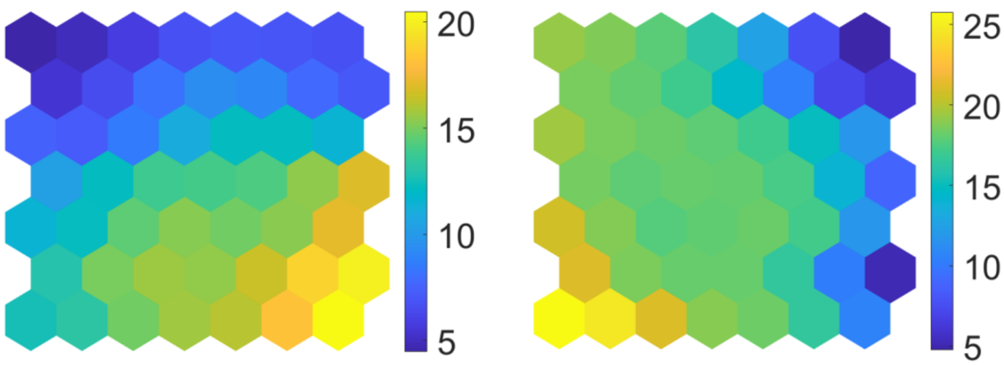




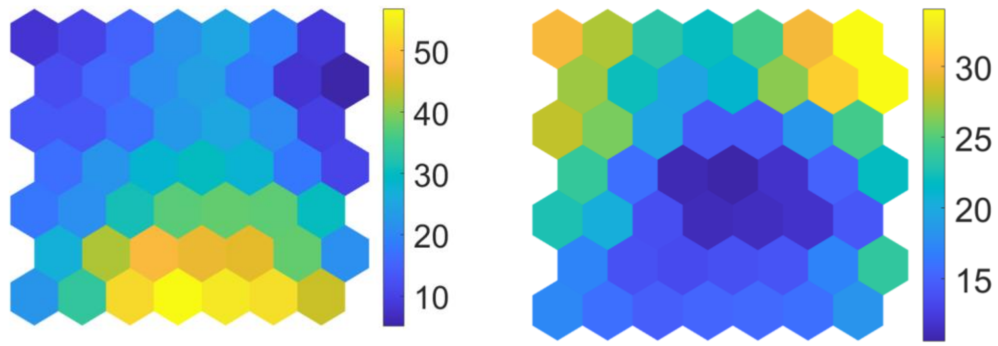
W3 W4

Figure 4.7: Input plots

In Figure 4.7 Weight 1(W1) refers to the weightage assigned to Goal 1 (maximize DCI value for component 235), Weight 2 (W2) refers to the weightage assigned to Goal 2 (maximize DCI value for component 237), Weight 3 (W3) refers to the weightage assigned to Goal 3 (maximize DCI value for component 353), and Weight 4 (W4) refers to the weightage assigned to for Goal 4 (maximize DCI value for total mass).



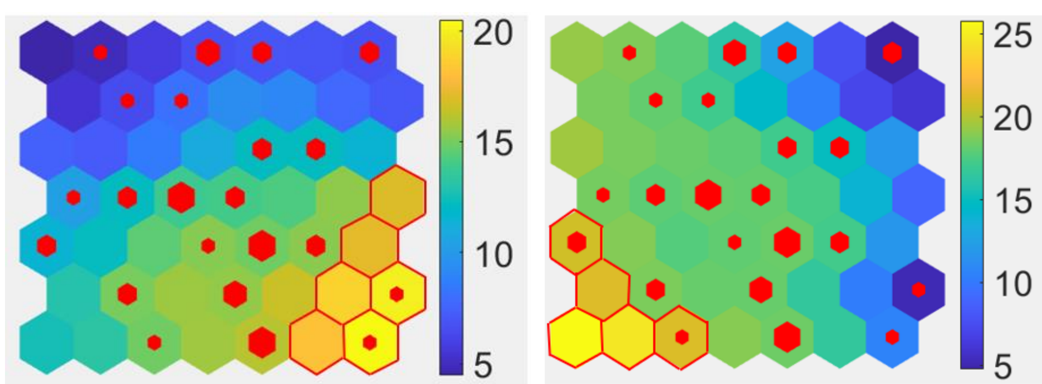
G1 G2



G3 G4

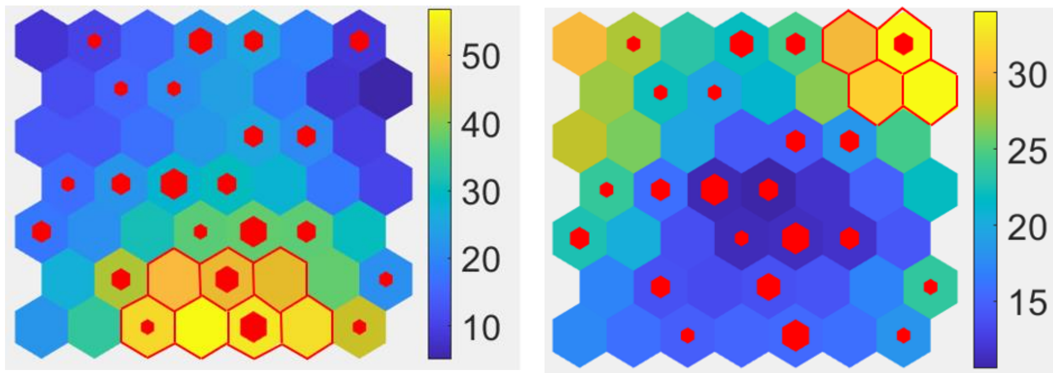
Figure 4.8: Output plots

The scales on the goal plots represent the achieved Design Capability Index (DCI) values. A high DCI value indicates that the average value of the goal is far from the Lower Requirement Limit (LRL) or Upper Requirement Limit (URL) and has minimal variance around its mean. From iSOM plots, the designer can observe the trends of the goals with varying weights. It is evident that high DCI values for Goal 1 are achieved when the weight W1 is high. Similarly, the DCI values for each goal increase with an increase in their respective weights. Analyzing the iSOM plots reveals the conflicting nature of the car side crash problem's goals, as regions satisfying the DCI values of one goal conflict with the high DCI regions for another goal. Using iSOM plots simplifies exploring and interpreting solutions when designers aim to maximize or minimize specific goals.



(a) DCI Goal 1 (Energy absorption for part 235)

(b) DCI Goal 2 (Energy absorption for part 237)



(c) DCI Goal 3 (Energy absorption for part 235) (d) DCI Goal 4 (Total mass)

Figure 4.9: iSOM plots highlighted for high DCI values

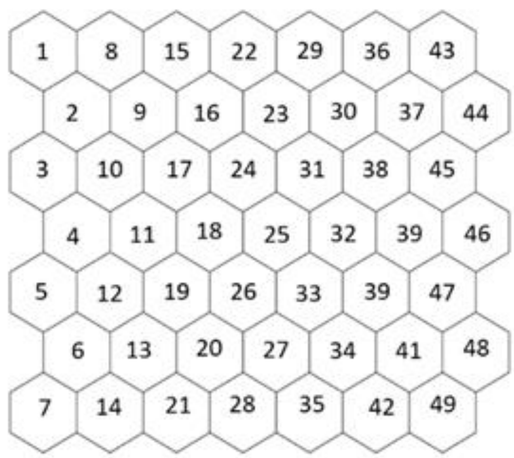


Figure 4.10: Node numbering

Figure 4.9 shows the iSOM plots for vehicular crashworthiness problems considering high DCI values (highlighted hexagons). The red dots in iSOM plots are called hits, they represent the samples mapped to the grids. The node numbers of the iSOM grid points are shown in Figure 4.10. From the iSOM plots in Figure 4.9, it can be concluded that there are no common satisficing solutions for all the four goals and hence the nodes with high DCI values are chosen as satisficing solutions shown in Table 4.6. The maximum DCI values for all goals considered are:

i. Goal 1 DCI values ≥ 20
ii. Goal 2 DCI values ≥ 25

iii. Goal 3 DCI values ≥ 50

iv. Goal 4 DCI values ≥ 30

Table 4.6: Weight scenarios and design variables with high DCI values

Goals	Grids	No of scenarios	Weight scenarios	Design variables (thickness)				
				Part 235(t_1)	Part 235(t_2)	Part 235(t_3)	Part 235(t_4)	Part 235(t_5)
Goal 1	49	1	24	0.794759	0.785839	0.397125	0.913072	0.383625
	48	1	1	0.999904	0.578394	0.377797	0.5714	0.407019
Goal 2	21	1	30	0.953159	0.776908	0.579186	1.01582	0.343664
	5	2	8	0.88638	0.822095	0.465883	0.735803	0.468546
			2	0.9725	0.826967	0.5295	0.600292	0.5115
Goal 3	27	3	36	0.988117	0.776135	0.582469	1.01565	0.343664
			33	0.957301	0.83545	0.3621080	0.73001	0.517434
			14	0.792591	0.785477	0.397125	0.913072	0.383625
	35	4	6	0.747891	0.800108	0.397125	0.823183	0.383625
			18	0.794759	0.785839	0.397125	0.913072	0.383625
			20	0.633364	0.804622	0.364031	0.808268	0.573625
			32	0.792591	0.785477	0.397125	0.913072	0.383625
	21	1	30	0.692604	0.835467	0.360528	0.563781	0.343664
Goal 4	43	2	4	0.583963	0.604725	0.355451	0.53114	0.437419
			6	0.632911	0.626524	0.357554	0.602653	0.344142

Taking into account high DCI values, all the design variable values listed in Table 4.6 are deemed satisfactory. Based on the requirements, the design variable values for each goal can be chosen. For instance, considering Goal 3, any of the identified design variable value sets mapped to weight scenarios corresponding to grids 27, 35, and 21 can be selected. However, upon examining the grids chosen for all four goals, it becomes apparent that there are no common regions for the given problem, indicating a trade-off between the conflicting objectives.

4.4 Limitations of the Proposed Decision Based Design (DBD) Framework

The proposed Decision-Based design (DBD) framework addresses the challenges in Complex Problem such as:

- Comprehensive Consideration of Goals and Uncertainties: The framework effectively incorporates many conflicting goals and uncertainties into the system, providing a more realistic representation of real-world design scenarios.
- Visualizing Trade-offs: The framework utilizes a machine learning based visualization technique, called iSOM to effectively communicate the trade-offs between different goals and effectively visualize many goals. This allows designers to make informed decisions based on their preferences and requirements.

Despite addressing the challenges in complex system, this framework has a limitation:

- Lack of a systematic approach for identifying and selecting common satisficing solutions for many goals: The framework does not provide a systematic approach for selecting satisficing solutions from the visualized plots for many goals. This leaves designers to make subjective decisions based on their interpretation of the visualizations.

The proposed framework's limitations lead to Research Question 2 (RQ2) in Chapter 1, which sought to address these limitations by developing a systematic approach method for choosing satisficing solutions. An example problem is used in Chapter 5 to test the effectiveness of the proposed systematic approach.

Chapter 5 : Designing of Composite Structure Problem

This chapter presents a test problem related to the design of composite structures. Section 5.1 provides a brief introduction to the problem and then establishes the mathematical foundation for designing composite structures in Section 5.2. In Section 5.3 the systematic approach proposed is discussed in detail. In this chapter the efficacy of the proposed systematic approach is evaluated through composite design problem.

5.1 Designing of Composite Structure: Problem Definition

Composite materials are a combination of two or more materials on a macroscopic level and each material exhibits different characteristics. Designing composite materials is a complex process that involves making numerous decisions at the microstructural level, such as determining the fiber orientation, fiber volume fraction, matrix material distribution, and porosity. These decisions have a significant impact on the overall properties of the composite material, such as its strength, stiffness, and weight. In addition to microstructural considerations, composite material designers must also consider a many conflicting objectives, such as maximizing strength while minimizing weight. Composites offer dual advantages first, they provide a remarkable combination of high strength and low weight second, they allow for customization to meet specific needs. Customization involves tailoring or modifying both the constituent materials and microstructural properties of the composites to achieve the structural requirements. Tailoring constituent materials entails selecting the matrix and fiber according to the specific requirements of the intended applications. Tailoring composite materials at the microstructural level is a complex task that involves managing various properties like fiber orientation, fiber volume fraction, matrix material distribution, and porosity. The design process is further complicated by the need to balance multiple and often conflicting objectives, such as maximizing strength while minimizing weight. Traditionally, composite material designers have relied on their experience and judgment to make these decisions. However, advancements in heuristics and theoretical models have provided valuable tools for tackling the challenges of composite material design. However, these approaches often have limitations, as they may be tailored to specific problems and may not be easily adaptable to new design configurations.

In this thesis a sandwich composite problem is considered as an example problem. Sandwich materials represent a class of composite structures characterized by two outer layers (skin) made of stiff and strong materials and an intermediate core layer shown in Figure. The skin serves as the primary load-bearing component, providing strength and rigidity to the structure by absorbing and transmitting external forces. The core, situated between the skin layers, plays a crucial role in supporting the skin against buckling, preventing excessive deflections under load, and aiding in dissipating shear stresses. Common skin materials include thin metallic sheets, such as aluminum or steel, or fiber-reinforced composites, where reinforcing fibers, like carbon fiber or glass fiber, are embedded within a matrix material, such as epoxy or polyester resin. Core materials, on the other hand, typically consist of honeycomb structures, characterized by a hexagonal arrangement of cells, or closed-cell foam structures, where gas bubbles are trapped within a solid matrix.

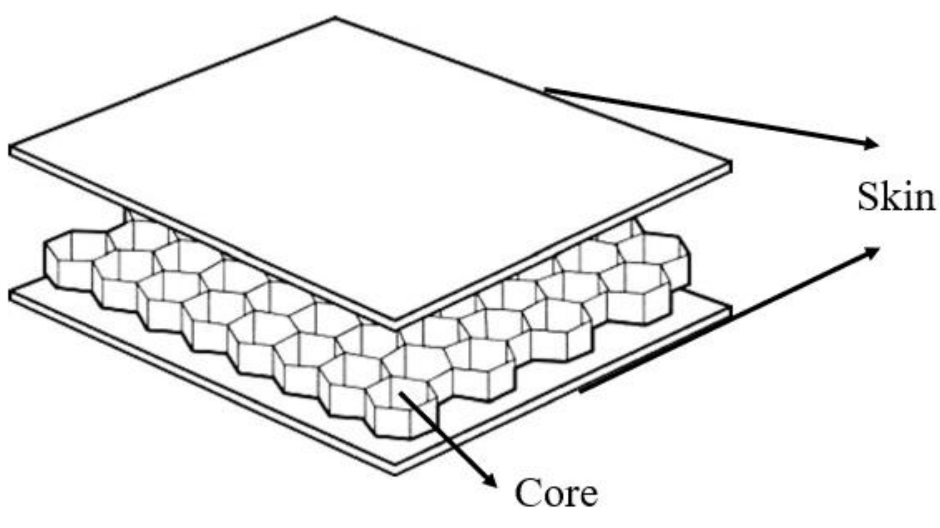


Figure 5.1: Composite sandwich structure

The mechanical performance of a sandwich beam is governed by two key properties: bending rigidity and shear rigidity. Bending rigidity, also known as flexural rigidity, determines the beam's resistance to bending deformation under applied loads. Shear rigidity, on the other hand, characterizes the beam's ability to withstand shear forces, which tend to cause the layers of the beam to slide relative to each other. Both bending and shear rigidities are influenced by the thickness and material properties of the skin and core layers.

Designing an effective sandwich structure involves striking a balance between many conflicting design goals. Increasing the thickness of the skin and core materials can enhance both bending and shear rigidities, leading to a stiffer and stronger beam. However, this approach comes at the cost of increased weight, which can be a critical consideration in many applications. Therefore, sandwich design often involves seeking satisficing solutions, where the design goals are met to a satisfactory level rather than achieving absolute optimality. Satisficing solutions become particularly relevant in applications where weight constraints are paramount. Sandwich materials offer a versatile and efficient solution for constructing lightweight yet strong and stiff structures. The interplay of skin and core materials, along with their respective thicknesses, dictates the mechanical performance of sandwich beams. Designing effective sandwich structures involves carefully considering the conflicting objectives of strength, stiffness, and weight, often leading to satisficing solutions that meet the design requirements within acceptable trade-offs.

In this thesis focus is on designing a sandwich composite beam with a predefined length of 1500mm and a width of 750mm, as illustrated in Figure 5.2. Throughout the design process, the same set of materials with their corresponding properties is employed for both the skin and core layers, as detailed in Table 5.1.

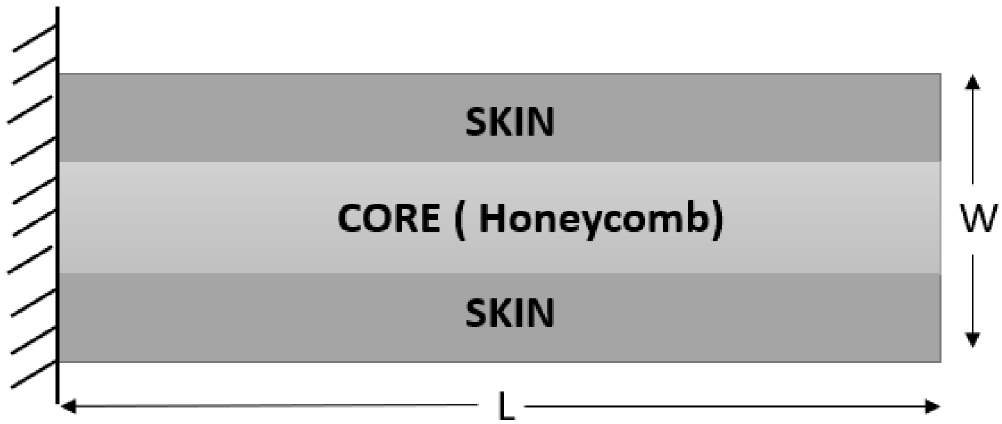


Figure 5.2: Sandwich composite beam

Table 5.1: Material description

Material	**Density** (kg/m^3)		**Modulus** (MPa)	
	Fiber	***Matrix***	***Fiber***	***Matrix***
Skin (carbon epoxy)	1760	1280	230000	3700
Core (Aluminum)	2700		26000	

In this thesis designing a composite beam that fulfills specific performance or design criteria by tailoring the microstructural characteristics of the composite laminate, as depicted in Figure 5.3 are discussed. These microstructural properties encompass volume fraction for the skin and wall angle, wall thickness, and wall height for the core. The design variables considered in this problem are enumerated in Table 5.2.

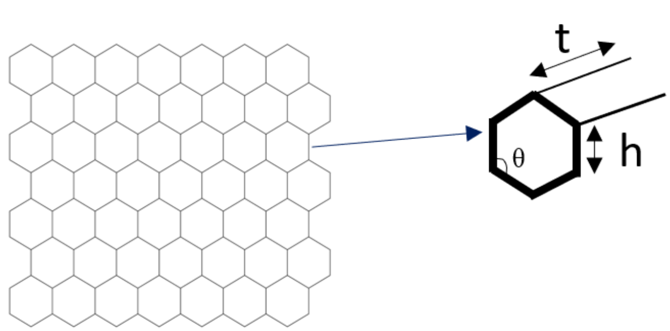


Figure 5.3: Microstructural properties

Table 5.2: Design variables for composite beam problem.

Material	Design variable
Skin	Volume fraction (V_f)
Core	Wall angle(θ)
	Wall length (h)
	Wall thickness (t)

This problem delves into enhancing the performance of a composite beam by tailoring the microstructural characteristics of both the skin and core layers. Four goals considered are: minimizing the density of both the skin and core (Goals 1 and 3) while concurrently maximizing the modulus and elastic and shear modulus of the skin and core, respectively (Goals 2 and 4). This problem at hand has been formulated using four design variables: volume fraction, wall angle, wall height, and wall thickness. For the design variables considered there are different sources of uncertainties such as, density of the fiber material variation due to impurities or manufacturing defects. This can affect the volume fraction of fibers in the skin layer, which in turn affects the beam's stiffness and strength, manufacturing tolerances can affect the wall angle of the core cells. This can affect the core's stiffness and strength.

The equations for density of skin, modulus of skin, density of core and shear modulus of core are given below,

$$\rho_s = \rho_f V_f + \rho_m\,(1 - V_f) \qquad \text{Equation 5.1}$$

$$E_s = V_f E_f + (1 - V_f)\,E_m \qquad \text{Equation 5.2}$$

$$\rho_c = \frac{2}{(1+cos\theta)sin\theta}\frac{t}{h}\,\rho \qquad \text{Equation 5.3}$$

$$G_c = \frac{1+cos^2\theta}{(1+cos\theta)sin\theta}\frac{t}{h}\,G \qquad \text{Equation 5.4}$$

Where, ρ_s is density of skin.

ρ_f is density of fiber.

V_f is Volume fraction.

ρ_m is density of matrix.

E_s is modulus of skin.

E_f is modulus of fiber.

E_m is modulus of matrix.

ρ_c is density of core.

G_c is shear modulus of core.

To tackle the complexities of composite material design, this thesis employs a Decision-Based Design framework introduced in Chapter 3. This framework begins by identifying the specific design requirements for the problem at hand. These requirements are then formulated using the compromise decision support problem (cDSP) construct, which explicitly considers uncertainty as discussed in detail in the following section.

5.2 Developing a compromise Decision Support Problem Construct (cDSP) for Designing of Composite Structure

The composite design problem is formulated using cDSP. Compromise decision support problem is a combination of both mathematical programming and goal programming. This differs from conventional mathematical programming, which models numerous goals a weighted function of only the system variables. cDSP helps in minimizing the deviation function, which is mathematically the weighted sum of deviation on of the goal values achieved from their target values. The four important keywords: Given, Find, Satisfy and Minimize capture the problem specific information. To address the issue of uncertainties in design variables a mathematical construct called DCI (Design capability index) is used. In this problem cDSP with DCI robust design metrics is used to address the issue of uncertainty present in design variables. DCI is employed as goal formulation in the cDSP for the composite beam design problem. The cDSP word and mathematical formulation for composite beam problem is shown in Table 5.3 and 5.4 respectively.

Table 5.3: Data file for designing of composite problem

```
NUMSYS : Number of system variables: real, integer, boolean
   4   0   0

SYSVAR  : System variable information
Vf        1  0.4   0.7  0.55
a         2  30  60  45
h         3  2  25  13
t         4  0.01   0.11  0.06

NUMCAG  : Number of constraints and goals
 0  4  0  0  4  :  nlinco,nnlinq,nnlequ,nlingo,nnlgoa

ACHFUN  : Achievment function
   1    : level
  1 4  : level 1, 4 terms
 (-1,0.03) (-2,0.48) (-3,0.35) (-4,0.14)

STOPCR  : Stopping criteria
1  0  100  0.05  0.05  : perfm cal, prt intereslts, Mcyles,sta dev, sta var

NLINCO  : Names of nonlinear constraints
DDsmin  1 : Minimum DCI density of skin
DEsmax  2 : Minimum DCI modulus of skin
DDcmin  3 : Minimum DCI density of core
DGcmax  4 : Minimum DCI modulus of core

NLINGO  : Names of the nonlinear goals
DCIDs 1 : DCI value for density of skin
DCIEs 2 : DCI value for modulus of skin
DCIDc 3 : DCI value for density of core
DCIGc 4 : DCI value for shear modulus of core

ALPOUT  : Output Control
    1   1   1   0   0   0   0   0  1  1

USRMOD  : User module flags
   1   0   0   0

OPTIMP  : Optimization parameters
  -0.05   0.5  0.005 : VIOLIM, REMO, STEP
```

Table 5.4: Fortran file for designing of composite problem

```
REAL     Vf, a, h, t
REAL     DCIDs, DCIEs, DCIDc, DCIGc
REAL     F1, F2, F3, F4, F4a, F4b, pi
REAL     A1, B1, C1, C2, C3, D1, D2, D3
REAL     C1a, C1b, C1c, D1a, D1b, D1c, D1d, D1e, D2a, D2b, D3a, D3b
REAL      delY1, delY2, delY3, delY4, delVf, dela, delh, delt, D1a1, D1a2

1.0 Set the values of the local design variables (optional)

Vf = DESVAR(1)
a = DESVAR(2)
h = DESVAR(3)
t = DESVAR(4)

 2.0 Perform analysis relevant to non-linear constraints and goals
pi= 3.14
delVf= 0.05
dela= 0.1
delh= 0.1
delt= 0.005

Goal 1: Maximize DCI density of skin (Min density of skin)
F1 = 1760*Vf + 1280 - 1280*Vf
A1 =  480
delY1 = (abs(A1))*delVf
DCIDs = (1700-F1)/delY1
DCIDs = (F1-1200)

Goal 2: Maximize DCI modulus of skin (Maximize modulus of skin)
F2 = Vf*230000+(1-Vf)*3700
B1 = 226300
delY2 = (abs(B1))* delVf
DCIEs = (F2 -60000)/delY2
DCIEs=(F2-60000)

Goal 3: Maximize DCI density of core (Minimize density of core)
F3 =((5400*t)/((sin(pi*a/180))*(1+cos(pi*a/180))*h))
C1a = 2*h*t*2700*(cos(pi*a/180)+cos(pi*a/180)**2)
C1b = 2*h*t*2700*(-sin(pi*a/180)**2)
C1c = (((1+cos(pi*a/180))**2)*(sin(pi*a/180)**2)*(h**2))
C1 = (C1a + C1b)/C1c
C2 = -(10800*t)/(h**2*(sin(pi*a/90)+2*sin(pi*a/180)))
C3 = (10800)/(h*(sin(pi*a/90)+2*sin(pi*a/180)))
delY3 = (abs(C1)*dela + abs(C2)*delh + abs(C3)*delt)
DCIDc = (30- F3)/delY3
DCIDc = (F3-15)
```

```
Goal 4: Maximize DCI shear modulus of core (Maximize shear modulus of core)
F4a = (26000*t*(1+cos(pi*a/180)**2))
F4b = ((1+cos(pi*a/180))*sin(pi*a/180)*h)
F4 = F4a/F4b
D1a1 = (h*t*26000)
D1a2 = (sin(pi*a/180)*(sin(2*pi*a/180))*(1+cos(pi*a/180)))
D1a = D1a1*D1a2
D1b = ((h*t*26000)*(1+cos(pi*a/180)**2)*(cos(pi*a/180)))
D1c = (cos(pi*a/180)**2)
D1d = (-sin(pi*a/180)**2)
D1e = (((1+cos(pi*a/180))**2)*(sin(pi*a/180)**2)*(h**2))
D1 = (D1a+D1b+D1c+D1d)/D1e
D2a = (26000*t*(cos(pi*a/180)**2+1))
D2b = ((h**2)*(sin(pi*a/180))*(cos(pi*a/180)+1))
D2 = D2a/D2b
D3a = (26000*(cos(pi*a/180)**2+1))
D3b = (h*(sin(pi*a/180))*(cos(pi*a/180)+1))
D3 = D3a/D3b
delY4= (abs(D1)*dela + abs(D2)*delh + abs(D3)*delt)
DCIGc = (F4-5)/delY4
DCIGc =(F4-100)

3.0 Evaluate non-linear constraints
DCI CONSTRAINTS
CONSTR(1) = DCIDs - 1
CONSTR(2) = DCIEs - 1
CONSTR(3) = DCIDc - 1
CONSTR(4) = DCIGc - 1

END IF

4.0 Evaluate non-linear goals

Density of skin
GOALS(1) = (DCIDs)/(10) - 1

Modulus of skin
GOALS(2) = (DCIEs)/(10) - 1

Density of core
GOALS(3) = (DCIDc)/(10) - 1

Shear modulus of core
GOALS(4) = (DCIGc)/(10) - 1

END IF
```

After obtaining the feasible solutions from cDSP, the next stage involves exploring and visualizing the design space to identify the Region of Interest (RoI). iSOM is employed to effectively visualize the solutions, and a systematic approach for selecting satisficing solutions from the visualized plots is proposed in the subsequent section.

5.3 Solution Space Visualization and Exploration and Results

Visualization plays a crucial role in assisting designers in systematically exploring the design space to identify reliable and satisfactory solutions problems with many goals. In this thesis, interpretable Self-Organizing Maps (iSOM), a machine learning-based visualization technique, to effectively visualize the solution space for many goals is employed. The ability to visualize the design space provides designers with a valuable tool for exploring and understanding the complex interplay between many goals. Through methodical exploration, designers can identify regions of interest (ROIs) that contain solutions that satisfy the design requirements. iSOM, with its ability to effectively visualize high-dimensional data, proves to be an efficient visualization technique.

The iSOM plots for the sandwich composite beam are shown in Figure 5.4. First set of plots (W1, W2, W3, W4) represent the weightage given to the goals. W1 represents the weightage give to goal 1 (to maximize the DCI density value for skin) and W2 represents the weightage given to goal 2 (to maximize the DCI modulus value for skin). Similarly, W3 indicates the weightage given to goal 3 (to maximize the DCI density value for core) and W4 indicates the weightage given to goal 4 (to maximize the DCI shear modulus value for core). Second set of plots represent the output plots (goals) and the scales of these output plots indicate the achieved DCI values. Higher DCI value indicate that the mean value of goal is away from lower requirement limit (LRL) and upper requirement limit (URL) which means that there is minimum deviation. From iSOM plots, we can comprehend the influence of input factors on output factors and gain insights into the interdependencies among many objectives.

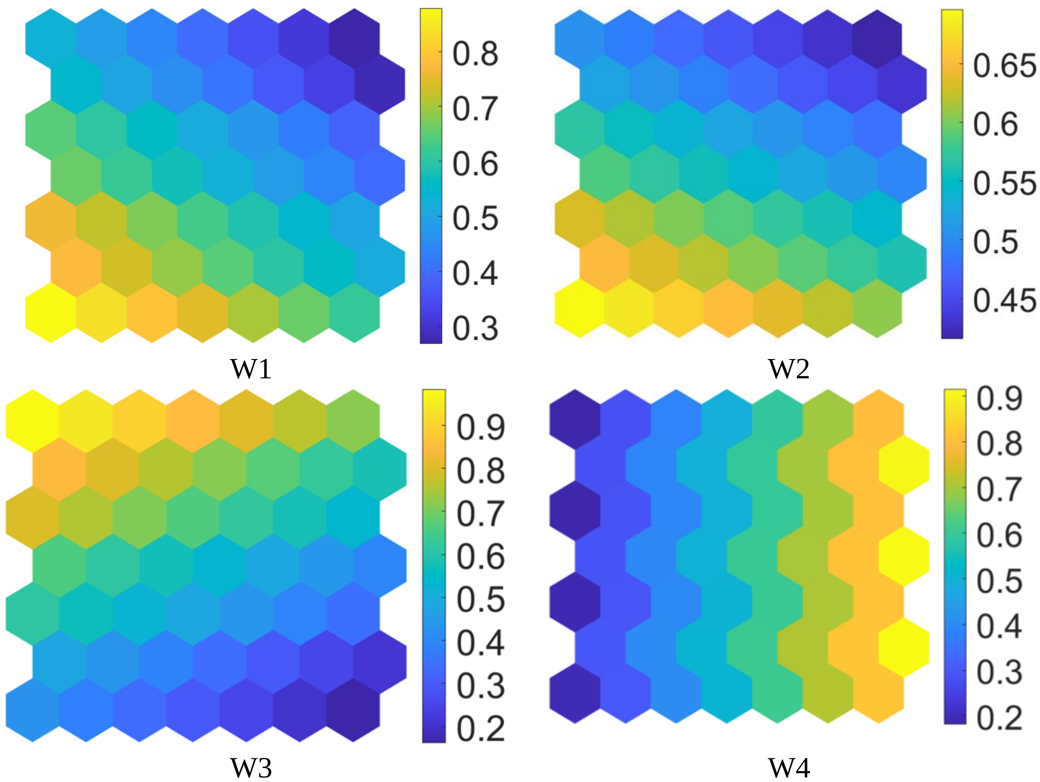
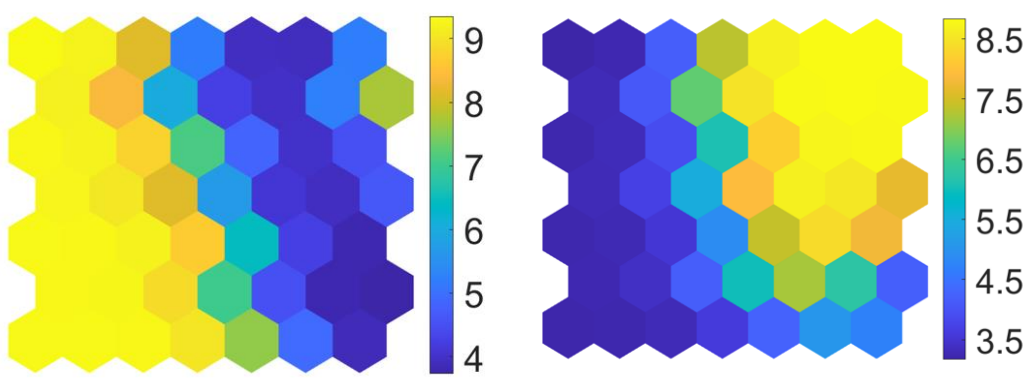
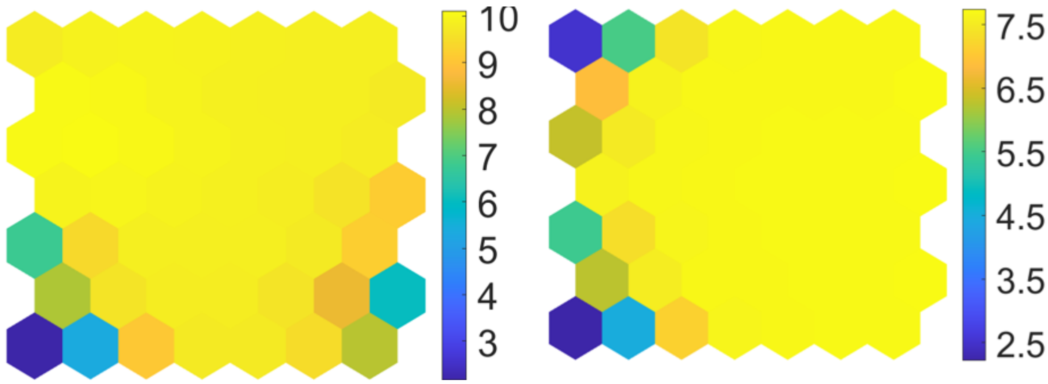


Figure 5.4: Input plots for composite problem



G1(DCI goal for density of skin) G2(DCI goal for elastic modulus of skin)



G3(DCI goal for density of core) G4(DCI goal for shear modulus of core)

Figure 5.5: Output plots for composite problem

Figure 5.5 shows the input plots for the composite problem, providing valuable insights into the interrelationships between the objectives. The first two plots reveal a conflicting nature between the goals, indicating that achieving one may necessitate compromising the other. This interplay is further evident in the effect of input parameters on output metrics. For instance, an increase in W1 leads to a decrease in the DCI value for both goals 3 and 4. Upon careful examination of the plots, it becomes apparent that there is no common satisficing solutions for all the four goals. This highlights the inherent trade-offs involved in designing composite problem.

5.3.1 Systematic Approach for Identifying Common Satisficing Solutions

A systematic approach is proposed to identify the common satisficing solutions for all the four goals of sandwich composite problem shown in Figure 5.6. The proposed approach commences by identifying satisfying solutions with high DCI (Design Capability Index) values for each individual goal, as depicted in Step A. High DCI values are preferred when designers seek solutions that exhibit greater robustness. This step also involves identifying the node numbers for the selected grids, as illustrated in Figure 5.7. Subsequently, Step B involves checking for grids that possess high DCI values and are common to most of the goals. If such grids are identified, the process proceeds to Step C; otherwise, it advances to

Step E. In Step E, the satisficing DCI limits are relaxed for specific goals, termed "appropriate goals," based on the designer's requirements. These appropriate goals represent objectives for which DCI limits can be relaxed without compromising the overall design. Step C entails verifying whether the design solutions corresponding to the identified grids meet the designer's specifications. If the designer's requirements are satisfied, these solutions are selected as satisficing solutions, and the corresponding weight scenarios and design variables are identified. If the designer's requirements are not met, the process returns to Step E, and the loop iterates until satisfactory solutions are found.

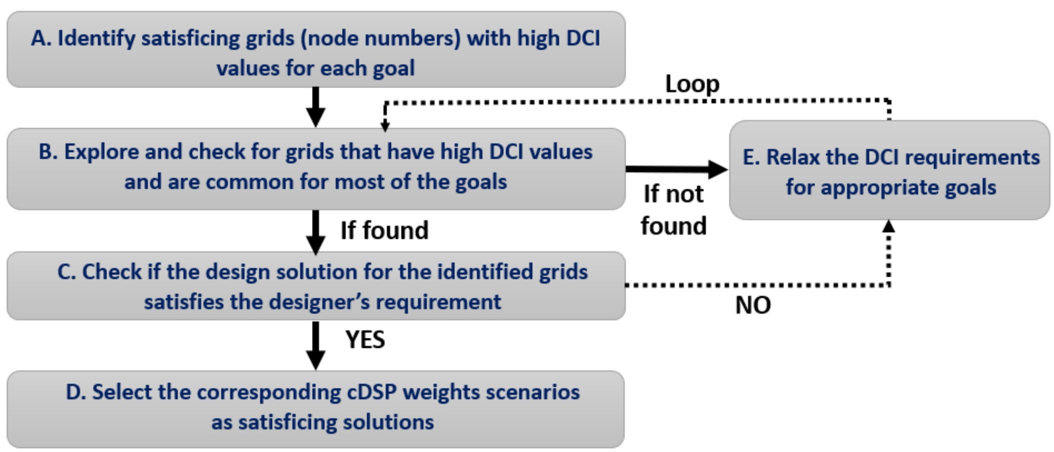


Figure 5.6: Systematic approach

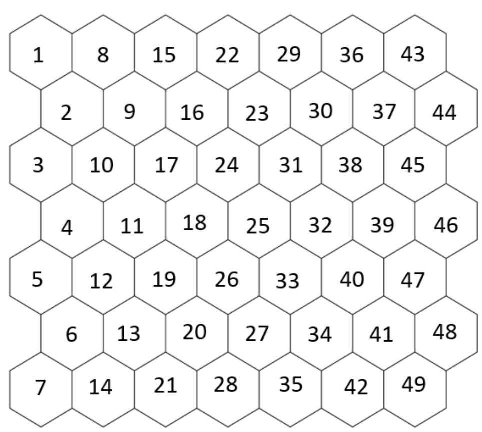


Figure 5.7: Node numbering

To demonstrate the effectiveness of this methodology, we employ a composite beam with three distinct scenarios as an illustrative example. In Scenario 1, the process begins with Step A, where grids with the highest DCI values for all four goals are considered. These grids are highlighted in red in Figure 5.8. The corresponding high DCI values for the four goals are: i) Goal 1 DCI values ≥ 8.5; ii) Goal 2 DCI values ≥ 8; iii) Goal 3 DCI values ≥ 9; iv) Goal 4 DCI values ≥ 7. The next step, Step B, involves identifying grids with high DCI values for most of the goals. In this case, a region highlighted in black in Figure 5.9 is identified, containing grids highlighted in red that exhibit high DCI values for goals 2, 3, and 4. Correspondingly, grids for goal 1 are chosen as shown in Figure 5.9. Grids 36 and 23 are common to all four goals, each mapped with two design scenarios. The corresponding weight scenarios and design variables are presented in Table 5.5. Since Step B is successful, the process moves to Step C, where the designer verifies whether the design solutions identified for the grids meet the specified requirements. If these requirements are satisfied, the solutions are selected as satisficing solutions, and the corresponding weight scenarios and design variables are identified as shown in Table 5.5.

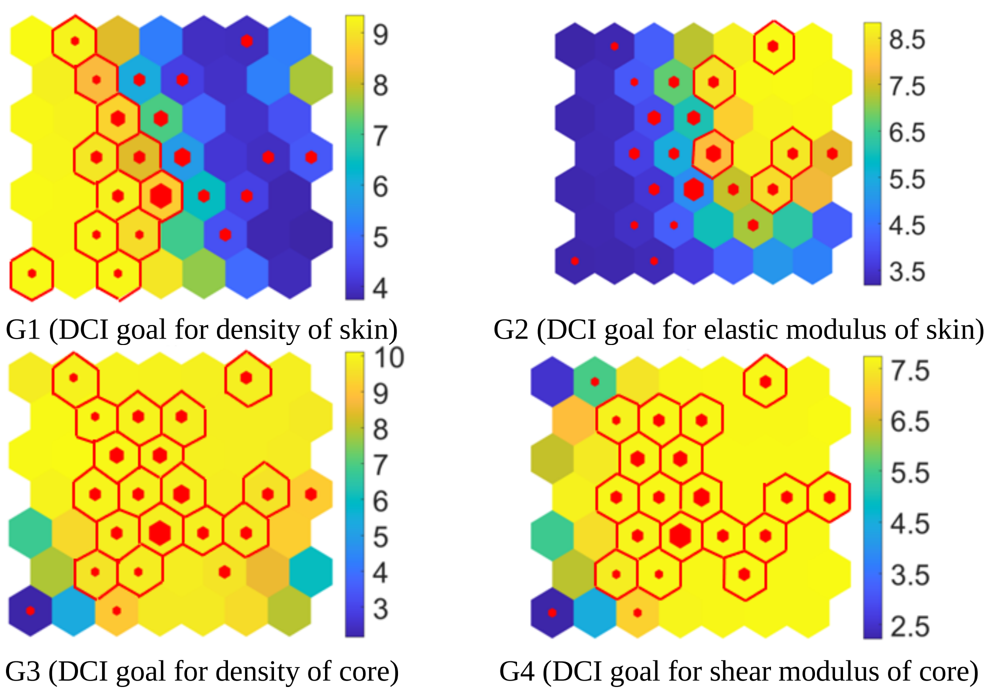
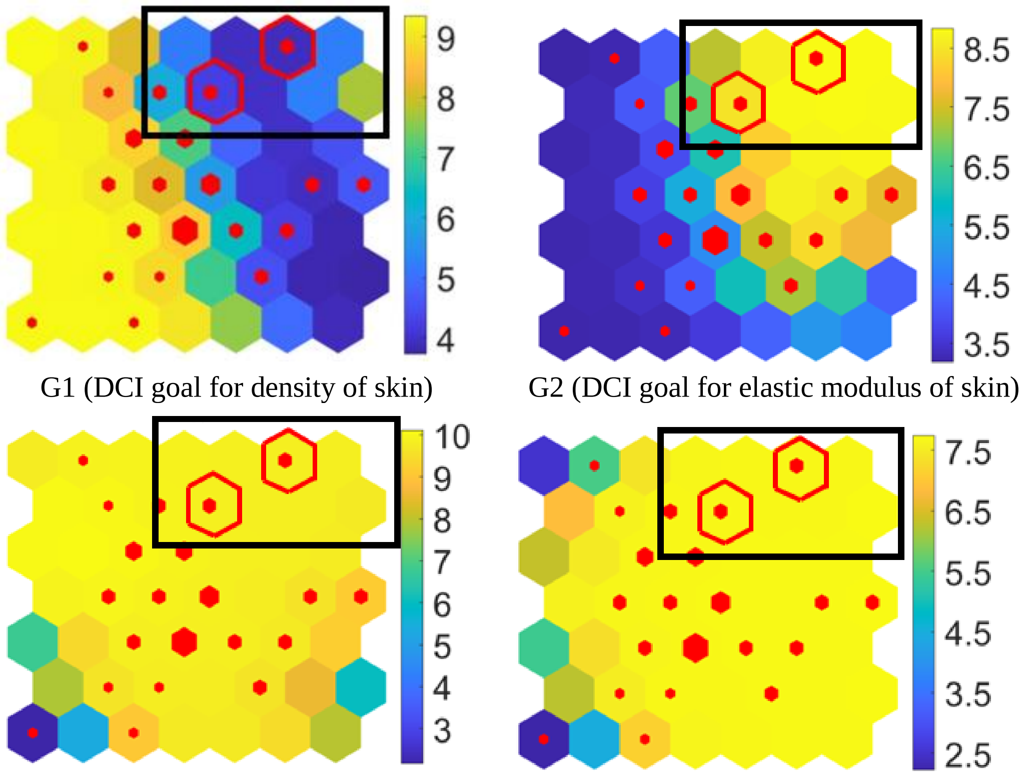


Figure 5.8: Highlighted grids for high DCI values



G1 (DCI goal for density of skin) G2 (DCI goal for elastic modulus of skin)

G3 (DCI goal for density of core) G4 (DCI goal for shear modulus of core)

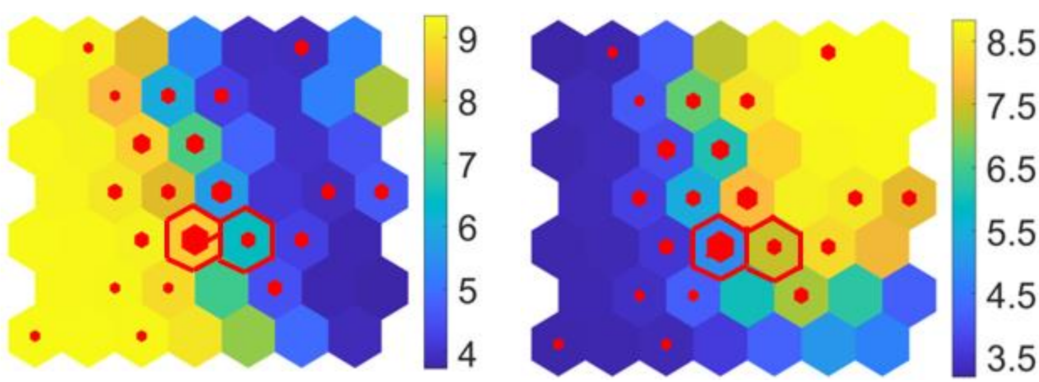
Figure 5.9: Highlighted grids with most common high DCI values

Table 5.5: Weight scenarios and design variables considering most common high DCI values.

Goals	**Grids**	**No of scenarios**	**Weight scenario**	**Design variables**			
				Volume fraction (V_f)	Wall angle (θ)	Wall length (h)	Wall thickness (t)
Goal 1, 2, 3 and 4	36	2	27	0.690625	59.062500	22.614200	0.105833
			39	0.690625	59.062500	22.614200	0.105833
	23	2	10	0.409375	59.062500	22.614200	0.105833
			43	0.690625	59.062500	22.614200	0.105833

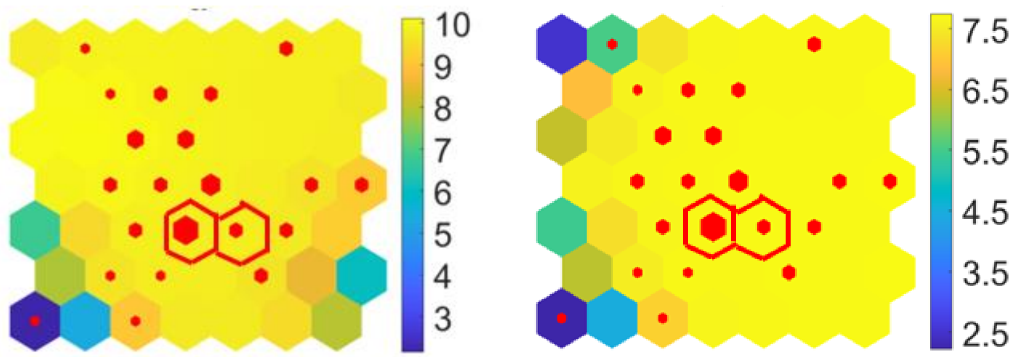
Moving on to Scenario 2, we apply Steps A and B of the proposed approach. If Step B is not successful, meaning there are no commonly found grids with high DCI values, we proceed to Step E. In this step, the designer has the flexibility to relax the DCI requirements for specific goals, termed "appropriate goals," based on the characteristics of each goal. This relaxation of DCI limits allows for the exploration of a broader range of design solutions. The loop from Step B through Step D is continued until the satisficing solutions are found.

Scenario 3 involves executing Steps A to C of the proposed approach. If Step C is unsuccessful, meaning the designer is not satisfied with the DCI values or the required compromises between goals, the process transitions to Step E. In this step, the DCI limits are specified for relaxation for each goal, guided by the priority of the goals and the need to satisfy the design requirements, as shown in Figure 5.10. For example, if the designer cannot compromise on the weight of the sandwich beam, particularly the density of the skin and core, then goals 1 and 3 should be considered for maximum DCI ranges, while reduced DCI limits can be chosen for goals 2 and 4. The relaxed DCI limits are as follows: i) Goal 1 DCI values ≥ 7; ii) Goal 2 DCI values ≥5.5; iii) Goal 3 DCI values ≥9; iv) Goal 4 DCI values ≥ 7. Upon relaxing the DCI limits, grids 26 and 33 emerge as common satisficing solutions for all four goals. Their corresponding weight scenarios and design variables are presented in Table 5.6. By utilizing iSOM, designers can effectively explore and visualize the design space, facilitating the interpretation of solutions for problems with many goals.



G1 (DCI goal for density of skin)

G2 (DCI goal for elastic modulus of skin)



G3 (DCI goal for density of core) G4 (DCI goal for shear modulus of core)

Figure 5.10: Highlighted grids after relaxing DCI limits

Table 5.6: Weight scenarios and design variables values after relaxing DCI limits

Goals	**Grids**	**No of scenarios**	**Weight scenario**	**Design variables**			
				Volume fraction (V_f)	Wall angle (θ)	Wall length (h)	Wall thickness (t)
Goal 1, 2, 3 and 4	26	7	14	0.409375	59.062500	22.614200	0.105833
			15	0.409375	59.062500	22.614100	0.105833
			21	0.690625	59.062500	22.614100	0.105833
			28	0.409375	59.062500	22.614100	0.105833
			29	0.409375	59.062500	22.614200	0.105833
			32	0.690625	59.062500	22.614200	0.105833
			36	0.690625	59.062500	22.614200	0.105833
	33	2	23	0.409375	59.062500	22.614100	0.105833
			30	0.690625	59.062500	22.614100	0.105833

The systematic approach proposed in this Chapter facilitates the designers in choosing the common satisficing solutions effectively from visualized iSOM plots. This enables the informed decision-making. The approach proposed is tested by a composite beam problem.

The subsequent chapter delves into the detailed functionalities of the decision-based design framework and the systematic approach, outlining the potential areas for future research and development within the context of this thesis.

Chapter 6: Summary of thesis, Closure and Way Forward

This chapter thoroughly examines the functionalities of the proposed decision-based design framework, revisits and elaborates on the research questions and hypothesis, and concludes by outlining directions for future research. In Section 6.1 the summary of the thesis and generic functionalities of the framework are discussed. In Section 6.2 the research questions and hypothesis are highlighted. The way forward and future research are discussed in Section 6.3.

6.1 Summary of the Thesis

Complex engineered systems often involve many conflicting goals, uncertainties, and intricate interactions, making them challenging to design effectively. This thesis addresses these challenges by introducing a decision-based design (DBD) framework that utilizes compromise decision-based design construct (cDSP) and machine learning-based visualization techniques to visualize and explore the design space efficiently.

The DBD framework provides a structured approach for handling many conflicting goals and uncertainties in the design process. It incorporates cDSP with robust design metrics accounting for uncertainties and interpretable self-organizing maps (iSOM), which effectively visualize the relationships between many goals. This visualization enables designers to explore the design space and identify solutions that meet their preferences and requirements.

The thesis also proposes a systematic approach for selecting satisfactory solutions from the visualized plots generated by the DBD framework. This approach involves defining satisficing solutions and developing an interactive selection approach to guide the designer's decision-making process.

The effectiveness of the DBD framework and the systematic selection method is demonstrated through the application to example problems involving many conflicting goals and uncertainties. The results show that the framework can effectively identify satisfactory solutions and enhance the decision-making process for complex design problems.

In Chapter 1, the foundation for the thesis is established. In this chapter, the need for realization of complex system and motivation for this thesis is discussed. The need for effective visualization is discussed in this chapter. Finally, the research gaps, questions and hypothesis are highlighted.

In Chapter 2, there is detailed discussion of the constructs and tools used in this thesis. The foundational construct compromise Decision Support Problem (cDSP) is discussed in detail. And the robust design metrics used in thesis that accounts uncertainty is explained in detail. Finally, the existing visualization techniques in use and their limitations are discussed.

In Chapter 3, the Decision Based Design framework is proposed, addressing research question 1. The need for this framework and its utility is discussed. The formulation of cDSP and working of machine learning based visualization technique iSOM is discussed in detail.

In Chapter 4, the proposed framework is tested considering a vehicular crashworthiness problem and the utility of the framework is discussed in detail.

In Chapter 5, the research question 2 is addressed by considering a test problem for designing composite beam. In this chapter a systematic approach is proposed to effectively choose the solutions from the visualized plots and validated with test problem.

In this chapter, a summary of the thesis is given first and then the functionality of the framework is discussed. The research questions and discussion on the research hypotheses are made. Further, the achievements and contributions made on the thesis are summarized. Finally, the future research is discussed.

6.1.1 Functionalities of the DBD Framework

In the field of designing complex systems, the existence of many goals and uncertainty in design factors can make it difficult to find the robust satisficing solutions. To address these issues, the Decision-Based Design (DBD) framework is proposed in this thesis. The functionality of the framework is discussed in this section.

Formulation of many (more than three) goals problems

- The DBD framework provides a systematic approach to formulate and address many-goal problems. In engineering design, systems are often characterized by more than three goals, leading to intricate interactions and conflicting goals. DBD offers a structured methodology for designers to define and navigate through a multitude of goals, fostering a holistic understanding of the design problem at hand.

Managing uncertainty with robust design metrics

- One of the key strengths of the DBD framework lies in its ability to address the issue of uncertainty in design variables and model. The Design Capability Index (DCI) is used in this thesis accounting uncertainties in design variables. DCI plays a crucial role in quantifying how well a variety of design criteria can be satisfied, considering uncertainties in the design variables themselves.

Solution space visualization and exploration using iSOM

- Visualizing and exploring the solution space for many conflicting goals requires advanced visualization and exploration techniques. The integration of Interpretable Self-Organizing Maps (iSOM) visualization technique within the DBD framework enhances designer’s capabilities to explore and understand the complex solution space. With the use of iSOM the intricate interrelations between design goals be understood. And also, how the priorities given to each goal affect the others.

DBD framework offers a unique capability to unravel the interdependencies, providing designers with valuable insights into the complex relationships between different design goals. By fostering a holistic understanding of how various goals interact, DBD enables designers to make informed decisions and strike a balance between many conflicting goals.

6.2 Functionality of the Systematic Approach

The systematic approach is proposed to facilitate the designers in selecting satisfactory solutions from the visualized plots. This approach helps designers to make informed decisions based on the visualization's plots. The functionality of the systematic approach is discussed below:

- Developing a systematic approach for choosing satisficing solutions is a crucial step towards understanding the system's performance. This approach offers designers a structured way to find common satisfying solutions in addition to helping them methodically identify solutions that balance all specified requirements. Therefore, use of the systematic approach helps designers easily choose the satisficing solutions from visualized plots.

6.3 Answering Research Question and Validating the Hypothesis

The two research questions addressed in this thesis can be broadly classified into two research areas such as,

i. Decision Based design Framework for complex system.
ii. Systematic approach for identifying common solutions.

6.3.1 Answering Research Question 1 (RQ 1)

The primary research question is formulated as:

Research Question (RQ1): What are the mathematical and computational foundations necessary for the formulation, visualization, and exploration of problems involving many (more than three) conflicting goals?

The primary research question in this thesis deals with modeling decisions in a complex system with many conflicting goals. A decision-based framework is proposed to address this research question. The hypothesis to this research question is given as:

Hypothesis H1: Problems involving many conflicting goals under uncertainty could be effectively formulated, visualized, and explored from a decision-based design perspective using compromise decision support problem construct, robust design metrics and an effective machine learning-based solution space visualization and exploration technique.

From the hypothesis, the framework proposed facilitates formulating the complex problem with many goals using cDSP technique which is discussed in detail in Chapter 2. The cDSP approach is particularly well-suited for early design stages, where uncertainties and incomplete information are prevalent. The robust design metrics play a crucial role in addressing uncertainties inherent in complex systems. These metrics quantify the sensitivity of design solutions to variations in input parameters and environmental conditions. By incorporating robust design metrics into the DBD framework, designers can identify solutions that are less susceptible to performance degradation under uncertain conditions which is discussed in detail in Chapter 3. The combination of cDSP, robust design metrics, and machine learning-based visualization techniques within the DBD framework provides a comprehensive approach for effectively formulating, visualizing, and exploring problems involving many conflicting goals under uncertainty. This framework is generic and can be applied to a wide range of complex design problems with many goals. This framework is tested using a vehicular crashworthiness problem discussed in Chapter 4.

Theoretical Structural Validation

Theoretical structural validation is the process of ensuring that the constructs used in a model are logically sound and that the model is well-defined and consistent. This involves checking that the constructs are clearly defined and that they are not circular or contradictory. It also involves checking that the relationships between the constructs are well-defined and that they are consistent with the theory being modeled.

Chapters 1, 2, and 3 focus on evaluating the internal consistency to establish the logical soundness of the design method. Chapter 1 provides an overview of design methodologies, complex systems, and the solution space visualization and exploration. It also outlines the scope of the work, including the research questions, proposed hypotheses, and the significance of the present work. Chapter 2 conducts an extensive literature review and discusses the mathematical tools employed. It specifically discusses different types of robust design, the compromise Decision Support Problem (cDSP) construct, and the Design Capability Index (DCI). Chapter 3 provides a detailed explanation of the decision-based design framework proposed in this thesis.

Empirical Structural Validation

Empirical structural validation is the process of evaluating the effectiveness of a model or framework by comparing its predictions or outputs to real-world data. It is a crucial step in the development and validation of any model or framework, as it provides evidence of its ability to accurately represent and predict real-world phenomena. In this thesis, empirical structural validation can be used to evaluate the effectiveness of the DBD framework in identifying and selecting suitable solutions for complex design problems. This can be done by comparing the solutions identified by the DBD framework to real-world solutions that have been implemented and evaluated.

Chapter 4 delves into the suitability of the chosen test problems for showcasing and validating the decision-based design framework. The outcomes from example problems are presented and discussed in this chapter, emphasizing the method's validity and effectiveness.

A vehicular side crash example problem is formulated in Chapter 4 validating the effectiveness of DBD framework.

Empirical Performance Validation

Empirical performance validation is the process of evaluating the effectiveness of a model, framework, or technique by comparing its predictions or outputs to real-world data. It is a crucial step in the development and validation of any model, framework, or technique, as it provides evidence of its ability to accurately represent and predict real-world phenomena.

Chapter 4 the suitability of the selected comprehensive test problem for demonstrating and validating the design method is considered. In Chapter 4, vehicular side crash design is presented as an example design problem. This is followed by a discussion of DSP-based mathematical formulations and the DCI mathematical construct for solving the problem.

6.3.2 Answering Research Question 2 (RQ 2)

The secondary research question is formulated as:

Research Question (RQ2): How can designers effectively interpret and select satisficing solutions for many (more than three) goals?

Designing with many goals can be a complex task, especially when the goals are conflicting. To effectively navigate this challenge, designers can employ a satisficing approach, seeking solutions that meet a minimum level of satisfaction for each goal rather than aiming for an unattainable optimization of all goals simultaneously. This approach involves several key steps: clearly defining and prioritizing the goals, identifying trade-offs between them, exploring the solution space through various design concepts, evaluating these concepts against the goals, selecting solutions that meet the minimum satisfaction criteria, and continuously iterating and refining the designs. By employing a systematic approach designers can effectively interpret and select satisficing solutions that address the complexities of many conflicting goals.

Hypothesis H2: It is hypothesized that this could be addressed by proposing a systematic approach to evaluate and identify common satisficing design scenarios for many goals through solutions space visualization, and exploration.

Navigating the complexities of designing with many conflicting goals can be overcome by proposing a systematic approach that emphasizes identifying common satisficing design scenarios. The DBS framework involves visualizing the solution space to understand the trade-offs between goals, exploring the solution space to locate regions containing satisficing solutions, and identifying common patterns or clusters of satisficing solutions within the solution space. By employing this approach, designers can effectively tackle the challenge of many conflicting goals and identify common satisficing solutions that are both practical and achievable.

Theoretical Structural Validation

Theoretical structural validation is the process of ensuring that the constructs used in a model are logically sound and that the model is well-defined and consistent. This involves checking that the constructs are clearly defined and that they are not circular or contradictory. It also involves checking that the relationships between the constructs are well-defined and that they are consistent with the theory being modeled.

The importance of effective visualization is emphasized in Chapter 1. Chapter 2 delves into solution space visualization and exploration, examining existing visualization techniques, their limitations, and Chapter 3 discusses the need to use iSOM (interpretable self-organizing maps) in this thesis.

Empirical Structural Validation

Empirical structural validation entails evaluating the effectiveness of a model or framework by comparing its predictions or outputs to real-world data. This critical step in developing and validating any model or framework provides evidence of its capability to accurately represent and predict real-world phenomena. Within this thesis, empirical structural validation can be employed to assess the effectiveness of the DBD framework in identifying and selecting appropriate solutions for intricate design problems. This can be accomplished by comparing the solutions identified by the DBD framework to real-world solutions that have been implemented and evaluated.

Chapter 5 delve into the suitability of the chosen test problems for showcasing and validating the systematic approach. The outcomes from example problems are presented and discussed in this chapter, emphasizing the method's validity and effectiveness. A composite design problem is formulated in Chapter 5 validating the systematic approach.

Empirical Performance Validation

Empirical performance validation serves as an essential step in evaluating the effectiveness of a model, framework, or technique. This process involves comparing the model's predictions or outputs to real-world data, providing insights into its ability to accurately represent and predict real-world phenomena. By comparing the model's predictions or outputs to actual observations, empirical performance validation helps to establish its validity and usefulness in practical applications.

Chapter 5 the suitability of the selected comprehensive test problem for demonstrating and validating the design method is considered. Chapter 5 explores a decision problem in composite structure design to validate the systematic approach. This step evaluates the practical effectiveness of the framework or design method in real-world complex engineered systems. A comprehensive discussion of the method's validity and usefulness is also provided.

Theoretical Performance Validation (TPV)

The proposed approach embraces speculative elements while remaining firmly grounded in the established principles of theoretical structural validation (TSV), empirical structural validation (ESV), and empirical performance validation (EPV). The verification of the proposed method draws upon insights from all three validation quadrants (TSV, ESV, and

EPV). The validation of the method is rooted in the concept of its extensibility beyond the specific examples explicitly addressed in the thesis, thus demonstrating its utility in a wider range of applications. Chapter 6 focuses on establishing confidence in the generalizability of the framework. This chapter presents and discusses the functionality and the broader applicability of the framework.

6.4 Way forward

This section of the thesis aims to expand the discussion on the future of product development, delving into the implications of this research for shaping design methodologies and innovation. Building upon the foundation of the work presented, this section explores the potential impact of this research on the future. Some of the futuristic scope of this thesis is discussed below:

- In the current scope of research primarily focuses on uncertainties in the input parameters of the design problem. To further broaden the applicability and robustness of the Decision-Based Design framework, it is crucial to consider other forms of uncertainties that may arise in the design process. To address this challenge, future research can explore the incorporation of model uncertainty into the Decision-Based Design framework with incorporating EMI robust metrics that address the uncertainty in the model itself. This could involve developing methods for quantifying and propagating model uncertainty through the EMI calculations, enabling a more comprehensive assessment of solution satisficing under various model scenarios.
- The proposed framework has demonstrated its effectiveness in addressing single-level design problems. However, in many real-world design scenarios, multiple stakeholders with different perspectives and priorities collaborate to develop a product or system. Co-design scenarios involve the active participation of these stakeholders in the design process, requiring a framework that can effectively manage interactions and integrate diverse inputs. Expanding the framework to handle multiple levels of decision-making would facilitate the flow of information

across different levels of decision-making, account for interdependencies and trade-offs between decisions made at different levels by ensure consistency and coherence between decisions made at different levels.

- The current framework uses the machine learning based visualization technique to visualize and explore solution space. This can be extended to evolving cyber-physical systems. In dynamic environments where system characteristics and requirements may change over time, the framework could benefit from predictive capabilities to anticipate potential changes in solutions. Extending the framework with predictive features would enable proactive adaptation to ensure the continued effectiveness of the selected solutions. To achieve this predictive capability, iSOM can be employed as a forecasting tool. By analyzing the trends and patterns present in the iSOM plots, potential changes in the solution space can be identified. This information can then be used to preemptively update the framework's internal models and decision-making processes, ensuring that the selected solutions remain valid and effective even as the Cyber Physical System environment evolves.

References

Acar, E., Guler, M., Gerceker, B., Cerit, M. and Bayram, B., 2011, "Multi-Objective Crashworthiness Optimization of Tapered Thin-Walled Tubes with Axisymmetric Indentations," *Thin-Walled Structures*, vol. 49, no. 1, pp. 94-105.

Aird, T. J. and Rice, J. R., 1977, "Systematic Search in High Dimensional Sets," *SIAM Journal on Numerical Analysis*, vol. 14, no. 2, pp. 296-312.

Allen, J. K., Seepersad, C. C., Mistree, F., Savannah, G. and Savannah, G., 2006, "A Survey of Robust Design with Applications to Multidisciplinary and Multiscale Systems," *Journal of Mechanical Design*, vol. 128, no. 4, pp. 832-843.

Aspenberg, D., Jergeus, J. and Nilsson, L., 2013, "Robust Optimization of Front Members in a Full Frontal Car Impact," *Engineering Optimization*, vol. 45, no. 3, pp. 245-264.

Bascaran, E., Mistree, F. and Bannerot, R. B., 1987, "Compromise: An Effective Approach for Solving Multiobjective Thermal Design Problems," *Engineering Optimization*, vol. 12, no. 3, pp. 175-189.

Bedford, T. and Cooke, R., 2001, "Probabilistic Risk Analysis: Foundations and Methods", Cambridge University Press.

Beitz, W., Pahl, G. and Grote, K., 1996, "Engineering Design: A Systematic Approach," *MRS Bulletin*, vo. 71.

Blanchard, C. and Stiglitz, M., 1992, "IEEE Standard Dictionary of Electrical and Electronic Terms, NY," *USA: Wiley-Interscience*.

Bloebaum, C. L. and McGowan, A.-M. R., 2012"The Design of Large-Scale Complex Engineered Systems: Present Challenges and Future Promise," *Proc. 12th aiaa atio conference and 14th aiaa/issmo ma&o conference, aiaa paper*, pp. 2012-5517.

Borror, C. M., Montgomery, D. C. and Myers, R. H., 2002, "Evaluation of Statistical Designs for Experiments Involving Noise Variables," *Journal of Quality Technology*, vol. 34, no. 1, pp. 54-70.

Box, G. E. and Draper, N. R., 1987, Empirical Model-Building and Response Surfaces, John Wiley & Sons.

Budiharto, W., 2013, "The Development of an Expert Car Failure Diagnosis System with Bayesian Approach," *Journal of Computer Science*, vol. 9, no. 10, pp. 1383.

Chen, W., Allen, J. K., Mavris, D. N. and Mistree, F., 1996, "A Concept Exploration Method for Determining Robust Top-Level Specifications," *Engineering Optimization*, vol. 26, no. 2, pp. 137-158.

Chen, W., Allen, J. K. and Mistree, F., 1997, "A Robust Concept Exploration Method for Enhancing productivity in Concurrent Systems Design," *Concurrent Engineering*, vol. 5, no. 3, pp. 203-217.

Chen, W., Allen, J. K., Tsui, K.-L. and Mistree, F., 1996, "A Procedure for Robust Design: Minimizing Variations Caused by Noise Factors and Control Factors." ASME Journal of Mechanical Design, pp. 951012.

Chen, W., Meher-Homji, C. B. and Mistree, F., 1994, "Compromise: An Effective Approach for Condition-Based Maintenance Management of Gas Turbines," *Engineering Optimization*, vol. 22, no. 3, pp. 185-201.

Choi, H.-J., Austin, R., Allen, J. K., McDowell, D. L., Mistree, F. and Benson, D. J., 2005, "An Approach for Robust Design of Reactive Power Metal Mixtures Based on Non-Deterministic Micro-Scale Shock Simulation," *Journal of Computer-Aided Materials Design*, vol. 12, pp. 57-85.

Choi, H.-J., Mcdowell, D. L., Allen, J. K. and Mistree, F., 2008, "An Inductive Design Exploration Method for Hierarchical Systems Design under Uncertainty," *Engineering Optimization*, vol. 40, no. 4, pp. 287-307.

Choi, H., McDowell, D. L., Allen, J. K., Rosen, D. and Mistree, F., 2008, "An Inductive Design Exploration Method for Robust Multiscale Materials Design."

Du, X. and Chen, W., 2004, "Sequential Optimization and Reliability Assessment Method for Efficient Probabilistic Design," *J. Mech. Des.*, vol. 126, no. 2, pp. 225-233.

Fang, H., Rais-Rohani, M., Liu, Z. and Horstemeyer, M., 2005, "A Comparative Study of Metamodeling Methods for Multiobjective Crashworthiness Optimization," *Computers & structures*, vol. 83, no. 26, pp. 2121-2136.

Feng, H., Grifoll, M., Yang, Z., Zheng, P. and Martin-Mallofre, A., 2020, "Visualization of Container Throughput Evolution of the Yangtze River Delta Multi-Port System: The Ternary Diagram Method," *Transportation Research Part E: Logistics and Transportation Review*, vol. 142, pp. 102039.

Forrester, A., Sobester, A. and Keane, A., 2008, Engineering Design via Surrogate Modelling: A Practical Guide, John Wiley & Sons.

Fotopoulos, C., Kafetzopoulos, D. and Gotzamani, K., 2011, "Critical Factors for Effective Implementation of the HACCP System: a Pareto analysis," *British Food Journal*, vol. 113, no. 5, pp. 578-597.

Frohlich, C., 1992, "Triangle Diagrams: Ternary Graphs to Display Similarity and Diversity of Earthquake Focal Mechanisms," *Physics of the Earth and Planetary interiors*, vol. 75, no. 3, pp. 193-198.

Fuchs, W., Karandikar, H., Mistree, F. and Eschenauer, H., 1990, "Compromise: An Effective Approach for Designing Composite Conical Shell Structures."

Gan, N. and Gu, J., 2019, "Hybrid Meta-Model-Based Design Space Exploration Method for Expensive Problems," *Structural and Multidisciplinary Optimization*, vol. 59, no. 3, pp. 907-917.

Gu, X., Dai, J., Huang, X. and Li, G., 2017, "Reliable Optimisation Design of Vehicle Structure Crashworthiness under Multiple Impact Cases," *International Journal of Crashworthiness*, vol. 22, no. 1, pp. 26-37.

Gu, X., Sun, G., Li, G., Huang, X., Li, Y. and Li, Q., 2013, "Multiobjective Optimization Design for Vehicle Occupant Restraint System under Frontal Impact," *Structural and Multidisciplinary Optimization*, vol. 47, pp. 465-477.

Hamza, K. and Shalaby, M., 2014, "A Framework for Parallelized Efficient Global Optimization with Application to Vehicle Crashworthiness Optimization," *Engineering Optimization*, vol. 46, no. 9, pp. 1200-1221.

Hernandez, G. and Mistree, F., 2000, "Integrating Product Design and Manufacturing: a Game Theoretic Approach," *Engineering Optimization+ A35*, vol. 32, no. 6, pp. 749-775.

Holden, C. and Keane, A., "Visualization Methodologies in Aircraft Design," *Proc. 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, pp. 4449.

Horstemeyer, M., Ren, X., Fang, H., Acar, E. and Wang, P., 2009, "A Comparative Study of Design Optimisation Methodologies for Side-Impact Crashworthiness, Using Injury-Based versus Energy-Based Criterion," *International Journal of Crashworthiness*, vol. 14, no. 2, pp. 125-138.

Hou, S., Li, Q., Long, S., Yang, X. and Li, W., 2009, "Crashworthiness Design for Foam Filled Thin-Wall Structures," *Materials & design*, vol. 30, no. 6, pp. 2024-2032.

Ibrahim, A., Rahnamayan, S., Martin, M. V. and Deb, K., "3D-RadVis: Visualization of Pareto Front in Many-Objective Optimization," *Proc. 2016 IEEE Congress on Evolutionary Computation (CEC)*, IEEE, pp. 736-745.

Isukapalli, S. S., Spendiff, M., Georgopoulos, P. G. and Krishnan, K., 2010, "Uncertainty, Variability, and Sensitivity Analyses in Simulation Models," *Quantitative Modeling in Toxicology*, pp. 429-458.

Jin, R., Chen, W. and Simpson, T. W., 2001, "Comparative Studies of Metamodelling Techniques under multiple Modelling Criteria," *Structural and Multidisciplinary Optimization*, vol. 23, pp. 1-13.

Jin, Y., Gruna, R. and Sendhoff, B., 2009, "Pareto Analysis of Evolutionary and Learning Systems," *Frontiers of Computer Science in China*, vol. 3, pp. 4-17.

Karandikar, H. and Mistree, F., 1993, "Modeling Concurrency in The Design of Composite Structures," *Progress in Astronautics and Aeronautics*, vol. 150, pp. 769-769.

Kennedy, M. C. and O'Hagan, A., 2001, "Bayesian Calibration of Computer Models," *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, vol. 63, no. 3, pp. 425-464.

Koch, P. N., Evans, J. P. and Powell, D., 2002, "Interdigitation for Effective Design Space Exploration using iSIGHT," *Structural and Multidisciplinary Optimization*, vol. 23, no. 2, pp. 111-126.

Kyprioti, A. P., Zhang, J. and Taflanidis, A. A., 2020, "Adaptive Design of Experiments for Global Kriging Metamodeling through Cross-Validation Information," *Structural and Multidisciplinary Optimization*, vol. 62, pp. 1135-1157.

Lanzi, L., Castelletti, L. and Anghileri, M., 2004, "Multi-Objective Optimisation of Composite Absorber Shape under Crashworthiness Requirements," *Composite Structures*, 65(3-4), pp. 433-441.

Leiva, J. P., "Structural Optimization Methods and Techniques to Design Efficient Car Bodies," *Proc. Proceedings of the International Automotive Body Congress, Troy, MI, USA*, pp. 9-10.

Lewis, K. and Mistree, F., "Designing Top-Level Aircraft Specifications-A Decision-Based Approach to a Multiobjective, Highly Constrained Problem," *Proc. 36th Structures, Structural Dynamics and Materials Conference*, p. 1431.

Lewis, K., Smith, W. and Mistree, F., 1999, "Ranged Set of Top-Level Specifications for Complex Engineering Systems," *Simultaneous Engineering: Methodologies and Applications*, pp. 279-303.

Lin, Y., Krishnapur, K., Allen, J. K. and Mistree, F., "Robust design: Goal Formulations and A Comparison of Metamodeling Methods," *Proc. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. 1355-1367.

López-Rubio, E., 2013, "Improving the Quality of Self-Organizing Maps by Self-Intersection avoidance," *IEEE transactions on neural networks and learning systems*, vol. 24, no. 8, pp. 1253-1265.

McDowell, D. L., Panchal, J., Choi, H.-J., Seepersad, C., Allen, J. and Mistree, F., 2009, Integrated Design of Multiscale, Multifunctional Materials and Products, Butterworth-Heinemann.

McKay, M. D., 1995, "Evaluating Prediction Uncertainty," Nuclear Regulatory Commission, vol. 26, no. 14, pp. 26051087

Meixner, G., Paternò, F. and Vanderdonckt, J., 2011, "Past, Present, and Future of Model-Based user Interface Development," *i-com*, vol. 10, no. 3, pp. 2-11.

Miñón, R., Paternò, F., Arrue, M. and Abascal, J., 2016, "Integrating Adaptation Rules for People with Special Needs in Model-Based UI Development Process," *Universal Access in the Information Society*, vol. 15, pp. 153-168.

Mistree, F. and Allen, J. K., 1997, "Position Paper Optimization in Decision-Based Design," *Optimization in Industry, Palm Coast, FL, Mar*, pp. 23-27.

Mistree, F. and Bras, B., "Axioms, Models, And Marine Applications." Research Gate

Mistree, F., Hughes, O. F. and Bras, B., 1993, "Compromise Decision Support Problem and the Adaptive Linear Programming Algorithm," *Progress in Astronautics and Aeronautics*, vol. 150, pp. 251-251.

Mistree, F. and Kamal, S., 1985, DSIDES: Decision Support in the Design of Engineering Systems, University of Houston.

Mistree, F., Muster, D., Srinivasan, S. and Mudali, S., 1990, "Design of Linkages: A Conceptual Exercise in Designing for Concept," *Mechanism and Machine Theory*, vol. 25, no. 3, pp. 273-286.

Mistree, F., Patel, B. and Vadde, S., "On Modeling Multiple Objectives and Multi-Level Decisions in Concurrent Design," *Proc. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. 151-161.

Mistree, F., Smith, W. and Bras, B., 1993, "A Decision-Based Approach to Concurrent Design," _Concurrent Engineering: Contemporary issues and modern design tools_, Springer, pp. 127-158.

Mistree, F., Smith, W., Bras, B., Allen, J. and Muster, D., 1990, "Decision-Based Design: A Contemporary Paradigm for Ship Design," _Transactions, Society of Naval Architects and Marine Engineers_, vol. 98, pp. 565-597.

Mudali, S., 1987, "The Dimensional Synthesis of Mechanical Linkages using Compromise Decision Support Problems," PhD dissertation, University of Houston.

Muster, D. and Mistree, F., 1988, "The Decision Support Problem Technique in Engineering Design," _International Journal of Applied Engineering Education_, vol. 4, no. 1, pp. 23-33.

Nagar, D., Pannerselvam, K. and Ramu, P., 2022, "A Novel Data-Driven Visualization of n-Dimensional Feasible Region using Interpretable Self-Organizing Maps (iSOM)," _Neural Networks_, vol. 155, pp. 398-412.

Nellippallil, A. B., 2018, "The Integrated Realization of Materials, Products and Associated Manufacturing Processes," PhD dissertation, University of Oklahoma.

Panchal, J. H., Choi, H.-J., Allen, J. K., McDowell, D. L. and Mistree, F., "Robust Multiscale Simulation-Based Design of Multifunctional Materials," _Proc. International Conference on Advanced Materials Design & Development. Notices of the AMS Goa, India_.

Parrish, A., Rais-Rohani, M. and Najafi, A., 2012, "Crashworthiness Optimisation of Vehicle Structures with Magnesium Alloy Parts," _International Journal of Crashworthiness_, vol. 17, no. 3, pp. 259-281.

Perona, M., 1998, "Manufacturing Conformity Assessment through Taguchi's Quality Loss Function," _International Journal of Quality & Reliability Management_, vol. 15, no. 8, pp. 931-946.

Phadke, M. S., 1986, "Design Optimization Case Studies," _AT&T technical journal_, vol. 65, no. 2, pp. 51-68.

Phadke, M. S., 1989, "Quality Engineering using Design of Experiments," _Quality Control, Robust Design, and the Taguchi method_, pp. 31-50.

Reddy, R., Smith, W., Mistree, F., Bras, B., Chen, W., Malhotra, A., Badhrinath, K., Lautenschlager, U., Pakala, R. and Vadde, S., 1996, "DSIDES User Manual," _Systems Realization Laboratory, Woodruff School of Mechanical Engineering, Georgia Institue of Technology, Atlanta, Georgia_.

Ruiz, J., Serral, E. and Snoeck, M., 2019, "Evaluating User interface Generation Approaches: Model-Based versus Model-Driven Development," *Software & Systems Modeling*, vol. 18, no. 4, pp. 2753-2776.

Seepersad, C. C., Pedersen, K., Emblemsvåg, J., Bailey, R., Allen, J. K. and Mistree, F., 2006, "The validation Square: How Does One Verify and Validate a Design Method," *Decision Making in Engineering Design*, pp. 303-314.

Sen, P. and Yang, J.-B., 2012, Multiple Criteria Decision Support in Engineering Design, Springer Science & Business Media.

Shan, S. and Wang, G. G., 2004, "Space Exploration and Global Optimization for Computationally Intensive Design Problems: A Rough Set Based Approach," *Structural and Multidisciplinary Optimization*, vol. 28, pp. 427-441.

Sharma, G., Allen, J. K. and Mistree, F., "Classification and Execution of Coupled Decision Problems in Engineering Design for Exploration of Robust Design Solutions," *Proc. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, p. V02AT03A019.

Sharma, G., Allen, J. K. and Mistree, F., 2021, "A Method for Robust Design in a Coupled Decision Environment," *Design Science*, vol. 7, pp. e23.

Shupe, J., Muster, D. and Mistree, F., "Decision-Based Design: Some Concepts."

Shupe, J. A. and Mistree, F., 1987, "Compromise: An Effective Approach for the Design of Damage Tolerant Structural Systems," *Computers & structures*, vol. 27, no. 3, pp. 407-415.

Simon, H. A., 1969, "The sciences of the Artificial Cambridge," *Mass. and London: MIT*.

Simpson, T. W., Chen, W., Allen, J. K. and Mistree, F., 1999, "Use of the Robust Concept Exploration Method to Facilitate the Design of a Family of Products," *Simultaneous engineering: Methodologies and applications*, vol. 6, pp. 247-278.

Simpson, T. W., Maier, J. R. and Mistree, F., 2001, "Product Platform Design: Method and Application," *Research in Engineering Design*, vol. 13, pp. 2-22.

Simpson, T. W., Rosen, D., Allen, J. K. and Mistree, F., "Metrics for Assessing Design Freedom and Information Certainty in the Early Stages of Design," *Proc. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, p. V004T004A004.

Simpson, T. W., Seepersad, C. C. and Mistree, F., 2001, "Balancing Commonality and Performance within the Concurrent Design of Multiple Products in a Product Family," _Concurrent Engineering_, vol. 9, no. 3, pp. 177-190.

Sinha, A., Bera, N., Allen, J. K., Panchal, J. H. and Mistree, F., 2013, "Uncertainty Management in the Design of Multiscale Systems," _Journal of Mechanical Design_, vol. 135, no. 1, pp. 011008.

Sinha, K. and Suh, E. S., 2018, "Pareto-Optimization of Complex System Architecture for Structural Complexity and Modularity," _Research in Engineering Design_, vol. 29, pp. 123-141.

Soto, C., 2004, "Structural Topology optimization for Crashworthiness," _International Journal of Crashworthiness_, vol. 9, no. 3, pp. 277-283.

Taguchi, G., 1986, Introduction to Quality Engineering: Designing Quality into Products and Processes, pp. 01411752.

Theissler, A., Ulmer, D. and Dear, I., "Interactive Knowledge Discovery in Recordings from Vehicle Tests," _Proc. Proc. 33rd FISITA World Automotive Congress. Budapest, Hungary_

Thole, S. P. and Ramu, P., 2020, "Design Space Exploration and Optimization using Self-Organizing Maps," _Structural and Multidisciplinary Optimization_, vol. 23, no. 3, pp. 1071-1088.

Thunnissen, D. P., "Uncertainty Classification for the Design and Development of Complex Systems," _Proc. 3rd annual predictive methods conference_, Newport Beach CA.

Triantaphyllou, E. and Sánchez, A., 1997, "A Sensitivity Analysis Approach for Some Deterministic Multi-Criteria Decision-Making Methods," _Decision sciences_, vol. 28, no. 1, pp. 151-194

Vadde, S. V., 1995, "Modeling multiple Objectives and Multilevel Decisions in Concurrent Design of Engineering Systems."

Wang, J., Liu, X., Shen, H.-W. and Lin, G., 2016, "Multi-Resolution Climate Ensemble Parameter Analysis with Nested Parallel Coordinates Plots," _IEEE transactions on visualization and computer graphics_, vol. 23, no. 1, pp. 81-90.

Xie, S., Peacock, D., Milthorpe, J. and Smith, W., "Modelling and Exploring Hierarchical Systems Engineering Models with a Preliminary Ship Design Focus," _Proc. 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization_, p. 5413.

Youn, B. D., Choi, K., Yang, R.-J. and Gu, L., 2004, "Reliability-Based Design Optimization for Crashworthiness of Vehicle Side impact," *Structural and Multidisciplinary Optimization*, vol.26, pp. 272-283.

Zhang, Y., Sun, G., Li, G., Luo, Z. and Li, Q., 2012, "Optimization of Foam-Filled Bitubal Structures for Crashworthiness Criteria," *Materials & design*, vol.38, pp. 99-109.

Zhou, H., Yuan, X., Qu, H., Cui, W. and Chen, B., "Visual Clustering in Parallel Coordinates," *Proc. Computer graphics forum*, Wiley Online Library, pp. 1047-1054.

APPENDIX

Surrogate Models – Vehicular Crashworthiness problem

Function	Response Surface Model
Energy Absorption for Part 235 (N-mm)	E_{235} = (0.19739 + 0.14887x t_1 +0.02670x t_2 – 0.05124x t_3– 0.08292 x t_4 + 0.01850 x t_5– 0.08935x t_1^2+ 0.02951 x t_1x t_2- 0.00579 x t_1 x t_3 + 0.07720 x t_1 x t_4 – 0.02256 x t_1 x t_5 – 0.02076 x t_2^2 – 0.01874x t_2 x t_3 – 0.00806 x t_2 x t_4 + 0.00479x t_2 x t_5 +0.03565x t_3^2 + 0.04727 x t_3 x t_4– 0.01389 x t_3 x t_5+ 0.06025 x t_4^2– 0.0511 x t_4 x t_5+ 0.04532 x t_5^2) x 10^6
Energy Absorption for Part 237 (N-mm)	E_{237} = (0.13666 + 0.00607x t_1 +0.09939x t_2 + 0.321x t_3 + 0.02319x t_4 - 0.02941x t_5 + 0.005624x t_1^2 − 0.04545 x t_1x t_2 + 0.21449 x t_1 x t_3 - 0.0618 x t_1 x t_4 + 0.02878 x t_1 x t_5 + 0.06637 x t_2^2 – 0.0321x t_2 x t_3 + 0.31315 x t_2 x t_4 - 0.00037x t_2 x t_5 -0.01601x t_3^2 + 0.001301 x t_3 x t_4– 0.00013 x t_3 x t_5 − 0.01339 x t_4^2 + 0.01777 x t_4 x t_5+ 0.003009 x t_5^2) x 10^6
Energy Absorption for Part 329 (N-mm)	E_{329} = (0.015484 - 0.02342x t_1 +0.024195x t_2 + 0.238752x t_3 − 0.0417x t_4 + 0.011807x t_5 + 0.005085x t_1^2 − 0.013 x t_1x t_2 − 0.0312 x t_1 x t_3 + 0.0513 x t_1 x t_4 + 0.01696 x t_1 x t_5 + 0.001905 x t_2^2 – 0.03205x t_2 x t_3 + 0.011237x t_2 x t_4 + 0.000675x t_2 x t_5 - 0.03904x t_3^2 + 0.080816 x t_3 x t_4 + 0.041151 x t_3 x t_5 + 0.000641 x t_4^2 −0.06095 x t_4 x t_5 − 0.00502 x t_5^2) x 10^6
Energy Absorption for Part 353 (N-mm)	E_{353} = (0.080989 - 0.05628x t_1 -0.03778x t_2 + 0.09809x t_3 + 1.05099x t_4 + 0.114237x t_5 − 0.02441x t_1^2 − 0.02385 x t_1x t_2 −

	0.087018 x t_1 x t_3 + 0.0509 x t_1 x t_4 + 0.116148 x t_1 x t_5 + 0.015803 x t_2^2 + 0.060604x t_2 x t_3 - 0.0333x t_2 x $t_4 -$ 0.00434x t_2 x t_5 - 0.11714x t_3^2 - 0.08324 x t_3 x $t_4 -$ 0.02029 x t_3 x $t_5 -$ 0.48092 x t_4^2 $-$0.14211 x t_4 x $t_5 -$ 0.7409 x t_5^2) x 10^6
Energy Absorption for Part 357 (N-mm)	E_{357} = (0.124979 - 0.17563x t_1 - 0.01787x t_2 - 0.0144x $t_3 -$ 0.06605x t_4 + 0.2426x $t_5 -$ 0.00679x t_1^2 + 0.05797 x t_1x t_2 + 0.012006x t_1 x t_3 + 0.140194 x t_1 x t_4 + 0.071128 x t_1 x t_5 + 0.012219x t_2^2 – 0.00968x t_2 x t_3 - 0.0443x t_2 x $t_4 -$ 0.04542x t_2 x t_5 + 0.0568x t_3^2 + 0.1267 x t_3 x t_4 + 0.055 x t_3 x t_5 + 0.0384 x t_4^2 $-$0.1261 x t_4 x $t_5 -$ 0.0072x t_5^2) x 10^6
Total mass of all parts	Mass = 6.90989 – 3.708636 x t_1 – 1.850079 x t_2 + 6.84746 x t_3 + 13.566x t_4 + 1.251x t_5 + 4.57118x t_1^2 + 0.007921 x t_1x t_2 + 5.538308x t_1 x t_3 -1.3646 x t_1 x t_4 – 1.6955 x t_1 x t_5 + 3.1740 x t_2^2 + 1.0138x t_2 x t_3 + 2.582x t_2 x $t_4 -$ 1.7904x t_2 x t_5 – 6.903x t_3^2 – 7.156 x t_3 x t_4 + 6.10069 x t_3 x t_5 + 2.253031x t_4^2 + 7.952 x t_4 x $t_5 -$ 4.5043x t_5^2